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Non Newtonian Fluid Model for the Effect of Resistance Parameter on Different Portion of Arteries of Blood Flow Through an Arterial Stenosis

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Abstract: The present study deals with a mathematical model describing the resistance to flow across mild stenosis situated symmetrically on steady blood flow through arteries with uniform or non-uniform cross-section. This mathematical model involves the usual assumption that the blood is Non-Newtonian, incompressible and homogeneous fluid.

Key words: Herschel-bulkley fluid, wall parameter, blood flow, arterial stenosis, resistance parameter, India

INTRODUCTION

Arteries throughout the body may be affected by hardening, which causes symptoms because hardened arteries cannot carry enough blood to the body. Narrowing or hardening of the arteries that feed the heart (the coronary arteries) can lead to a heart attack. Due to these serious consequences, attention has been given in studies of blood flow in stenotic region under different conditions.

By assuming the artery to be circularly cylindrical in shape, Mishra and Panda (2005c) discussed characteristics of blood in stenosed artery and the stenosis to be symmetric about the axis of artery. Mishra and Panda (2005a) studied the flow of blood in stenosed artery for a power law fluid.

Large number of researchers Smith *et al.* (2002), Tu and Deville (1996), Misra *et al.* (1993), Tu *et al.* (1992), Misra and Chakravarty (1986), Siouffi *et al.* (1984), Young and Tsai (1973a, b), Mishra (2003), Mishra and Panda (2005b) and Mishra *et al.* (2008) have contributed a lot in developing a mathematical model for blood flow in atherosclerosis.

By assuming that blood to be Non-Newtonian, incompressible and homogeneous fluid, cylindrical polar co-ordinate is used with the axis of symmetry of artery taken as Z-axis (Fig. 1). The stenoses are mild and the motion of the fluid is laminar and steady. The inertia term is neglected as the motion is slow. No body force acts on the fluid and there is no slip at the wall.

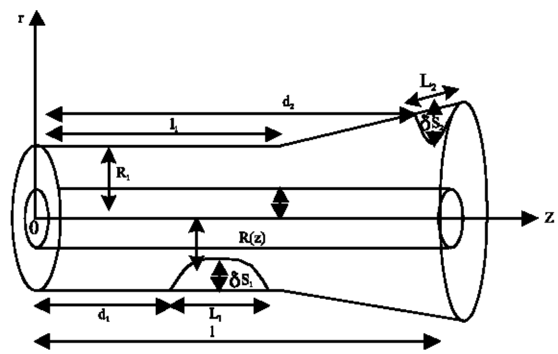

Fig. 1:Physical model and coordinate system

MATERIALS AND METHODS

Development of the model: For a Herschel Bulkley Fluid, the relationship between stress and strain is given by:

$$\tau = \tau_0 + \mu e^n \quad (1)$$

$$\left[e = \frac{-du}{dr} \right] \quad (2)$$

Where:

- n = Measure of how the fluid deviates from the Newtonian fluid
- τ_0 = Measure of yield stress
- τ = Stress tensor
- e = Strain rate [e = -du/dr]
- u = Velocity of fluid
- r = Radius of the artery
- μ = Viscosity of blood

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For the steady flow through circular artery,

$$\tau = \frac{r\frac{dp}{dz}}{2} = \frac{rG}{2} \tag{3}$$

Where, G = (dp/dz) is pressure gradient. From Eq. 1-3

$$\tau = \mu\left(\frac{du}{dr}\right)^n + \tau_0, \tau \geq \tau_0$$

$$\frac{rG}{2} = \mu\left(\frac{du}{dr}\right)^n + \tau_0$$

$$\left[\frac{\left(\frac{rG}{2}\right) - \tau_0}{\mu}\right]^{1/n} = \frac{du}{dr} \tag{4}$$

Now,

$$\frac{rG}{2} - \tau_0 = R \text{ (say)} \tag{5}$$

$$dR = \frac{Gdr}{2}$$

$$dr = \frac{2dR}{G} \tag{6}$$

From Eq. 4-6, we have,

$$\left[\frac{R}{\mu}\right]^{1/n}\frac{2dR}{G} = du$$

$$\left[\frac{2}{G\mu^{1/n}}\right]\int_R^{R(z)} R^{1/n}dR = \int_u^0 du$$

$$\left[\frac{2}{G\mu^{1/n}}\right]\left[\frac{n}{n+1}\right]\left[R(z)^{\frac{n+1}{n}} - R^{\frac{n+1}{n}}\right] = -u$$

or,

$$u = \left[\frac{2}{G\mu^{1/n}}\right]\left[\frac{n}{n+1}\right]\left[R^{\frac{n+1}{n}} - R(z)^{\frac{n+1}{n}}\right] \tag{7}$$

Now, Total flow flux

$$Q = \int_0^{R(z)} 2\pi urdr$$

$$Q = \left[\frac{4\Pi}{G\mu^{1/n}}\right]\left[\frac{n}{n+1}\right]\int_0^{R(z)}\left\{R^{\frac{n+1}{n}} - R(z)^{\frac{n+1}{n}}\right\}\left\{\frac{2(R+\tau_0)}{G}\right\}\left\{\frac{2dR}{G}\right\}$$

$$Q = \left[\frac{16\Pi}{G^3\mu^{1/n}}\right]\left[\frac{n}{n+1}\right]\int_0^{R(z)}\left\{\begin{matrix} R^{\frac{2n+1}{n}}dR + \tau_0 R^{\frac{n+1}{n}} \\ -RR(z)^{\frac{n+1}{n}} - \tau_0 R(z)^{\frac{n+1}{n}}\end{matrix}\right\}dR$$

$$= A\left[\begin{matrix}\frac{nR(z)^{\frac{3n+1}{n}}}{3n+1} + \tau_0\frac{n}{2n+1}R(z)^{\frac{2n+1}{n}} \\ -\frac{R(Z)^{\frac{3n+1}{n}}}{2} - \tau_0 R(z)^{\frac{2n+1}{n}}\end{matrix}\right]$$

Where:

$$A = \left[\frac{16\Pi}{G^3\mu^{1/n}}\right]\left[\frac{n}{n+1}\right] \tag{8}$$

$$Q = A\left[R(z)^{\frac{3n+1}{n}}\left\{\frac{n}{3n+1} - \frac{1}{2}\right\} + \tau_0 R(z)^{\frac{2n+1}{n}}\left\{\frac{n}{2n+1} - 1\right\}\right]$$

$$Q = \text{-}A(n+1)R(z)^{\frac{2n+1}{n}}\left\{\frac{R(z)}{6n+2} + \frac{\tau_0}{2n+1}\right\}$$

$$Q = \left[\frac{16\Pi n}{G^3\mu^{1/n}}\right]R(z)^{\frac{2n+1}{n}}\left\{\frac{R(z)}{6n+2} + \frac{\tau_0}{2n+1}\right\} \tag{9}$$

$$\frac{dp}{dz} = \left[\frac{16\Pi n}{\mu^{\frac{1}{n}}Q}\right]^{\frac{1}{3}} R(z)^{\frac{2n+1}{3n}}\left\{\frac{R(z)}{6n+2} + \frac{\tau_0}{2n+1}\right\}^{\frac{1}{3}} \tag{10}$$

Now,

$$\lambda = \frac{\Delta p}{Q} = \left[\frac{16\Pi n}{\mu^{\frac{1}{n}}Q}\right]^{\frac{1}{3}}\frac{1}{Q}\int_0^1 R(z)^{\frac{2n+1}{3n}}\left\{\frac{R(z)}{6n+2} + \frac{\tau_0}{2n+1}\right\}^{\frac{1}{3}}dz \tag{11}$$

where, λ is resistance to flow at the wall for the flow of blood and

$$\lambda_0 = \frac{\Delta p}{Q}\left[\frac{16\Pi n}{\mu^{\frac{1}{n}}Q}\right]^{\frac{1}{3}}\frac{1}{Q}\int_0^1 R_1^{\frac{2n+1}{3n}}\left\{\frac{R_1}{6n+2} + \frac{\tau_0}{2n+1}\right\}^{\frac{1}{3}}dz \tag{12}$$

Where:

λ_0 = Resistance to flow at the wall for the flow of blood in uniform portion of artery

R_1 = Radius of uniform portion of the artery

The resistance parameter $\lambda' = \lambda/\lambda_0$ is given by the expression:

$$\lambda' = \frac{1}{R_1^{\frac{2n+1}{3n}}\left\{\frac{R_1}{6n+2} + \frac{\tau_0}{2n+1}\right\}^{\frac{1}{3}}} \int_0^1 R(z)^{\frac{2n+1}{3n}}\left\{\frac{R(z)}{6n+2} + \frac{\tau_0}{2n+1}\right\}^{\frac{1}{3}}dz \tag{13}$$

We assume one stenosis each in uniform and non-uniform portion of the artery. The surface of stenosis as obtained by Young and Tsai (1973a, b) is:

$$R(Z) = R_{s1}(z) = R_1; 0 \le z \le d_1 \,\&\, d_1 + L_1 \le z \le l_1$$

$$R(z) = R_{s1}(z) = R_1 - \delta S_1/2.\left[1+\cos\left(2\prod/L_1\right)\left(z-d_1-L_1/2\right)\right]; \quad d_1 \le z \le d_1 + L_1 \qquad (14)$$

$$R(z) = R_{s2}(z) = R_2(z); l_1 \le z \le d_2 \,\&\, d_2 + L_2 \le z \le l$$

$$R(z) = R_{s2}(z) = R_2(z) - \delta S_2/2.\left[1+\cos\left(2\Pi/L_2\right)\left(z-d_2-L_2/2\right)\right]; \quad d_2 \le z \le d_2 + L_2$$

To observe explicity the effect of various parameters resistance to the flow, the following function has been assumed for the artery radius for the portion of the artery which is non-uniform.

$$R(z) = R_1 e^{K(z-l_1)^2}; l_1 \le z \le l$$

Where:

K	=	Wall exponent parameter
$R_{sn}(z)$	=	Radius of obstructed portion due to the nth stenosis of artery
l	=	Length of artery
l_1	=	Length of uniform portion of artery
δS_n	=	Amplitude of nth stenosis
l_n	=	Length of nth stenosis
d_n	=	Location of nth stenosis

Using Eq. 14 in Eq. 13 and integrating, for n = 1/3, we get the expression of resistance parameter as:

$$\lambda' = \left(l'_1 - L'_1\right) + L'_1\left[\left(1-\frac{5\delta S'_1}{3}\right)+\frac{5\delta S'_1 \Pi^2}{36}\right]$$
$$+\frac{L'_1}{12d}\left[\left(1-\frac{8\delta S'_1}{3}\right)+\frac{8\delta S'_1 \Pi^2}{36}\right]+$$
$$R_1^{5/3}\left[d'_2 - l'_1 + \frac{5K}{9}\left(d'_2 - l'_1\right)^3\right]+$$
$$\frac{R_1^{\frac{8}{3}}}{12d}\left[d'_2 - l'_1 + \frac{8K}{9}\left(d'_2 - l'_1\right)^3\right] + L'_2\left[\left(1+\frac{5\delta S'^2_2}{36}\right) + \frac{5\delta S'^3_2}{648}\right]+$$
$$\frac{L'_2}{12d}\left[\left(1+\frac{5\delta S'^2_2}{9}\right)+\frac{5\delta S'^3_2}{81}\right] + R_1^{\frac{5}{3}}\left[1-d'_2-L'_2\right]+$$
$$R_1^{\frac{5}{3}}\left[\frac{5K}{9}\left\{\left(1-l'_1\right)^3-\left(d'_2+L'_2-l'_1\right)^3\right\}\right]+$$
$$\frac{R_1^{\frac{8}{3}}}{12d}\left[\left(1-d'_2-L'_2\right)+\frac{8K}{9}\left\{\left(1-l'_1\right)^3-\left(d'_2+L'_2-l'_1\right)^3\right\}\right]$$

Where:

$$d_n' = \left(d_n/l\right), L_n' = \left(L_n/l\right), l_1' = \left(l_1/l\right), \delta S_n' = \left(\delta S_n/R_1\right), R'(z) = \left(R(z)/R_1\right), R_1' = R_1/l$$

RESULTS AND DISCUSSION

Physiological insight: In order to get the effect of stenosis on the resistance to flow for pulmonary artery, the following values are taken:

Length of the first stenosis, L_1 = 0.05 cm; Length of the second stenosis, L_2 = 0.05 cm; Height of the first stenosis δS_1 = 0.03 cm (Initially); Height of the second stenosis δS_1 = 0.03 cm (Initially); Length of the artery, l = 5 cm; Length of the uniform portion of the artery, l_1 = 4.0 cm; Radius of the uniform portion of the artery, R_1 = 1.5 cm; Position of second stenosis d_2 = 5 cm.

An investigation has been done for the resistance to flow across mild stenosis situated symmetrically on steady blood flow through arteries with uniform or non-uniform cross section by assuming the blood to be Non-Newtonian, incompressible and homogeneous fluid. We have obtained an analytical solution for Herschel-Bulkley fluid.

It can be observed from the Tables (1-3) that in the divergence of artery (K<0), uniform portion of the artery (K = 0) and the convergence of artery (K>0) the resistance parameter λ' increases as the height of the stenosis in the uniform or non-uniform or both portions of the artery increases.

Table 1: Variation of λ' against $\delta S'_1$ (when n = 1/3) for various value of K

	λ'		
$\delta S'_1$	K = -0.001	K = 0	K = 0.001
00.04	0.577807534	0.577818275	0.577829236
00.05	0.577879144	0.577889575	0.577900856
00.06	0.577951324	0.577961965	0.577972822
00.07	0.578023757	0.578034695	0.578045436
00.08	0.578096773	0.578107674	0.578118415
00.09	0.578170524	0.578187065	0.5781911976
00.10	0.578229284	0.578240135	0.578250956

Table 2: Variation of λ' against $\delta s'_1$ (when n = 1/3) for various value of K

	λ'		
$\delta s'_1$	K = -0.001	K = 0	K = 0.001
0.04	0.567807524	0.567818375	0.567829226
0.05	0.567879124	0.567889975	0.567900826
0.06	0.567951124	0.567961975	0.567972826
0.07	0.568023744	0.568034595	0.568045446
0.08	0.568096783	0.568107634	0.568118485
0.09	0.568170224	0.568187075	0.5681911926
0.10	0.568229274	0.568240125	0.568250976

Table 3: Variation of λ' against $\delta s'_1$ and $\delta s'_2$ (when n = 1/3) for various value of K

		λ'		
$\delta s'_1$	$\delta s'_2$	K = -0.001	K = 0	K = 0.001
0.04	0.04	1.55967298	1.01079198	1.78523198
0.05	0.05	1.57612321	1.0136148	1.7872743
0.06	0.06	1.5796876	1.01902876	1.79346876
0.07	0.07	1.5749123	1.02301728	1.7954198
0.08	0.08	1.57614558	1.02726558	1.80170558
0.09	0.09	1.577917243	1.03316458	1.80370216
0.10	0.10	1.5789123	1.0341718	1.8046178

CONCLUSION

The study reveals that as the height of the stenosis increases in the uniform or non-uniform or both portions of the artery the resistance to the flow also increases.

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