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Non-Linear Hyperbolastic Growth Models for Describing Growth Curve in Classical Strain of Broiler Chicken

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Abstract: Mathematical models describing growth kinetics are important tools to predict biological phenomena such as BW at a specific age, a maximum growth response and/or growth rates. Classical models such as Gompertz and Richards have been extensively used to describe broiler studies. Recently, for accurate prediction or describtion of growth behavior in biological system, a group of flexible growth models known as hyperbolastic with 3 or 4 parameters are introduced. These models may predict variety of growth behaviors for continuous output as occurs in cancer and stem cell growth studies. In the present study, three new flexible hyperbolastic growth models, called H1, H2 and H3 were evaluated to determine their strength in describing the relationship of BW and age of broilers or compatibility with two classical growth models of Gompertz and Richards. A growth data set of 217 male broiler chickens raised for 170 days were used to test and compare the fitness of the growth models. Goodness of fit for the models were determined by Mean Square Error (MSE), R^2, Residual Standard Deviation (RSD), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Based on our overall calculated goodness of fit criteria, it is revealed that the H3 model provides the most promising fit and it was reduced when the H2, Richards, Gompertz and H1 models were used, respectively. The more precise fitness observed with the H3 model might be due to its more flexibility. The hyperbolastic growth models in particular the H3 model is considered to be a more accurate tool for predicting broiler growth curve. However, it suggested to examine the fitness of different models to hire the best fit.

Key words: Broiler chicken, growth models, hyperbolastic, gompertz, richards

INTRODUCTION

A growth model conveniently summarizes the information obtained from an animal study into a small set of parameters that can be interpreted biologically and also used to derive other relevant growth traits. The use of growth functions is usually empirical and the form of the function is chosen by its fitness to the data set. A growth model may be characterized by some underlying physiological or biochemical mechanism or constraint (France and Thornley, 1984). A growth function is expressed in a rate term, as a function of state form, in which the instantaneous growth rate is a function of the organism's size. Unlike equations in which growth rate is purely an empirical function, an equation in this form is usually interpreted biologically and it can be ascribed by its parameters (Lopez *et al*., 2000). A number of nonlinear growth models have been used to describe growth in fish, poultry and mammals (Lopez *et al*., 2000; France *et al*., 1996). Gompertz and Richards (1825) models are the most common models to describe broiler growth curve (Aggrey, 2002; Darmani Kuhi *et al*., 2003). In these models, the growth curves are asymmetric around the point of maximum growth rate. However, differences between them is the points of inflection that in the Gompertz model is a fixed and in the Richards model is variable (flexible) proportion of their asymptotic growth values (Tabatabai *et al*., 2005).

Tabatabai *et al*. (2005) introduced models with three or four parameter models called hyperbolastic models to predict a self-limited growth behavior that occurs in tumors and stem cells. These models are called hyperbolastic because the outcome is a function of inverse hyperbolic sine function (arcsinh). These models are a family of flexible growth models that may predict variety of growth behaviors for continuous outputs in the biological systems. The hyperbolastic growth models are 3 types includingtype 1 or H1 (generalizes logistic

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growth model), type 2 or H2 (stand alone) and type 3 or H3 (generalizes Weibull growth model).

The purpose of this study was to test the fitness of H1, H2 and H3 models to broiler growth data obtained from an empirical experiment using classical strain of broiler chickens. In addition the growth models output was compared to those obtained with Gompertz and Richards.

MATERIALS AND METHODS

Data source: The average live BW of 217 male broilers (Unselected, randomly mated Athens-Canadian poultry population) reported by Aggrey (2002) for 170 days was used in this study.

Growth models: Five growth models were fitted to data using NLIN procedure (Marquart algorithm) of the SAS (1999). The models are as:

H1 model:

$$W(t) = \frac{M}{1+\alpha \exp\left[-M\beta t - \theta \operatorname{arcsinh}(t)\right]}$$

Where,

$$\alpha = \frac{M - W_0}{W_0} \exp\left[M\beta\, t_0 + \theta \operatorname{arcsinh}(t_0)\right]$$

H2 model:

$$W(t) = \frac{M}{1+\alpha \operatorname{arcsinh}\left[\exp(-M\beta t^{\gamma})\right]}$$

Where,

$$\alpha = \frac{M - W_0}{W_0 \operatorname{arcsinh}\left[\exp(-M\beta t_0{}^{\gamma})\right]}$$

H3 model:

$$W(t) = M - \alpha \exp\left[-\beta t^{\gamma} - \operatorname{arcsinh}(\theta t)\right]$$

Where,

$$\alpha = (M - W_0)\exp\left[\beta t_0{}^{\gamma} + \operatorname{arcsinh}(\theta t_0)\right]$$

Gompertz model: (Gompertz, 1825)

$$W(t) = M \exp\left[-\alpha \exp(-M\beta t)\right]$$

Where,

$$\alpha = LN\left(\frac{M}{W_0}\right) \exp(M\beta t_0)$$

Richards model: (Richards, 1959)

$$W(t) = \frac{M}{\left[1+\alpha \exp(-M\beta t)\right]^{\gamma}}$$

Where,

$$\alpha = \left[\left(\frac{M}{W_0}\right)^{\frac{1}{\gamma}} - 1\right]\exp(M\beta t_0)$$

In all models, $W(t)$ is live weight (g) at age t, β is the intrinsic growth rate, θ and γ are parameters and M represents the asymptotic or maximum growth response (potential final weight), which is assumed to be constant, though final weight may usually change over time. In each model α is defined as a function of the other parameters (M, β and initial observed value W_0 at time t_0) which allows to reduce the number of estimated parameters and also anchors the first predicted value to the original value observed at the initial point of time.

A quantitative verifying of the fit of the predictive models was made using error measurement indices commonly used to evaluate forecasting models. The accuracy of models (goodness of fit) was determined by Mean Squared Error (MSE), R^2 value and Residual Standard Deviation (RSD). Forecasting error measurements were based on the value differences between models predicted and empirical BW (Oberstone, 1990). Two information criteria were used to test the models goodness of fit.

- The akaike information criterion (AIC) (Akaike, 1974):

$$AIC = 2k - 2\ln L$$

where,
k : The number of parameters.
L : The value of likelihood function.

- The bayesian information criterion (BIC) (Schwarz, 1978):

$$BIC = -2\ln L + k\ln(n)$$

where,
n : The number of observations.
k and L : Similar to those described for AIC.

RESULTS

Empirical and 5 growth predicted models for growth curves are shown in Fig. 1-5. It is appeared that all growth models may finely be fitted to the empirical values, although, these models were sensitive to selected initial values.

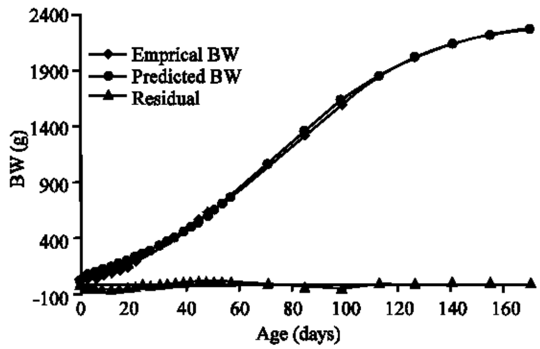

Fig. 1: Empirical and predicted hyperbolastic model type H1 for growth curve

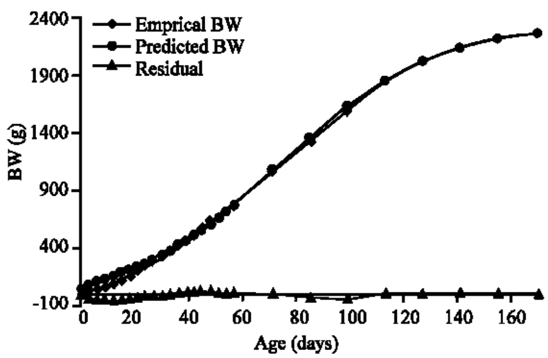

Fig. 2: Empirical and predicted hyperbolastic model type H2 for growth curve

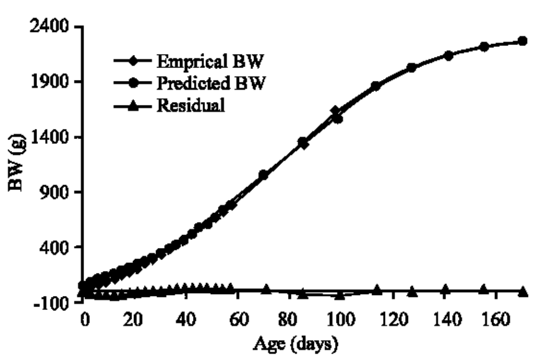

Fig. 3: Empirical and predicted hyperbolastic model type H3 for growth curve

Estimated parameters and standard error obtained with H1, H2, H3, Gompertz and Richards growth models are summarized in Table 1. The lowest estimated potential final weight (M) obtained for male Athens-Canadian broiler chicken population with H3 (2347 g) and followed by H1 (2351 g), H2 (2470 g), Richards (2513 g) and Gompertz (2540 g). There are relatively great differences between predicted weights by 5 models.

The empirical and 5 predicted BW values with models goodness of fit and residuals are shown in Table 2. It is appeared that H3 (MSE = 171.0, R^2 = 0.99985, RSD = 13.2) and H2 (MSE = 303.3, R^2 = 0.99973, RSD = 17.5) models provide the best fit to empirical growth values and are followed by Richards (MSE = 376.3, R^2 = 0.99970, RSD = 19.7), Gompertz (MSE = 384.7, R^2 = 0.99965, RSD = 19.9) and H1 (MSE = 590.6, R^2 = 0.99947, RSD = 23.8) models. The calculated AIC and BIC for models are shown in Table 2. Based on AIC and BIC the best goodness of fit is found for the H3 model (AIC = 152.0, BIC = 157.3) and followed by H2 (AIC = 166.0, BIC = 170.0), Gompertz (AIC = 170.7, BIC = 173.3), Richards (AIC = 172.0, BIC = 176.0) and H1 (AIC = 184.7, BIC = 188.7) models.

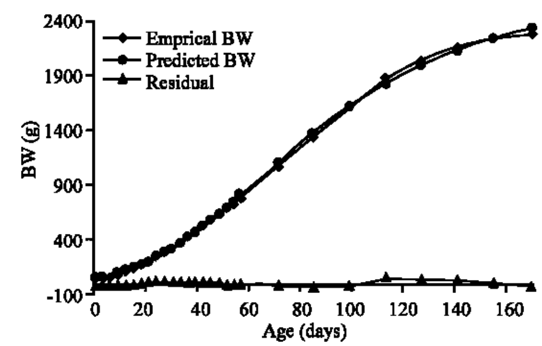

Fig. 4: Empirical and predicted Gompertz model for growth curve

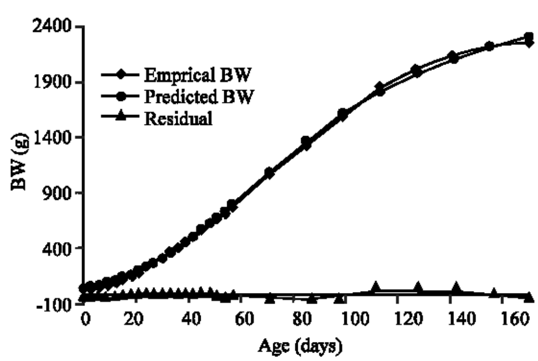

Fig. 5: Empirical and predicted Richards model for growth curve

Table 1: Estimated growth parameters ± standard error using different growth models

Model	M (Final weight) (g)	β (Intrinsic growth rate)	θ	γ
Hyperbolastic growth model				
H1	2351±25.0	$1.4\times10^{-5}\pm5.4\times10^{-7}$	0.33±0.011	-
H2	2470±27.2	$1.3\times10^{-4}\pm3.8\times10^{-6}$	0.60±0.009	-
H3	2347±21.6	$2.3\times10^{-5}\pm9.3\times10^{-6}$	0.002±0.0003	2.3±0.08
Gompertz	2540±27.2	$9\times10^{-6}\pm2.5\times10^{-7}$	-	-
Richards	2513±43.3	$9\times10^{-6}\pm7\times10^{-7}$	-	19.02±11.5

Table 2: Observed and predicted broiler BW using 5 models with model error measurements and determined residuals

		Hyperbolastic growth model type									
		H1		H2		H3		Gompertz		Richards	
Age (d)	Empirical BW (g)	P*	R**	P	R	P	R	P	R	P	R
0	37	37	0	37	0	37	0	45	-8	49	-12
3	42	74	-32	62	-20	55	-13	58	-16	62	-21
6	59	100	-41	82	-23	74	-15	74	-14	78	-19
9	80	125	-45	104	-24	96	-16	92	-12	96	-16
12	103	150	-47	127	-24	120	-17	114	-11	118	-15
15	132	175	-43	153	-21	147	-15	139	-7	142	-10
18	170	203	-33	182	-12	177	-6	167	3	170	0
21	207	232	-25	213	-6	209	-2	199	8	201	6
24	251	263	-12	247	4	244	7	234	17	235	15
27	285	296	-11	283	2	282	3	273	13	273	12
30	325	332	-7	322	3	323	2	314	11	314	11
33	373	370	2	364	9	366	7	359	14	358	14
36	417	411	6	409	9	412	6	407	10	406	12
39	469	455	14	456	13	460	9	458	12	455	14
42	520	501	19	506	14	510	10	510	9	508	12
45	577	550	28	558	19	562	15	565	12	563	15
48	634	601	33	613	21	617	17	622	11	619	14
51	667	655	13	669	-2	673	-6	681	-14	678	-11
54	717	711	7	727	-10	730	-13	741	-23	738	-20
57	786	769	18	787	-1	789	-3	801	-15	798	-12
71	1069	1059	10	1076	-7	1072	-3	1088	-19	1087	-18
85	1326	1357	-31	1361	-35	1353	-27	1362	-35	1364	-37
99	1590	1628	-39	1617	-27	1611	-21	1606	-17	1610	-21
113	1859	1850	9	1829	30	1831	29	1814	45	1818	41
127	2015	2016	-1	1997	19	2004	11	1983	32	1987	29
141	2142	2133	10	2124	19	2132	10	2117	25	2119	24
155	2221	2211	9	2218	3	2220	1	2222	-1	2220	0
170	2263	2265	-2	2290	-27	2279	-17	2307	-44	2302	-39
Goodness of fit criteria*											
R^2		0.99947		0.99973		0.99985		0.99965		0.99970	
MSE		590.60000		303.30000		171.00000		384.70000		376.30000	
Residual mean		-6.80000		-2.70000		-1.60000		-0.55000		-1.50000	
Residual SD		23.80000		17.50000		13.20000		19.90000		19.70000	
AIC		184.70000		166.00000		152.00000		170.70000		172.00000	
BIC		188.70000		170.00000		157.30000		173.30000		176.00000	

*Predicted body weight, **Residual, ***MSE = MS Error (standard deviation); AIC = Akaike Information Criterion; BIC = Bayesian Information Criterion

DISCUSSION

The overall calculated statistical values (MSE, R^2 and RSD) showed that the H3 and H2 models provide higher accuracy of fitness to the empirical data, followed by the Richards, Gompertz and H1 models. Similar results were observed with other flexible and generalized models used to find the best fitness to the empirical growth curve (Lopez *et al*., 2000; France *et al*., 1996; Gille and Salomon, 1995; Ricklefs, 1985).

Although, models are closely related, the parameter values may have appeared quite different when these models were fitted to a single set of data. The Gompertz model has 2 parameters (M and β) and may fit asymmetric growth, but it is not very flexible when compared with the H3, H2 and Richards models. The Richards (1959) model has three parameters (M, β and γ) and asymmetric with more flexibility than that of Gompertz model. The H1 model has one more parameter (θ) than that of Gompertz (1825) function, which allows more flexibility and may be fitted in asymmetric growth patterns. The H2 model has the same number of parameters as H1 and might be fitted in an asymmetric curve. The H3 model has the same flexibility as the H1 function at the expense of one more parameter (γ) similar to the Richards model. The high flexibility of the H3 and H2 model are resulted in more accurate prediction and better fit to the empirical broiler growth curve than other models (Darmani Kuhi *et al*., 2003). This means that the fixed point of inflection (less flexibility) in the models may act as a limitation for data fitting.

Despite unstable value, the asymptote may be regarded as the potential final weight (M). In this study, some bias was observed among models when M was estimated (Table 1). It is reported that the estimation of potential final weight in different species is a function of an algorithm fitting and the accuracy of judging is possible when precise final weight is available (Lopez *et al*., 2000; Brown *et al*., 1976).

In this study, the H3 model showed lower residuals distribution (in terms of RSD) than that of the Gompertz and Richards models. This is in agreement with Tabatabai *et al.* (2005) whom reported a fitness improvement when a flexible and accurate predictive hyperbolastic models were used to study the cancer, craniofacial and stem cell growth.

The AIC and BIC methodologies can be used to find the model that best explains the empirical growth data with a minimum of free parameters. The preferred model is the one with the lowest AIC and BIC values (Akaike, 1974; Schwarz, 1978). Based on these tow information criteria and other similar statistical tools (MSE, R^2 and RSD), the H3 and H2 models have the best fitness to the empirical broiler growth data than other models. However, the calculated values of AIC and BIC revealed that Gompertz (AIC = 170.7 and BIC = 173.3) model had a better fitness than that of Richards (AIC = 172.0 and BIC = 176.0) and H1 (AIC = 184.7 and BIC = 188.7) which is due to lower number of free parameters in Gompertz model. The new growth models, in particular the H3, clearly demonstrated a valid data fitting for broiler growth study and also better goodness of fit as compared to others models. Based on calculated R^2 and RSD for the same data set (Aggrey, 2002), it is revealed that the H3 model (R^2 = 0.99995 and RSD = 13.2) has more accurate prediction than the spline regression model for a broiler growth (R^2 = 0.9642 and RSD = 87.8).

CONCLUSION

The overall calculated statistical values (MSE, R^2 and RSD) have shown that the Hyperbolastic growth model types H3 and H2 may be used to provide the most accurate fit to the broiler growth curve set and followed by the Richards, Gompertz and H1 models. Hyperbolastic growth models can be used to fit the empirical broiler growth values. It is suggested to consider the flexible growth models (such as the H3) as an alternative to the classical models of Gompertz and Richards. The hyperbolastic growth models in particular the H3 model may be considered as a more accurate tool for modeling broiler growth. However, it is suggested to compare the models fitness to find the most accurate one.

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