Research Journal of Applied Sciences 13 (6): 369-371, 2018
ISSN: 1815-932X

The Fractional Complex Transform of Heat Equation for Polypropylene Textile Fabric

Hisham Mohammed Ali Hasan
College of Computer Science and Information Technology,
University of Al-Qadisiyah, Al Diwaniyah, Iraq

Abstract: The problem of heat transfer through fabric materials is theoretically solved by complex transform fractional differential equation method, the heat transfer and thermal behavior of polypropylene synthetic textile fiber as thermal insulators have been investigated by Maple Software simulation.

Key words: Heat equation, textile fiber, thermal conductivity, fractional complex transform, synthetic, polypropylene

INTRODUCTION

Recently, the wide applications of industrial for textile materials has been used as thermal insulators, the thermal insulation properties depend on thermal conductivity, density, specific heat of fabrics and heat flux by radiation, this can be represent by equation of energy with heat radiation, many research activities to solve this problem, the majority was concerned with exact solution and numerical or approximate solutions. In this study, fractional complex transform (Lakshmikantham and Vatsala, 2011) is proposed to convert the heat fractional differential equations into ordinary differential equations where the fractional complex transform makes the solution procedure extremely simple when contrasted with other analytical methods, fractional differential equations play important roles and serve as tools in physics, dynamical systems, control systems and engineering to create the mathematical model of many physical phenomena (Ibrahim, 2012).

MATERIALS AND METHODS

Fractional complex transform: For textile fabric the energy equation with heat radiation source term is given as follows (Siegel and Howell, 2002):

$$k\frac{\partial^{2\alpha}T}{\partial x^{2\alpha}} = \rho c_p \frac{\partial^\beta T}{\partial t^\beta}+R \tag{1}$$

Where:

$\frac{\partial^{2\alpha}T}{\partial x^{2\alpha}}$ and $\frac{\partial^\beta T}{\partial t^\beta}$ = Modified Riemann-Liouville derivative, $0<a\leq 1$, $0< a\leq 1$
k = Thermal conductivity
T = Temperature for the selected fabrics
ρ = The density as calculated as $\rho = \omega/L$ (ω is the basic weight of the sample, L is the fabric thickness)
c_p = Specific heat of the sample
R = The heat flux by radiation at any point within the fabric

$$R = 4\sigma T_0^3(T_1\text{-}T_2), 0 \leq x \leq L$$

Where:

σ = The Stephan-Boltzman constant and equals 5.67×10^{-8} W/m^{2}K^4
T_0 = The mean temperature ($T_\circ = 298$ K)
T_1, T_2 = The inner and the outer surface temperature, respectively
L = The thickness

By fractional complex transform (Zayed *et al*., 2016):

$$\xi = \frac{px^\alpha}{\Gamma(1+\alpha)}+\frac{qt^\beta}{\Gamma(1+\beta)} \tag{2}$$

where (p, q) are unknown constants for left hand side derivative of Eq. 1:

$$\begin{aligned}
&\frac{\partial^{2\alpha}T}{\partial x^{2\alpha}} = \frac{\partial^2 T}{\partial \xi^2}\left(\frac{\partial^\alpha \xi}{\partial x^\alpha}\right)^2 = \\
&T_{\xi\xi}\left[\frac{\partial^\alpha}{\partial x^\alpha}\left(\frac{px^\alpha}{\Gamma(1+\alpha)}+\frac{qt^\beta}{\Gamma(1+\beta)}\right)\right]^2 = \\
&T_{\xi\xi}\left[\frac{\partial^\alpha}{\partial x^\alpha}\frac{px^\alpha}{\Gamma(1+\alpha)}+\frac{\partial^\alpha}{\partial z^\alpha}\frac{qt^\beta}{\Gamma(1+\beta)}\right]^2 = \\
&T_{\xi\xi}\left[\frac{p}{\Gamma(1+\alpha)}\frac{\partial^\alpha}{\partial x^\alpha}+0\right]^2 = \\
&T_{\xi\xi}\left[\frac{p}{\Gamma(1+\alpha)}\frac{\Gamma(1+k\alpha)x^{(k-1)\alpha}}{\Gamma(1+(k\text{-}1)\alpha)}\right]^2, k=1 \\
&\frac{\partial^{2\alpha}T}{\partial z^{2\alpha}} = T_{\xi\xi}\left[\frac{p}{\Gamma(1+\alpha]}+\frac{\Gamma(1+\alpha)}{\Gamma(1)}x^0\right] = T_{\xi\xi}p^2
\end{aligned} \tag{3}$$

For left hand side the derivative of Eq. 1:

$$\frac{\partial^{\beta}T}{\partial t^{\beta}} = \frac{\partial T}{\partial \xi}\frac{\partial^{\beta}\xi}{\partial t^{\beta}} =$$

$$T_{\xi}\left[\frac{\partial^{\beta}}{\partial t^{\beta}}\left(\frac{px^{\alpha}}{\Gamma(1+\alpha)}+\frac{qt^{\beta}}{\Gamma(1+\beta)}\right]=$$

$$T_{\xi}\left[\frac{\partial^{\beta}}{\partial t^{\beta}}\frac{px^{\alpha}}{\Gamma(1+\alpha))}+\frac{\partial^{\beta}}{\partial t^{\beta}}\frac{qt^{\beta}}{\Gamma(1+\beta)}\right]= \quad (4)$$

$$T_{\xi}\left[0+\frac{q}{\Gamma(1+\beta)}\frac{\Gamma(1+k\beta)}{\Gamma(1+(k-1)\beta)}t^{(k-1)\beta}\right], k=1$$

$$T_{\xi}\left[\frac{q}{\Gamma(1+\beta)}\frac{\Gamma(1+\beta)}{\Gamma(1)}t^{0}\right] = T_{\xi}q$$

By substituting Eq. 3 and 4 in Eq. 1, we obtain:

$$kp^2T_{\xi\xi} = \rho c_p qT_{\xi}+R$$
$$kp^2T_{\xi\xi}-\rho c_p qT_{\xi} = R \quad (5)$$

$$kp^2\lambda^2-\rho c_p q\lambda = 0$$
$$\lambda(kp^2\lambda-\rho c_p q) = 0$$

Then:

$$\lambda = 0 \text{ or } \lambda = \frac{\rho c_p q}{kp^2}$$

$$y_c = c_1e^{\lambda_1\xi}+c_2e^{\lambda_2\xi}$$

$$y_c = c_1e^{0}+c_2e^{\frac{\rho c_p q}{kp^2}}$$

Let:

$$y_p = \frac{1}{\phi(D)}R, \ \phi(D) = kp^2D^2-\rho c_p qD$$

$$y_p = \frac{1}{kp^2D^2-\rho c_p qD}R$$

$$y_p = \frac{1}{(kp^2D-\rho c_p q)}\frac{1}{D}R$$

$$y_p = \frac{1}{\left(\frac{kp^2}{\rho c_p q}D-1\right)}R\xi$$

By Taylor series:

$$y_p = -\left[1+\frac{kp^2D}{\rho c_p q}+\left(\frac{kp^2D}{\rho c_p q}\right)^2+, ...,\right]R\xi = -R\xi-\frac{kp^2R}{\rho c_p q}$$

$$T(\xi) = y_c+y_p = c_1+c_2e^{\lambda_2\xi}\cdot R\xi-\frac{kp^2}{\rho c_p q}R$$

$$T(x,t) = c_1+c_2e^{\left[\frac{\rho c_p q}{kp^2}\left(\frac{px^{\alpha}}{\Gamma(1+\alpha)}+\frac{qt^{\beta}}{\Gamma(1+\beta)}\right)\right]}-R\left(\frac{px^{\alpha}}{\Gamma(1+\alpha)}+\frac{qt^{\beta}}{\Gamma(1+\beta)}\right)-\frac{kp^2R}{pc_p q} \quad (6)$$

For initial conditions $T(0, 0) = T_0$ and $T_L = T(L, 0)$, we finally have integral constants c_1 and c_2 are:

$$c_1 = T_0-\frac{(T_L-T_0)+\frac{RL^{\alpha}}{\Gamma(1+\alpha)}}{e^{\frac{\rho c_p}{k}\left(\frac{L^{\alpha}}{\Gamma(1+\alpha)}\right)}-1}-\frac{kR}{\rho c_p} \quad (7)$$

$$c_2 = \frac{(T_L-T_0)+\frac{RL^{\alpha}}{\Gamma(1+\alpha)}}{e^{\frac{\rho c_p}{k}\left(\frac{L^{\alpha}}{\Gamma(1+\alpha)}\right)}-1} \quad (8)$$

RESULTS AND DISCUSSION

The heat transfer and thermal behavior of polypropylene synthetic textile fiber (Le Bozec *et al*., 2000; Park and Baik, 1997) as thermal insulators, the mathematical Eq. 6-8 of the fractional complex transform plotted using Maple Software, the temperature variation with test time through the textile fabrics thickness during the heat exchange process between the hot air inlet and the fabric sample shown in Fig. 1.

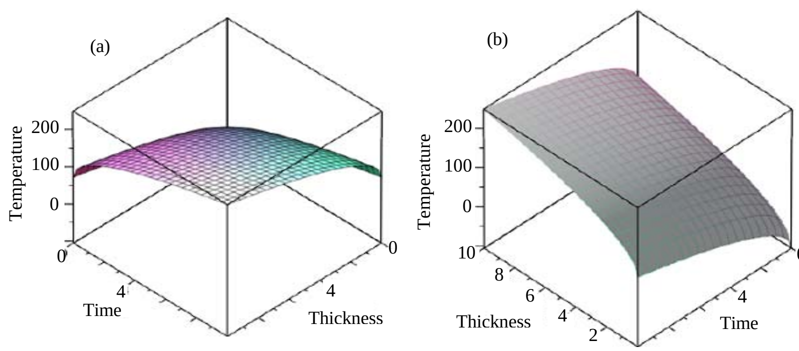

Fig. 1: a, b) The relation between temperature, thickness and time for polypropylene textile fiber

In fractional complex transform of heat equation is easy to convert partial differential equations to ordinary differential equations, the exact solutions can be easily obtained. The calculations have shown that selected textile fabrics can be used as good thermal insulators exposure temperatures up to 250°C which shows that the selected fabrics have high thermal performance and thermal response as insulators also fabric thickness affect the transient fabric temperatures as fabric thickness increase the temperature decreases.

CONCLUSION

A theoretical model of the combined conductive and radiative heat flow through fibrous insulating materials is presented and compared to experimental values of the thermal resistances of several synthetic fiber battings and of a down and feather mixture.

REFERENCES

Ibrahim, R.W., 2012. Fractional complex transforms for fractional differential equations. Adv. Difference Equations, Vol. 2012. 10.1186/1687-1847-2012-192.

Lakshmikantham, V. and A.S. Vatsala, 2008. Basic theory of fractional differential equations. Nonlinear Anal., 69: 2677-2682.

Le Bozec, Y., S. Kaang, P.J. Hine and I.M. Ward, 2000. The thermal-expansion behaviour of hot-compacted polypropylene and polyethylene composites. Compos. Sci. Technol., 60: 333-344.

Park, S.I. and D.H. Baik, 1997. Heat and mass transfer analysis of fabric in the tenter frame. Textile Res. J., 67: 311-316.

Siegel, R. and J.R. Howell, 2002. Thermal Radiation Heat Transfer. 4th Edn., Taylor and Francis, USA.

Zayed, E.M., Y.A. Amer and R.M. Shohib, 2016. The fractional complex transformation for nonlinear fractional partial differential equations in the mathematical physics. J. Assoc. Arab Univ. Basic Appl. Sci., 19: 59-69.