Research Journal of Applied Sciences 13 (2): 125-130, 2018
ISSN: 1815-932X

Derivation of Approximation to Select Bayes Population of Median from Even Samples

Kawther Fawzi Alhasan
Department of Mathematics, University of Babylon, Hillah, Iraq

Abstract: Finding the median is very important in many fields of life, therefore, we have addressed in this study derived approximate selection of median using Bayesian approach with prior and posterior distribution, so that, the sample size of observation is even number.

Key words: Prior and posterior, observation, fields of life, Bayesian approach, distribution, selection of median

INTRODUCTION

In many circumstances, we select the median value (alternative) among the alternatives. In this case, we ask how the median can be found reasonably quickly. Just like the "faster selection", given an unsorted array, how quickly can one select the median element? Median finding is a special case of the more general selection problem which asks for the ψth element in an arranged order (Dor and Zwick, 1999).

The ranking and selection methodology is distinct. The ranking and selection methodology asks given the information regarding the distribution what is the probability that we can acceptably rank them from smallest to greatest? What is the probability pick the finest population maybe the one with the biggest populace mean or at smallest populaces? (Horrace, 2000).

R&S procedures are statistical methods specifically advanced to select the greatest method or a subset that encloses the greatest method design from a set of multi-competing alternatives for example altered treatments (medicines) for a specific sickness or several contenders for certain place. The methods known generally as ranking and selection procedures include techniques appropriate for many different aims, although, each different aim needs careful formulation of the corresponding problem.

Al-Hassan (2006) the fully optimal Bayesian sequential best selection procedure using the dynamic programming technique in conjunction with Bayesian decision-theoretic approach.

By Madhi and Kawther (2007) addressed Bayesian fixed sample size for selected worst in multinomial distribution. Later by Sultani (2014) complemented her research in 2007 by using functional analysis for the purposes of approximation.

Al Hassan (2010) find a new approach: median selection where sample size is odd and found median selection when sample size is even (Al-Hassan, 2016). Al-Hassan also found approximate of odd median by functional analysis. In this study, we introduce Bayes approximate to find median stopping risk in multinomial distribution where the size of sample is even with prior and posterior distribution.

Selection Bayes median (Al-Hassan, 2016): In statistics and probability theory, the median is the arithmetic rate sorting out the upper half of a data sample, cells from the lesser half. The median of a finite list of values can be found by ordered all the observations from last value to utmost value and picked the mid one. If there is an even number of observations, then there are two mid values we can calculate the median from average to their values (Horrace, 2000).

Now, we present the problem of selecting the median category out of R mutually exclusive categories when each element in a single population is classified into one of these R categories. For each of the R types (cells) there is a certain probability that any element will be classified $p_1, p_2, ..., p_R$. The median cell may be defined as the one with the half biggest probability or alternatively as the one with the half lowest probability, dependent on the state.

Before discussing the R-nomial ranking and selection problem in more detail, we initial develop the R-nomial distribution model. Let an arbitrary experiment or process where each outcome is classified into one of R possible mutually exclusive possibilities which we call categories (or cells). Let, $P = (p_1,..., p_\psi, p_{\psi+1}, ..., p_R)$ where, p_i is the probability of the event E_i $(1 \le i \le R)$ with:

$$\sum_{i=1}^{R} p_i = 1$$

when m is even observation, the median dependent on n_ψ, $n_{\psi+1}$ and $p_\psi, p_{\psi+1}$, where $\psi = R/2$. Let, $n_1, ..., n_\psi, n_{\psi+1}, ..., n_R$ be respective frequencies in R cells of the distribution with, it is assumed that the values of p_i and of the $p_{[j]}$ ($1 \leq i, j \leq R$) is completely unknown. We will the probability of the median event is $\not\subset\subset_{[med]}$ and the expected value of median cell is:

$$\underset{\tau(\underline{p}/\underline{n})}{E}\left[p_{[med]}\right]$$

and let, $p_{[1]} \leq, ..., \leq p_{[\psi]}$, $p_{[\psi+1]} \leq, ..., \leq p_{[R]}$ denote the ordered values of the $p_i(i = 1, ..., R)$ such that $\underline{p} = (p_1, p_2, ..., p_R)$ the probability p_i of an observation in the cell i:

$$P\left(\underline{n}|\underline{p}\right) = \frac{\psi!}{n_1!n_2!, ..., n_R!}\prod_{i=1}^{R} p_i^{n_i}, \sum_{i=1}^{R} n_i = \psi$$

In the Bayesian procedure we depended on the prior and posterior distribution. Prior distribution of p_i is conjugate to the R-nomial distribution Dirichlet distribution with prior density function is:

$$\tau\left(\underline{p}\right) = \frac{\Gamma\left(\sum_{i=1}^{R} n_i^{'}\right)}{\prod_{i=1}^{R}\Gamma\left(n_i^{'}\right)}\prod_{i=1}^{R} p_i^{n_i^{'}-1}$$

Such that:

$$\psi^{'} = \sum_{i=1}^{R} n_i^{'}$$

and the posterior probability:

$$\tau\left(\underline{p}|\underline{n}\right) \propto p_1^{n_1+n_1^{'}-1}, ..., p_\psi^{n_\psi+n_\psi^{'}-1} p_\psi^{n_{\psi+1}+n_{\psi+1}^{'}-1}, ..., p_R^{n_R+n_R^{'}-1}$$

Since:

$$P\left(\underline{n}|\underline{p}\right) \propto p_1^{n_1}, ..., p_\psi^{n_\psi} p_{\psi+1}^{n_{\psi+1}}, ..., p_R^{n_R}$$

And:

$$\tau\left(\underline{p}\right) \propto p_1^{n_1^{'}-1}, ..., p_\psi^{n_\psi^{'}-1} p_{\psi+1}^{n_{\psi+1}^{'}-1}, ..., p_R^{n_R^{'}-1}$$

Which is a member of the Dirichlet family with parameters $n_i^{"} = n_i^{'}+n_i$ and $\psi^{"} = \psi^{'}(1, 2, ..., R)$ with mean $\hat{p}_i = n_i^{"}/\psi^{"}$ will be termed the posterior frequency in the cell i.

BAYES APPROXIMATE OF EVEN MEDIAN STOPPING RISK

The posterior expected loss $S_i(n_1^{'}, n_\psi^{'}, n_{\psi+1}^{'}, ..., n_R^{'}, \psi^{'})$ or Bayes risk of the terminal decision d_i can be approximate as follows:

$$S_i\left(n_1^{"}, ..., n_\psi^{"}, n_{\psi+1}^{"}, ..., n_R^{"} : \psi^{"}\right) = \underset{\tau(\underline{p}|\underline{n})}{E}\left[L\left(d_i, \underline{p}^*\right)\right] = mc+\omega\left[\underset{\tau(\underline{p}|\underline{n})}{E}\left([med]\right)-\hat{p}_i\right]$$

The value of $\underset{\tau(\underline{p}|\underline{n})}{E}[p_{med}]$ is derived as follows:

$$\underset{\tau(\underline{p}|\underline{n})}{E}[p_{med}] = \int_0^1 p_{[e]} \times g\left(p_{[e]}\right) dp_{[e]}$$

But:

$$\underset{\tau(\underline{p}|\underline{n})}{E}\left[f\left(p_{[\psi]}, p_{[\psi+1]}\right)\right] = \int_0^1\int_0^1 \left\{ \begin{array}{l} \left(\frac{p_{[\psi]}+p_{[\psi+1]}}{2}\right)\left(\frac{R!}{(\psi-1)!(R-\psi-1)!}\right)\left[\sum_{j=n_{[h]}^{"}}^{m\psi^{"}-1}\frac{(\psi^{"}-1)!}{j!(\psi^{"}-1-j)!}\times p_{[\psi]}^{\ j}\left(1-p_{[\psi]}\right)^{\psi^{"}-1-j}\right]^{R-1} \\ \left[1-\left\langle\sum_{I=n_{[h+1]}^{"}}^{m^{"}-1}\frac{(\psi^{"}-1)!}{I!(\psi^{"}-1-I)!}\times p_{[\psi+1]}^{\ I}\left(1-p_{[\psi+1]}\right)^{m^{"}-1-I}\right\rangle\right]^{R-\psi-1}\left\langle\frac{(\psi^{"}-1)!}{\left(n_{[R]}^{"}-1\right)!\left(\psi^{"}-n_{[R]}^{"}-1\right)!}p_{[\psi]}^{n_{[R]}^{"}-1}\left(1-p_{[\psi]}\right)^{\psi^{"}-n_{[R]}^{"}-1}\right\rangle \\ \left\langle\frac{(\psi^{"}-1)!}{\left(n_{[\psi+1]}^{"}-1\right)!\left(\psi^{"}-n_{[\psi+1]}^{"}-1\right)!}p_{[\psi+1]}^{n_{[\psi+1]}^{"}-1}\left(1-p_{[\psi+1]}\right)^{\psi^{"}-n_{[\psi+1]}^{"}-1}\right\rangle \end{array} \right\} dp_{[\psi]}dp_{[\psi+1]} =$$

$$\int_0^1\int_0^1 \left\{ \begin{array}{l} \left(\frac{R!}{2[\psi-1]!(R-\psi-1)!}\right)\left(p_{[\psi]}\ p_{[\psi+1]}\right)\left\langle\frac{(\psi^{"}-1)!}{\left(n_{[\psi]}^{"}-1\right)!\left(\psi^{"}-n_{[\psi]}^{"}-1\right)!}p_{[\psi]}^{n_{[\psi]}^{"}-1}\left(1-p_{[\psi]}\right)^{\psi^{"}-n_{[\psi]}^{"}-1}\right\rangle\left\langle\frac{(\psi^{"}-1)!}{\left(n_{[\psi+1]}^{"}-1\right)!\left(\psi^{"}-n_{[\psi+1]}^{"}-1\right)!}p_{[\psi+1]}^{n_{[\psi+1]}^{"}-1}\left(1-p_{[\psi+1]}\right)^{\psi^{"}-n_{[\psi+1]}^{"}-1}\right\rangle \\ \left[\sum_{j=n_{[\psi]}^{"}}^{\psi-1}\frac{(\psi^{"}-1)!}{j!(\psi^{"}-1-j)!}\times p_{[\psi]}^{\ j}\left(1-p_{[\psi]}\right)^{\psi-1-j}\right]^{\psi-1}\left[1-\left\langle\sum_{1=n_{[\psi+1]}^{"}}^{\psi^{"}-1}\frac{(\psi^{"}-1)!}{1!(\psi^{"}-1-1)!}\times p_{[\psi+1]}^{\ 1}\left(1-p_{[\psi+1]}\right)^{m^{"}-1-j}\right\rangle^{R-\psi-1}\right] \end{array} \right\} dp_{[\psi]}dp_{[\psi+1]} =$$

$$\int_0^1\int_0^1 \left(\frac{(\psi^{"}-1)!}{\left(n_{[\psi]}^{"}-1\right)!\left(\psi^{"}-n_{[\psi]}^{"}-1\right)!}\right)\left(\frac{(\psi^{"}-1)!}{\left(n_{[\psi+1]}^{"}-1\right)!\left(\psi^{"}-n_{[\psi+1]}^{"}-1\right)!}\right)\left(\frac{R!}{2[\psi-1]!(R-\psi-1)!}\right)$$

$$\left\{\left[\sum_{j=n_{[\psi]}^{"}}^{\psi^{"}-1}\frac{(\psi^{"}-1)!}{j!(\psi^{"}-1-j)!}\times p_{[\psi]}^{\ j}\left(1-p_{[\psi]}\right)^{\psi^{"}-1-j}\right]^{\psi-1}\left[1-\left\langle\sum_{1=n_{[\psi+1]}^{"}}^{\psi^{"}-1}\frac{(\psi^{"}-1)!}{1!(\psi^{"}-1-1)!}\times p_{[\psi+1]}^{\ 1}\left(1-p_{[\psi+1]}\right)^{\psi^{"}-1-1}\right\rangle\right]^{R-\psi-1}\right\}$$

$$\left(p_{[\psi]}^{n''[\psi]}p_{[\psi+1]}^{n''_{[\psi+1]}-1}\left(\left(1-p_{[\psi]}\right)^{\psi''-n''_{[\psi]}-1}\left(1-p_{[\psi+1]}\right)^{\psi''-n''_{[\psi+1]}-1}\right)+p_{[\psi]}^{n''_{[\psi]}-1}p_{[\psi+1]}^{n''_{[\psi+1]}}\left(1-p_{[\psi]}\right)^{\psi''-n''_{[\psi]}-1}\left(1-p_{[\psi+1]}\right)^{\psi''-n''_{[\psi1]}-1}\right)$$

$$dp_{[\psi]}dp_{[\psi+1]}=\left(\frac{(\psi''-1)!}{\left(n''_{[\psi]}-1\right)!\left(\psi''-n''_{[\psi]}-1\right)!}\right)\left(\frac{(\psi''-1)!}{\left(n''_{[\psi+1]}-1\right)!\left(\psi''-n''_{[\psi+1]}-1\right)!}\right)\left(\frac{R!}{2(\psi-1)!(R-\psi-1)!}\right)$$

$$\left\{\int_0^1\int_0^1\left[\sum_{1=n''_{[\psi]}}^{\psi''-1}\frac{(\psi''-1)!}{j!(\psi''-1-j)!}\times p_{[\psi]}{}^{j}\left(1-p_{[\psi]}\right)^{\psi''-1-j}\right]^{\psi-1}\left[1-\left\langle\sum_{1=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{I!(\psi''-1-l)!}\times p_{[\psi+1]}{}^{l}\left(1-p_{[\psi+1]}\right)^{\psi''-1-l}\right\rangle\right]^{R-\psi-1}\right.$$

$$\left.p_{[\psi]}^{n''[\psi]}p_{[\psi+1]}^{n''_{[\psi+1]}-1}\left\langle 1-p_{[\psi]}\right\rangle^{\psi''-n''_{[\psi]}-1}\left\langle 1-p_{[\psi+1]}\right)^{\psi''-n''_{[\psi+1]}-1}\right\rangle dp_{[\psi]}dp_{[\psi+1]}\Bigg\}+\left\{\int_0^1\int_0^1\left[\sum_{j=n''_{[\psi]}}^{\psi''-1}\frac{(\psi''-1)!}{j!(\psi''-1-j)!}\times p_{[\psi]}{}^{j}\left(1-p_{[\psi]}\right)^{\psi''-1-j}\right]^{\psi-1}\right.$$

$$\left.\left[1-\left\langle\sum_{1=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{I!(\psi''-1-l)!}p_{[\psi+1]}{}^{l}\left(1-p_{[\psi+1]}\right)^{\psi''-1-l}\right\rangle\right]^{R-\psi-1}p_{[\psi]}^{n''[\psi]}p_{[\psi+1]}^{n''_{[\psi+1]}}\left\langle 1-p_{[\psi]}\right\rangle^{\psi''-n''_{[\psi]}-1}\left\langle 1-p_{[\psi+1]}\right)^{\psi''-n''_{[\psi+1]}-1}\right\rangle\right\}dp_{[\psi]}dp_{[\psi+1]}$$

Now, we can approximate there formulas:

$$\left[\sum_{j=n''_{[\psi]}}^{\psi''-1}\frac{(\psi''-1)!p_{[\psi]}{}^{j}\left(1-p_{[\psi]}\right)^{\psi''-1-j}}{j!(\psi''-1-j)!}\right]^{\psi-1}\text{ and }\left[1-\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!p_{[\psi+1]}{}^{j_2}\left(1-p_{[\psi+1]}\right)^{\psi''-1-j_2}}{j_2!(\psi''-1-j_2)!}\right]^{\psi-1}$$

By functional analysis (Bhayaa, 2003a):

$$\left[\sum_{j=n''_{[\psi]}}^{\psi''-1}\frac{(\psi''-1)!p_{[\psi]}{}^{j}\left(1-p_{[\psi]}\right)^{\psi''-1-j}}{j!(\psi''-1-j)!}\right]^{\psi''-1}=\left[\sum_{j_1=n''_{[\psi]}}^{\psi''-1}\frac{(\psi''-1)!\left[\frac{p_{[\psi]}}{1-p_{[\psi]}}\right]^{j}\left(1-p_{[\psi]}\right)^{(\psi''-1)}}{j!(\psi''-1-j)!}\right]^{\psi-1}\leq 2(\psi''-1)(\psi''-2)$$

$$\left\{\sum_{j_1=n''_{[1]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)!\left[\frac{p_{[\psi]}}{\left(1-p_{[\psi]}\right)}\right]^{j_1-1}\left(1-p_{[\psi'']}\right)^{(\psi''-1)}}{(j_1-1)!(\psi''-j_1)!}\right\rangle\frac{1}{2^{(R-2)(\psi''-1-j_1)}}+\left\langle\left[\frac{p_{[\psi]}}{\left(1-p_{[\psi]}\right)}\right]^{(\psi''-1)}\left(1-p_{[\psi]}\right)^{(\psi''-1)}\right\rangle^{\psi-1}\right\}$$

And:

$$\left[1-\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!p_{[\psi''+1]}{}^{j_2}\left(1-p_{[\psi+1]}\right)^{\psi''-1-j_2}}{j_2!(\psi''-1-j_2)!}\right]^{\psi-1}=\left[1-\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!\left(1-p_{[\psi+1]}\right)^{\psi''-1}}{j_2!(\psi''-1-j_2)!}\left(\frac{p_{[\psi+1]}}{1-p_{[\psi+1]}}\right)^{j_2}\right]^{R-\psi-1}$$

$$\leq\sum_{1=0}^{R-\psi-1}\binom{R-\psi-1}{1}(-1)^{1}\left\{2(\psi''-1)(1-1)\left[\begin{array}{l}\displaystyle\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\left(\frac{p_{[\psi+1]}}{(1-p_{[\psi+1]})}\right)^{j_2-1}\left(1-p_{[\psi+1]}\right)^{\psi''-1}\Bigg)^{1}\\ \displaystyle\frac{1}{2(1-1)(\psi''-1-j_2)}+\left(\frac{p_{[\psi+1]}}{1-p_{[\psi+1]}}\right)^{\psi''-1}\left(1-p_{[\psi+1]}\right)^{\psi''-1}\end{array}\right]\right\}$$

Now, from above equations we get:

$$E\left(p_{[med]}\right)=\int_0^1\int_0^1\left(\frac{(\psi''-1)!}{\left(n''_{[\psi]}-1\right)!\left(\psi''-n''_{[\psi]}-1\right)!}\right)\left(\frac{(\psi''-1)}{n''_{[\psi+1]}-1!\left(\psi''-n''_{[\psi+1]}-1\right)!}\right)\left(\frac{R!}{2(\psi-1)!(R-\psi-1)!}\right)$$

$$\left\{\left[\sum_{j=n_{[\psi]}}^{\psi''-1}\frac{(\psi''-1)!}{j!(\psi''-1-j)!}\times p_{[\psi]}{}^{j}\left(1-p_{[\psi]}\right)^{\psi''-1-j}\right]^{\psi-1}\left[1-\left\langle\sum_{l=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{l!(\psi''-1-l)!}\times p_{[\psi+1]}{}^{l}\left(1-p_{[\psi+1]}\right)^{\psi''-1-l}\right\rangle\right]^{R-\psi-1}\right\}$$

$$\left(p_{[\psi]}{}^{n''[\psi]}p_{[\psi+1]}^{n''_{[\psi+1]}-1}\left(\left(1-p_{[\psi]}\right)^{\psi''-n''[\psi]-1}\left(1-p_{[\psi+1]}\right)^{\psi''-n''[\psi+1]-1}\right)+\left\{\left[\sum_{j=n''_{[\psi'']}}^{\psi''-1}\frac{(\psi''-1)!}{j!(\psi''-1-j)!}\times p_{[\psi]}{}^{j}\left(1-p_{[\psi]}\right)^{\psi''-1-j}\right]^{\psi-1}\right.\right)$$

$$\left.\left[1-\left\langle\sum_{l=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{l!(\psi''-1-l)!}\times p_{[\psi+1]}{}^{l}\left(1-p_{[\psi+1]}\right)^{\psi''-1-l}\right\rangle\right]^{R-\psi-1}\right\}p_{[\psi]}^{n''[\psi]-1}p_{[\psi+1]}^{n''_{[\psi+1]}}\left(1-p_{[\psi]}\right)^{\psi''-n''_{[\psi]}-1}\left(1-p_{[\psi+1]}\right)^{\psi''-n''[\psi+1]-1}\Big\}dp_{[\psi]}dp_{[\psi+1]}$$

$$E\left(p_{[med]}\right)\le\int_0^1\int_0^1\left(\frac{(m''-1)!}{\left(n''_{[\psi]}-1\right)!\left(m''-n''_{[\psi]}-1\right)!}\right)\left(\frac{(m''-1)}{n''_{[\psi+1]}-1!\left(m''-n''_{[\psi+1]}-1\right)!}\right)\left(\frac{R!}{2(\psi-1)!(R-\psi-1)!}\right)$$

$$\left\{2^{(\psi''-1)(\psi-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)!\left[\frac{p_{[\psi]}}{\left(1-p_{[\psi]}\right)}\right]^{j_1-1}\left(1-p_{[\psi]}\right)^{(\psi''-1)}}{(j_1-1)!(\psi''-j_1)!}\right\rangle\frac{1}{2^{(R-2)(\psi''-1-j_1)}}+\left\langle\left[\frac{p_{[\psi]}}{\left(1-p_{[\psi]}\right)}\right]^{(\psi''-1)}\left(1-p_{[\psi]}\right)^{(\psi''-1)}\right\rangle^{(\psi-1)}\right\}$$

$$\left\{\sum_{l=0}^{R-\psi-1}\binom{R-\psi-1}{l}(-1)^{l}\,2^{\left((\psi''-1)(l-1)\right)}\left[\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\left(\frac{p_{[\psi+1]}}{\left(1-p_{[\psi+1]}\right)}\right)^{j_2-1}\left(1-p_{[\psi+1]}\right)^{\psi''-1}\right]^{l}\times\frac{1}{2^{(l-1)(\psi''-1-j_2)}}+\left(\frac{\left(p_{[\psi+1]}\right)^{\psi''-1}}{\left(1-p_{[\psi+1]}\right)}\left(1-p_{[\psi+1]}\right)^{\psi''-1}\right)^{l}\right\}$$

$$\left(p_{[\psi]}{}^{n''[\psi]}p_{[\psi+1]}^{n''_{[\psi+1]}-1}\left(1-p_{[\psi]}\right)^{\psi''-n''[\psi]-1}\left(1-p_{[\psi+1]}\right)^{\psi''-n''[\psi+1]-1}\right)dp_{[\psi]}dp_{[\psi+1]}+$$

$$\left\{2^{(\psi''-1)(h-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)!\left[\frac{p_{[\psi]}}{\left(1-p_{[\psi]}\right)}\right]^{j_1-1}\left(1-p_{[\psi]}\right)^{(\psi''-1)}}{(j_1-1)!(\psi''-j_1)!}\right\rangle\frac{1}{2^{(R-2)(\psi''-1-j_1)}}+\left\langle\left[\frac{p_{[\psi]}}{\left(1-p_{[\psi]}\right)}\right]^{(\psi''-1)}\left(1-p_{[\psi]}\right)^{(\psi''-1)}\right\rangle^{(\psi-1)}\right\}$$

$$\left\{\sum_{l=0}^{R-\psi-1}\binom{R-\psi-1}{l}(-1)^{l}\,2^{\left((\psi''-1)(l-1)\right)}\left[\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\left(\frac{p_{[\psi+1]}}{\left(1-p_{[\psi+1]}\right)}\right)^{j_2-1}\left(1-p_{[\psi+1]}\right)^{\psi''-1}\right]^{l}\times\frac{1}{2^{(l-1)(\psi''-1-j_2)}}\right\}+$$

$$\left.\left(\frac{\left(p_{[\psi+1]}\right)^{\psi''-1}}{\left(1-p_{[\psi+1]}\right)}\left(1-p_{[\psi+1]}\right)^{\psi''-1}\right)^{l}\right\}\left(p_{[\psi]}{}^{n''[\psi]}p_{[\psi+1]}^{n''_{[\psi+1]}}\left(1-p_{[\psi]}\right)^{\psi''-n''[\psi]-1}\left(1-p_{[\psi+1]}\right)^{\psi''-n''[+1\psi]-1}\right)dp_{[\psi]}dp_{[\psi+1]}$$

$$E\left(p_{[med]}\right) \leq \left(\frac{(\psi''-1)!}{\left(n''_{[\psi]}-1\right)!\left(\psi''-n''_{[\psi]}-1\right)!}\right)\left(\frac{(\psi''-1)!}{n''_{[\psi+1]}-1!\left(\psi''-n''_{[\psi+1]}-1\right)!}\right)\left(\frac{R!}{2(\psi-1)!(R-\psi-1)!}\right)$$

$$\int_0^1\int_0^1\left\{2^{(\psi''-1)(\psi-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)}{(j_1-1)!(\psi''-j_1)!}\right\rangle^{\psi-1}\frac{1}{2^{(\psi-2)(\psi''-1-j_1)}}\left\langle\begin{matrix}p_{[\psi]}^{n''_{[\psi]}+(j_1-1)(\psi-1)}\left(1-p_{[\psi]}\right)^{(\psi-1)(\psi''-j_1)+\left(\psi''-n_{[\psi]}1\right)}+\\ p_{[\psi]}^{n''_{[\psi]}+(\psi''-1)(\psi-1)}\left(1-p_{[\psi]}\right)^{\left(\psi''-n_{[\psi]}1\right)}\end{matrix}\right\rangle\right\}$$

$$\left\{\sum_{1=0}^{R-\psi-1}\binom{R-\psi-1}{1}(-1)^1 2^{(\psi''-1)(1-1)}\left(\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\right)^1\frac{1}{2^{(1-1)(\psi''-1-j_2)}}\left(p_{[\psi]}^{n''_{[\psi]}-1+1(j_2-1)}\left(1-p_{[\psi+1]}\right)^{1(\psi''-j_2)+\left(\psi''-n''_{[\psi+1]}-1\right)}\right)+\right.$$

$$\left.\left(p_{[\psi]}^{n''_{[\psi]}-1}\left(1-p_{[\psi+1]}\right)^{1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}-1\right)}\right)dp_{[\psi]}dp_{[\psi+1]}+\int_0^1\int_0^1\left\{2^{(\psi''-1)(\psi-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)!}{(j_1-1)!(\psi''-j_1)!}\right\rangle^{\psi-1}\frac{1}{2^{(\psi''-2)(\psi''-1-j_1)}}\right.$$

$$\left\langle p_{[\psi]}^{\left(j_1\psi-j_1-\psi+n''_{[1]}\right)}\left(1-p_{[\psi]}\right)^{\left(2\psi-\psi''+j_1-n''_{[\psi]}-2\right)}+p_{[\psi]}^{n''_{[\psi]}+\psi''\psi-\psi''-R}\left(1-p_{[\psi]}\right)^{\left(\psi''n''_{[\psi]}-1\right)}\right\rangle$$

$$\left\{\sum_{1=0}^{R-\psi-1}\binom{R-\psi-1}{1}(-1)^1 2^{(\psi''-2)}\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\left(\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\right)^1\frac{1}{2^{(1-1)(\psi''-1-j_2)}}\left(p_{[\psi]}^{n''_{[\psi+1]}-1+1(j_2-1)}\left(1-p_{[\psi+1]}\right)^{\psi''j_2+j_2-n''_{[\psi+1]}}\right)\right\}+$$

$$p_{[\psi]}^{n''_{[\psi+1]}}\left(1-p_{[\psi+1]}\right)^{1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}-1\right)}dp_{[\psi]}dp_{[\psi+1]}$$

$$E\left(P_{[med]}\right) \leq \left(\frac{(\psi''-1)!}{\left(n''_{[\psi]}-1\right)!\left(\psi''-n''_{[\psi]}-1\right)!}\right)\left(\frac{(\psi''-1)!}{n''_{[\psi+1]}-1!\left(\psi''-n''_{[\psi+1]}-1\right)!}\right)\left(\frac{R!}{2(\psi-1)!(R-\psi-1)!}\right)$$

$$\left\{2^{(\psi''-1)(\psi-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)!}{(j_1-1)!(\psi''-j_1)!}\right\rangle^{\psi-1}\frac{1}{2^{(\psi-2)(\psi''-1-j_1)}}\left\langle\frac{\Gamma\left(n''_{[\psi]}+j_1\psi-j_1-\psi+2_1\right).\Gamma\left((\psi''-1)(1-j_1)+\left(\psi''-n''_{[\psi]}\right)\right)}{.\Gamma\left(n''_{[\psi]}+j_1\psi-j_1-\psi+2+\psi''+(\psi''-1)(1-j_1)\right)}\right\rangle+\right.$$

$$\frac{\Gamma\left(n''_{[\psi]}+\psi''\psi-\psi''-\psi+2_1\right).\Gamma\left(\psi''-n''_{[\psi]}\right)}{.\Gamma\left(n''_{[\psi]}+\psi''\psi-2\psi''-\psi''+2\right)}\left\{\sum_{1=0}^{R-\psi''-1}\binom{R-\psi''-1}{1}(-1)^1 2^{(\psi''-1)(1-1)}\left(\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\right)\right.$$

$$\frac{1}{2^{(1-1)(\psi''-1-j_2)}}\left\langle\frac{\Gamma\left(n''_{[\psi+1]}+1(j_2-1)\right)\Gamma\left((\psi''-1)(j_2-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}{\Gamma\left(n''_{[\psi+1]}+1(j_2-1)+(\psi''-1)(j_2-1)+\left(\psi''-n''_{[\psi''+1]}\right)\right)}\right\rangle+\frac{\Gamma\left(n''_{[\psi+1]}\right)\Gamma\left(1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}{\Gamma\left(n''_{[\psi+1]}+1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}+$$

$$\left\{2^{(\psi''-1)(\psi-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)}{(j_1-1)!(\psi''-j_1)!}\right\rangle^{\psi-1}\frac{1}{2^{(\psi-2)(\psi''-1-j_1)}}\left\langle\frac{\Gamma\left(n''_{[\psi]}+j_1\psi-j_1-\psi''+1\right)\Gamma\left(\left(2\psi''-\psi''j_1+j_1-n''_{[\psi]}-1\right)\right)}{\Gamma\left(j_1\psi-\psi+2\psi''-\psi''j_1\right)}\right\rangle+\right.$$

$$\frac{\Gamma\left(n''_{[\psi]}+\psi''\psi-\psi''-\psi''++1_1\right)\Gamma\left(\psi''-n''_{[\psi]}\right)}{\Gamma(\psi''\psi-\psi+1)}\left\{\sum_{1=0}^{R-\psi''-1}\binom{R-\psi-1}{1}(-1)^1 2^{(\psi''-1)(1-1)}\sum_{j_2=n''_{[\psi''+1]}}^{\psi''-1}\left(\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\right)^1\right\}$$

$$\left\langle\frac{\Gamma\left(n''_{[\psi+1]}+1(j_2-1)\Gamma\left(\psi''j_2+j_2-n''_{[\psi+1]}+1\right)\right)}{\Gamma\left(j_2\psi''+j_2+2+1(j_2-1)\right)}+\frac{\Gamma\left(n''_{[\psi+1]}\right)\Gamma\left(1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}{\Gamma\left(1(\psi''-1)+\psi''+1\right)}\right\rangle$$

Then:

$$S_i\left(n''_1, n''_\psi, n''_{\psi+1},\ldots,n''_R:\psi''\right)=\psi c+\omega\left[\underset{\tau\left(\underline{p}/\underline{n}\right)}{E}\left(p_{[med]}\right)-\hat{p}_i\right]$$

$$
\leq \psi c+\omega \left[\begin{aligned}
&\left(\frac{(\psi''-1)!}{\left(n''_{[\psi]}-1\right)!\left(\psi''-n''_{[\psi]}-1\right)!}\right)\left(\frac{(\psi''-1)!}{n''_{[\psi+1]}-1!\left(\psi''-n''_{[\psi+1]}-1\right)!}\right)\left(\frac{R!}{2(\psi-1)!(R-\psi-1)!}\right)\left\{2^{(\psi''-1)(\psi-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)}{(j_1-1)!(\psi''-j_1)!}\right\rangle^{\psi-1}\frac{1}{2^{(\psi-2)(\psi''-1-j_1)}}\left\langle\frac{\Gamma\left(n''_{[\psi]}+j_1\psi-j_1-\psi+2\right).\Gamma\left((\psi''-1)(1-j_1)+\left(\psi''-n''_{[\psi]}\right)\right)}{.\Gamma\left(n''_{[\psi]}+j_1\psi-j_1-\psi+2+\psi''+(\psi''-1)(1-j_1)\right)}\right\rangle+\right.\\
&\frac{\Gamma\left(n''_{[\psi]}+\psi''\psi-\psi''-\psi''+2_1\right).\Gamma\left(\psi''-n''_{[m]}\right)}{.\Gamma\left(n''_{[\psi]}+\psi''\psi-2\psi''-\psi+2\right)}\left\{\sum_{1=0}^{R-\psi''-1}\binom{R-\psi-1}{1}(-1)^1 2^{(\psi''-1)(1-1)}\left(\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\right)\frac{1}{2^{(1-1)(\psi''-1-j_2)}}\left\langle\frac{\Gamma\left(n''_{[\psi+1]}+1(j_2-1)\right)\Gamma\left((\psi''-1)(j_2-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}{\Gamma\left(n''_{[\psi+1]}+1(j_2-1)+(\psi''-1)(j_2-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}\right\rangle+\right.\\
&\frac{\Gamma\left(n''_{[\psi+1]}\right)\Gamma\left(1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}{\Gamma\left(n''_{[\psi+1]}+1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}+\left\{2^{(\psi''-1)(\psi-2)}\sum_{j_1=n''_{[\psi]}+1}^{\psi''-1}\left\langle\frac{(\psi''-1)!}{(j_1-1)!(\psi''-j_1)!}\right\rangle^{\psi-1}\frac{1}{2^{(\psi-2)(\psi''-1-j_1)}}\left\langle\frac{\Gamma\left(n''_{[\psi]}+j_1\psi-j_1-\psi''+1\right)\Gamma\left(\left(2\psi''-\psi''j_1-n''_{[\psi]}-1\right)\right)}{\Gamma\left(j_1\psi-\psi+2\psi''-\psi''j_1\right)}\right\rangle+\right.\\
&\frac{\Gamma\left(n''_{[\psi]}+\psi''\psi-\psi-\psi''+1_1\right)\Gamma\left(\psi''-n''_{[\psi]}\right)}{\Gamma(\psi''\psi-\psi+1)}\left\{\sum_{1=0}^{R-\psi''-1}\binom{R-\psi-1}{1}(-1)^1 2^{(\psi''-1)(1-1)}\sum_{j_2=n''_{[\psi+1]}}^{\psi''-1}\left(\frac{(\psi''-1)!}{j_2!(\psi''-1-j_2)!}\right)^1\right\}\left\langle\begin{aligned}&\frac{\Gamma\left(n''_{[\psi+1]}+1(j_2-1)\Gamma\left(\psi''j_2+j_2-n''_{[\psi+1]}+1\right)\right)}{\Gamma\left(j_2\psi''+j_2+2+1(j_2-1)\right)}+\\&\frac{\Gamma\left(n''_{[\psi+1]}\right)\Gamma\left(1(\psi''-1)+\left(\psi''-n''_{[\psi+1]}\right)\right)}{\Gamma\left(1(\psi''-1)+\psi''+1\right)}\end{aligned}\right\rangle-\frac{n''_i}{\psi''}
\end{aligned}\right]
$$

CONCLUSION

In this study, we introduce Bayes approximate to find median stopping risk in multinomial distribution where the size of sample is even with prior and posterior distribution.

RECOMMENDATIONS

Some directions for future research are given as follows: can use sample size as a group of observation to build Bayesian sequential scheme for the selection problem. General loss functions may be tried where linear loss is considered as a special case. In some problems the experimenter might be interested in selecting a subset of the cells including the median cell. In this problem a correct selection is the selection of any subset including the cell with ith median probability. Bayesian approach can be used to solve such as a problem. We can find one of measure of dispersion.

REFERENCES

Al-Hassan, K.F., 2006. Bayesian procedures for multinomial selection problem. Master Thesis, University of Karbala, Karbala, Iraq.

Al-Hassan, K.F., 2016. The usage of dispersion measure in bayesian selection approach. J. Babylon Univ., 24: 590-598.

Al-Hassan, K.F.A., 2010. New approach: Bayesian selection to find median cell (value) in multinomial population. J. Karbala Univ., 8: 1-11.

Bhayaa, E.S., 2003. On constraint and uncon straint approximation. Ph.D Thesis, University of Baghdad, Baghdad, Iraq.

Dor, D. and U. Zwick, 1999. Selecting the median. SIAM. J. Comput., 28: 1722-1758.

Horrace, W.C., 2000. Selection procedures for order statistics in empirical economic studies. Master Thesis, University of Arizona, Tucson, Arizona.

Madhi, S.A. and F.H. Kawther, 2007. Bayesian fixed sample size procedure for selecting the least probable event in multinomal distribution. J. Coll. Educ. Univ. Babylon, 36: 391-396.

Sultani, K., 2014. Approximation bayesian for selecting the least cell in multinomial population by functional analysis. J. Karbala Univ., 9: 1-8.