Research Journal of Applied Sciences 13 (10): 582-586, 2018
ISSN: 1815-932X


# On the Total Irregularity Strength of M-Copy Cycles and M-Copy Paths

[1]Corry Corazon Marzuki1, [1]Fitria Nia Gianita, [1]Ramadana Fitri, [2]Abdussakir and [1]Fitri Aryani
[1]Departement of Mathematics, Faculty of Science and Technology,
Universitas Islam Negeri Sultan Syarif Kasim Riau, JL. HR. Soebrantas No. 155 Simpang Baru,
Panam, 28293 Pekanbaru, Indonesia
[2]Departement of Mathematics, Faculty of Sciences and Technology,
Universitas Islam Negeri Maulana Malik Ibrahim Malang, Malang, Indonesia

**Abstract:** Let G = (V, E) be a graph. A totally irregular total k-labeling f: $V \cup E \rightarrow \{1, 2, ..., k\}$ of a graph G is a total labeling such that for any different vertices x and y of G, their weights wt. (x) and wt. (y) are distinct and for any different edges $x_1x_2$ and $y_1y_2$ of G, their weights wt. $(x_1x_2)$ and wt $(y_1y_2)$ are distinct. The weight wt (x) of a vertex x is the sum of the label of x and the labels of all edges incident with x. The weight wt. $(x_1x_2)$ of an edge $x_1x_2$ is the sum of the label of edge $x_1$ $x_2$ and the labels of vertices $x_1$ and $x_2$. The minimum k for which a graph G has a totally irregular total k-labeling is called the total irregularity strength of G, denoted by ts(G). In this study, we determine the total irregularity strength of M-copy cycles and M-copy paths.

**Key words:** Totally irregular total k-labeling, total irregularity strength, M-copy cycles, M-copy paths, weight, irregular

## INTRODUCTION

A labeling of a graph is a function that carries graph elements to the numbers, usually to the positive or non-negative integers that satisfies certain requirements. The most common choices of domain are the set of all vertices (vertex labeling), the set of all edges (edge labeling) or the set of all vertices and edges (total labeling). Other domain are possible.

Baca *et al.* (2007) introduced two kinds of irregular total labelings, namely, vertex irregular total labeling and edge irregular total labeling. Let G = (V, E) be a graph. A vertex irregular total k-labeling f: $V \cup E \rightarrow \{1, 2, 3, ..., k\}$ of G is a total labeling such that for any two different vertices x and y of G, their weights wt (x) and wt (y) are distinct, where the weight wt (x) of a vertex x is the sum of the label of x and the labels of all the edges incident with x. The minimum k for which a graph G has a vertex irregular total k-labeling is called the total vertex irregularity strength of G, denoted by tvs(G). An edge irregular total k-labeling f: $V \cup E \rightarrow \{1, 2, ..., k\}$ of G is a total labeling such that for any two different edges $x_1x_2$ and $y_1y_2$ of G, their weights wt. $(x_1 x_2)$ and wt. $(y_1 y_2)$ are distinct where the weight wt. $(x_1x_2)$ of an edge $x_1x_2$ is the sum of the label of $x_1x_2$ and the labels of the vertices $x_1$ and $x_2$. The minimum k for which a graph G has an edge irregular total k-labeling is called the total edge irregularity strength of G, denoted by tes(G).

Baca *et al.* (2007) obtained lower bound and upper bound of total vertex irregularity strength of a graph G such as the following theorem.

## MATERIALS AND METHODS

**Theorem 1.1; Baca *et al.* (2007):** Let G be a graph (p, q) with minimum degree δ and maximum degree Δ, then:

$$\left\lceil \frac{p+\delta}{\Delta+1} \right\rceil \leq tvs(G) \leq p+\Delta-2\delta+1$$

Total vertex irregularity strength of t-copy of path has been determined by Nurdin *et al.* (2009) such as the following theorem.

**Theorem 1.2; Nurdin *et al.* (2009):** Let graph $tP_n$ be t-copy of path with n vertices where $t \geq 2$, then:

**Corresponding Author:** Corry Corazon Marzuki, Departement of Mathematics, Faculty of Science and Technology, Universitas Islam Negeri Sultan Syarif Kasim Riau, JL. HR. Soebrantas No. 155 Simpang Baru, Panam, 28293 Pekanbaru, Indonesia

$$\text{tvs}(tP_n) = \begin{cases} t, & \text{for } n = 1; \\ t+1, & \text{for } 2 \le n \le 3; \\ \left\lceil \frac{nt+1}{3} \right\rceil, & \text{for } n \ge 4 \end{cases}$$

Baca *et al*. (2007) also, determined lower bound and upper bound of total edge irregularity strength of a graph G as stated in the following theorem.

**Theorem 1.3; Baca *et al.* (2007):** Let G = (V, E) be a graph with the set of vertices V and the non empty set of Edges E:

$$\left\lceil \frac{|E|+2}{3} \right\rceil \le \text{tes}(G) \le |E|$$

In 2012, combination of the both labelings, namely totally irregular total k-labeling has introduced by Marzuki *et al*. (2013). Let G = (V, E) be a graph. A totally irregular total k-labeling f: V∪E→{1, 2, ..., k} of G is a total labeling such that for any two different vertices x and y of G, the weights wt. (x) and wt. (y) are distinct and for any two different edges $x_1x_2$ and $y_1y_2$ of G, the weights wt. ($x_1$ $x_2$) and wt. ($y_1y_2$) are distinct. The minimum k for which a graph G has a totally irregular total k-labeling is called the total irregularity strength of G, denoted by ts (G).

**Theorem 1.4; Marzuki *et al.* (2013):** For every graph G, we have maxftes(G); tυs(G)g≤ts(G). After that, Ramdani *et al*. (2015a, b) also, observed about the total irregularity strength of M-copy of stars by Ramdani (2014) total irregularity strength of three families of graph by and total irregularity strength of regular graph. Two theorems which have resulted by Ramdani *et al*. (2015) are:

**Theorem 1.5; Ramdani *et al.* (2015a, b):** Let $P_2$ be a path with 2 vertices. Then, ts($mP_2$) = m+1 for m≥1.

**Theorem 1.6; Ramdani *et al.* (2015 a, b):** Let Cn be a cycle of order n. For n≥3 and n ≡ (0mod3), ts($mC_n$) = ⌈mn+2/3⌉.

## RESULTS AND DISCUSSION

Graph $mP_3$ is a graph which is obtained by copying graph $P_3$ as much as m times in which the set of vertices of each copy are disjoint. Let the ith copy results of graph $P_3$ is $υ_i$, $1_{ei}$, $1υ_i$, $2_{ei}$, $2_{υi}$, 3 where $e_i$, j = $υ_{i,j}$ $υ_{i,j}$+1 for 1≤i≤m and 1≤j≤3. The following theorem will be discussed about total irregularity strength of graph $mP_3$, for positive integer m with m≥2.

**Theorem 2.1:** Let $mP_3$ be m-copy of path with three vertices. For m≥2, ts($mP_3$) = m+1.

**Proof:** Note that the number of edges of graph $mP_3$ is |E ($mP_3$)| = 2m. According to theorem (1.3), we have tes ($mP_3$)≥⌈2m+2/3⌉. While based on Theorem (1.1), we get tυs ($mP_3$)≥m+1.

According to theorem (1.4), we have ts($mP_3$)≥max {⌈2m+2/3⌉, m+1. Because of m+1>⌈2m+2/3⌉ for every m∈$Z^+$ then ts($mP_3$)≥m+1.

Next, we will prove that ts($mP_3$)≤m+1 by showing that there is a totally irregular total (m+1)-labeling on graph $mP_3$ such as the following:

$$\lambda(\upsilon_{i,j}) = \begin{cases} 1, & \text{for } 1\le i\le m \text{ and } j = 1; \\ m, & \text{for } 1\le i\le m \text{ and } j = 2; \\ m+1-i, & \text{for } 1\le i\le m \text{ and } j = 3; \end{cases}$$

$$\lambda(e_{i,j}) = \begin{cases} i, & \text{for } 1\le i\le m \text{ and } j = 1; \\ m+1, & \text{for } 1\le i\le m \text{ and } j = 2 \end{cases}$$

Easy to check that the weights of vertices and edges of graph $mP_3$ under labeling λ are as follows:

$$wt(e_{i,j}) = \begin{cases} m+1+i, & \text{for } 1\le i\le m \text{ and } j = 1; \\ 3m+2-i, & \text{for } 1\le i\le m \text{ and } j = 2; \end{cases}$$

$$wt(\upsilon_{i,j}) = \begin{cases} 1+i, & \text{for } 1\le i\le m \text{ and } j = 1; \\ 2m+2-i, & \text{for } 1\le i\le m \text{ and } j = 2; \\ 2m+2-i, & \text{for } 1\le i\le m \text{ and } j = 3 \end{cases}$$

Observe that λ is a labeling from V($mP_3$)∪E($mP_3$) into {1, 2, ..., m+1} such that no two vertices have the same weight and no two edges have the same weight. So, λ is a totally irregular total (m+1)-labeling. We conclude that ts($mP_3$)≤m+1.

Let $mC_n$ is a graph which is obtained by copying graph $C_n$ as much as m times in which the set of vertices of each copy are disjoint. In the following theorem, we will discuss about total irregularity strength of $mC_n$ for m = 2, 3 and n = 1 (mod3).

**Theorem 2.2:** Let $mC_n$ be m-copy of cycle with n vertices, then for m = 2, 3 and n ≡ 1 (mod3), ts($mC_n$) = ⌈mn+2/3⌉.

**Proof:** The number of edges of graph $mC_n$ is |E($mC_n$)| = mn. Based on Theorem (1.3), we have tes ($mC_n$)≥⌈mn+2/3⌉. Next, according to Theorem (1.1), we get tυs ($mC_n$)≥⌈mn+2/3⌉. While based on Theorem (1.4), we have ts($mC_n$)≥max{tes($mC_n$), tυs(mC )}. Because of tes ($mC_n$)≥⌈mn+2/3⌉ and tυs($mC_n$)≥⌈mn+2/3⌉ for every m = 2, 3 and n ≡ 1 (mod3), then ts ($mC_n$)≥⌈mn+2/3⌉.

Next, we will prove that $ts(mC_n) \geq \lceil mn+2/3 \rceil$ by showing that there is a totally irregular total $\lceil mn+2/3 \rceil$ labeling on graph $mC_n$.

Let the set of vertices of graph mCn is; $V(mC_n) = \{\upsilon_i | 1 \leq i \leq mn$ with $m = 2, 3$ and $n = 1 \pmod 3\}$ and the set of edges of graph mCn is; $E(mC_n) = \{\upsilon_i | 1 \leq i \leq mn$ with $m = 2, 3$ and $n = 1 \pmod 3\}$, construct a labeling f such as the following:

$$f(\upsilon_i) = f(e_i) = \begin{cases} \left\lceil \frac{i+1}{3} \right\rceil, & \text{for } 1 \leq i \leq 2n\text{-}1 \\ \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } 2n \leq i \leq 3n \end{cases}$$

Based on this labeling, we have the weights of vertices and edges of graph $mC_n$ are as follows:

$$wt(\upsilon_i) = wt(e_i) = \begin{cases} 3, & \text{for } i = 1; \\ \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i}{3} \right\rceil + \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } 2 \leq i \leq n\text{-}1; \\ 2\left\lceil \frac{n+1}{3} \right\rceil + \left\lceil \frac{n}{3} \right\rceil, & \text{for } i = n; \\ 2\left\lceil \frac{n+2}{3} \right\rceil + \left\lceil \frac{n+3}{3} \right\rceil, & \text{for } i = n+1; \\ \left\lceil \frac{2n}{3} \right\rceil + \left\lceil \frac{2n+1}{3} \right\rceil + \left\lceil \frac{2n+2}{3} \right\rceil, & \text{for } i = 2n\text{-}1; \\ 2n+2, & \text{for } i = 2n; \\ 2n+4, & \text{for } i = 2n+1; \\ \left\lceil \frac{i+2}{3} \right\rceil + \left\lceil \frac{i+1}{3} \right\rceil + \left\lceil \frac{i+3}{3} \right\rceil, & \text{for } i = 2n+2 \leq i \leq 3n\text{-}1; \\ 3n+3, & \text{for } i = 3n \end{cases}$$

The weights of vertices and edges of graph $mC_n$ for $m = 2, 3$ and $n \equiv 1 \pmod 3$ are:

- For i = 1, the weights of vertices and edges of graph $mC_n$ are 3
- For $2 \leq i \leq n\text{-}1$, the weights of vertices and edges of graph $mC_n$ are consecutive integer from 4 until $\lceil n/3 \rceil + \lceil n\text{-}1/3 \rceil + \lceil n+1/3 \rceil = n+1$
- For i = n, the weights of vertices and edges of graph $mC_n$ are $2\lceil n+1/3 \rceil + \lceil n/3 \rceil = n+2$
- For i = n+1, the weights of vertices and edges of graph $mC_n$ are $2\lceil n+2/3 \rceil + \lceil n+3/3 \rceil = n+3$
- For $n+2 \leq i \leq 2n\text{-}2$, the weights of vertices and edges of graph mCn are consecutive integer from $\lceil n+3/3 \rceil + \lceil n+2/3 \rceil + \lceil n+4/3 \rceil = n+4$ until $\lceil 2n\text{-}1/3 \rceil + \lceil 2n/3 \rceil = 2n$
- For i = 2n-1, the weights of vertices and edges of graph $mC_n$ are $\lceil 2n/3 \rceil + \lceil 2n\text{-}1/3 \rceil + \lceil 2n+2/3 \rceil = 2n+2$
- For i = 2n, the weights of vertices and edges of graph mCn are 2n+3
- For i = 2n+1, the weights of vertices and edges of graph $mC_n$ are 2n+4
- For $2n+2 \leq i \leq 3n\text{-}1$, the weights of vertices and edges of graph $mC_n$ are consecutive integer from $2\lceil 2n+4/3 \rceil + \lceil 2n+5/3 \rceil = 2n+5$ until 3n+2
- For i = 3n, the weights of vertices and edges of graph $mC_n$ are 3n+3

So, the weights of vertices and edges of graph $mC_n$ for m = 2, 3 and $n \equiv 1 \pmod 3$ are consecutive integer from 3 until 2n and consecutive integer from 2n+2 until 3n+3.

Observe that f is a labeling from $V(mC_n) \cup E(mC_n)$ into $\{1, 2, ..., \lceil mn+2/3 \rceil\}$ such that no two vertices have the same weight and no two edges have the same weight. So, f is a totally irregular total $\lceil mn+2/3 \rceil$-labeling. We conclude that $ts(mC_n) \leq \lceil mn+2/3 \rceil$ for m = 2, 3 and $n \equiv 1 \pmod 3$.

Let $mC_4$ is a graph which is obtained by copying graph $C_4$ as much as m times in which the set of vertices of each copy are disjoint. Let the ith copy results of graph $C_4$ is $\upsilon_{4i}\text{-}3$, $e_{4i}\text{-}3$, $\upsilon_{4i}\text{-}2$, $e_{4i}\text{-}1$, $\upsilon_{4i}$, $e_{4i}$, $\upsilon_{4i}\text{-}1$, $e_{4i}\text{-}2$, $\upsilon_{4i}\text{-}3$ for $1 \leq i \leq m$. The following theorem will be discussed about total irregularity strength of graph $mC_4$ for positive integer m with $m \leq 4$.

**Theorem 2.3:** Let $mC_4$ be m-copy of cycle of order 4. For $m \leq 4$, $ts(mC_4) = \lceil 4m+2/3 \rceil$.

**Proof:** According to Theorem (1.1), $t\upsilon s(mC_4) \leq \lceil 4m+2/3 \rceil$. While based on Theorem (1.3), $t\upsilon s(mC_4) \leq \lceil 4m+2/3 \rceil$. By Theorem (1.4), we have $t\upsilon s(mC_4) \leq \lceil 4m+2 \rceil$. Next, we will prove that $t\upsilon s(mC_4) \leq \lceil 4m+2 \rceil$ by showing there is a totally irregular total $\lceil 4m+2 \rceil$-labeling on $mC_4$ such as following: For $m \equiv 1 \pmod 3$ and $m \equiv 2 \pmod 3$:

$$\lambda(\upsilon_i) = \lambda(e_i) = \begin{cases} \left\lceil \frac{i+2}{3} \right\rceil, & \text{for } i \equiv 8 \pmod{12}; \\ \left\lceil \frac{i+1}{3} \right\rceil, & \text{for } i \equiv 9 \pmod{12};\ i \equiv 10 \pmod{12}, \\ & \text{atau } i \equiv 0 \pmod{12}; \\ \left\lceil \frac{i+5}{3} \right\rceil, & \text{for } i \equiv 11 \pmod{12}; \\ \left\lceil \frac{i+1}{3} \right\rceil, & \text{for others} \end{cases}$$

For m≡ 0 (mod 3):

$$\lambda(v_i)=\lambda(e_i)=\begin{cases}\left\lceil\frac{i+2}{3}\right\rceil, & \text{for } i=8 \text{ or } i=20;\\ \left\lceil\frac{i-1}{3}\right\rceil, & \text{for } i=9,10,12 \text{ or}\\ & i\equiv 6(\text{mod}12)\text{ with } i\geq 30;\\ \left\lceil\frac{i+5}{3}\right\rceil, & \text{for } i=11;\\ \left\lceil\frac{i+4}{3}\right\rceil, & \text{for } i=11(\text{mod}12)\text{ and } i\neq 11;\\ \left\lceil\frac{i+3}{3}\right\rceil, & \text{for } i\equiv 2(\text{mod}12)\text{ with } i\geq 26\\ & \text{or } i\equiv 4(\text{mod}12)\text{ with } i\geq 28;\\ \left\lceil\frac{i+1}{3}\right\rceil, & \text{for } i\equiv 7(\text{mod}12)\text{ with } i\geq 31;\\ \left\lceil\frac{i+3}{3}\right\rceil, & \text{for others}\end{cases}$$

By using this labeling, we get the weights of vertices and edges such as follows: For m = 1 (mod 3) and m = 2 (mod3):

$$wt(v_i)=wt.(e_i)=\begin{cases}2\left\lceil\frac{i-1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil, & \text{for } i\equiv 9(\text{mod}12);\\ 2\left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil, & \text{for } i\equiv 1(\text{mod }12)\text{ or}\\ & i\equiv 5(\text{mod}12);\\ \left\lceil\frac{i-1}{3}\right\rceil+\left\lceil\frac{i-2}{3}\right\rceil+\left\lceil\frac{i+6}{3}\right\rceil, & \text{for } i\equiv 10(\text{mod }12);\\ \left\lceil\frac{i+5}{3}\right\rceil+\left\lceil\frac{i-2}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil, & \text{for } i\equiv 11(\text{mod }12);\\ \left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+\left\lceil\frac{i+3}{3}\right\rceil, & \text{for } i\equiv 7(\text{mod }12);\\ \left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil, & \text{for } i\equiv 2(\text{mod }12),\ i\equiv 3\\ & (\text{mod }12),\ i\equiv 6(\text{mod }12);\\ 2\left\lceil\frac{i+2}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil, & \text{for } i\equiv 8(\text{mod }12);\\ 2\left\lceil\frac{i-1}{3}\right\rceil+\left\lceil\frac{i+4}{3}\right\rceil, & \text{for } i\equiv 0(\text{mod }12);\\ 2\left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil, & \text{for } i\equiv 4(\text{mod }12)\end{cases}$$

For m≡ 0 (mod3):

$$\begin{matrix}wt(v_i)\\=wt(e_i)\end{matrix}=\begin{cases}2\left\lceil\frac{i-1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil, & \text{for } i=9;\\ 3\left\lceil\frac{i+1}{3}\right\rceil+1, & \text{for } i\equiv 1(\text{mod}12)\text{ with } i\geq 25;\\ 2\left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil, & \text{for } i=4,\ i=4, i=16 \text{ or}\\ & i\equiv 5(\text{mod}12)\text{ with } i\geq 29;\\ 2\left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil, & \text{for } i=1,5,13,17 \text{ or}\\ & (i\equiv 9(\text{mod}12)\text{ and } i\neq 9);\\ \left\lceil\frac{i-1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+\left\lceil\frac{i+1}{3}\right\rceil+2, & \text{for } i\equiv 6(\text{mod}12)\text{ with } i\geq 30;\\ \left\lceil\frac{i-1}{3}\right\rceil+\left\lceil\frac{i-2}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+2, & \text{for } i=10;\\ \left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil+1, & \text{for } i\equiv 10(\text{mod}12)\text{ and } i\neq 10;\\ & \text{or } i\equiv 11(\text{mod}12)\text{ and } i\neq 11;\\ 2\left\lceil\frac{i}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil+1, & \text{for } i\equiv 2(\text{mod}12)\text{ with } i\geq 26;\\ \left\lceil\frac{i+2}{3}\right\rceil+\left\lceil\frac{i-2}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+1, & \text{for } i=11;\\ 2\left\lceil\frac{i+1}{3}\right\rceil+2\left\lceil\frac{i}{3}\right\rceil+1, & \text{for } i=7 \text{ or } i=19;\\ \left\lceil\frac{i}{3}\right\rceil+\left\lceil\frac{i-2}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil+2, & \text{for } i\equiv 7(\text{mod }12)\text{ with } i\geq 31;\\ 2\left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil+1, & \text{for } i\equiv 3(\text{mod }12)\text{ with } i\geq 27;\\ & \text{or } i\equiv 8(\text{mod }12)\text{ and } i\geq 32;\\ \left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+\left\lceil\frac{i+2}{3}\right\rceil, & \text{for } i=2,3,6,14,15 \text{ or } 18;\\ 2\left\lceil\frac{i+2}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil, & \text{for } i=8 \text{ or } 20;\\ 2\left\lceil\frac{i-1}{3}\right\rceil+\left\lceil\frac{i+4}{3}\right\rceil, & \text{for } i=12;\\ 3\left\lceil\frac{i}{3}\right\rceil+2, & \text{for } i=49(\text{mod}12)\text{ with } i\geq 28;\\ 2\left\lceil\frac{i+1}{3}\right\rceil+\left\lceil\frac{i}{3}\right\rceil+1, & \text{for } i\equiv 0(\text{mod }12)\text{ and } i\neq 12\end{cases}$$

## CONCLUSION

Observe that λ is a labeling from V ($mC_4$) ∪ E ($mC_4$) into {1, 2, ..., ⌈4m+2/3⌉} such that no two vertices have the same weight and no two edges have the same weight. So, $\lambda$ is a totally irregular total $\lceil 4m+2/3 \rceil$-labeling. We conclude that $tvs\ (mC_4) \leq \lceil 4m+2/3 \rceil$.

## ACKNOWLEDGEMENTS

The reserchers would like to thank the Faculty of Science and Technology, Universitas Islam Negeri Sultan Syarif Kasim (UIN Suska) Pekanbaru, Riau, Indonesia for support.

## REFERENCES

Baca, M., M. Miller and J. Ryan, 2007. On irregular total labellings. Discrete Math., 307: 1378-1388.

Marzuki, C.C., A.N.M. Salman and M. Miller, 2013. On the total irregularity strength of cycles and paths. Far East J. Math. Sci., 82: 1-21.

Nurdin, Salman, A.N.M., N.N. Gaos and E.T. Baskoro, 2009. On the total Vertex-irregular strength of a disjoint union of t copies of a path. J. Comb. Math. Comb. Comput., 71: 227-233.

Ramdani, R., 2014. The totally irregularity strength of M-copy stars. J. Math. Sci. Technol., 8: 1-16.

Ramdani, R., A.N.M. Salman and H. Assiyatun, 2015b. On the total irregularity strength of regular graphs. J. Math. Fundam. Sci., 47: 281-295.

Ramdani, R., A.N.M. Salman, H. Assiyatun, A. Semanicova-Fenovcikova and M. Baca, 2015a. Total irregularity strength of three families of graphs. Math. Comput. Sci., 9: 229-237.