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Total Vertex Irregularity Strength of Comb Product Graph of P_m and C_n

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Abstract: A vertex irregular total k-labeling of a graph G (V, E) with a non-empty set V of vertices and a set E of edges is a labeling $\lambda: V \cup E \rightarrow \{1,2, ..., k\}$ such that for every two distinct vertices have different weight. The weight of a vertex v, under a total labeling λ is the sum of label of vertex v and all labels of edges that incident with v. In other word, $wt(x) = \lambda(x)+\Sigma_{ux\in E}\lambda(ux)$. The total vertex irregularity strength, denoted by tvs(G) is the minimum biggest label that use to label graph G with the vertex irregular total labeling. Some classes of graphs have been obtained its total vertex irregularity strength. In this study, researcher observe about the total vertex irregularity strength of comb product graph of P_m and C_n, denoted by TVs ($P_m \triangleright C_n$). The result of this research is $tvs(Pm \triangleright Cn) = \lceil (n-1)\, m+2/3 \rceil$ for $m \geq 3$ forand odd number m.

Key words: Total labeling, comb product, vertex irregularity strength, vertices, graphs, weight, edges

INTRODUCTION

Let G = (V, E) is a simple graph. Graph labeling is a function that carries graph elements to the numbers, usually to the positive or non-negative integers that satisfies a certain requirement. The most common choices of domain are the set of all vertices (vertex labeling), the set of all edges (edge labeling) or the set of all vertices and edges (total labeling). Other domain are possible.

The labeling of graphs was introduced by Sedlacek in 1963. Based on the weight of the graph elements, the labeling divided into several types, they are graceful labeling, magic labeling, antimagic labeling and irregular labeling, etc. Irregular labeling has been introduced by Chartrand. But, their study titled "Irregular Network" was published in 1988.

Baca *et al*. (2007) introduced two kinds of irregular total labeling, namely, vertex irregular total labeling and edge irregular total labeling. Let G = (V, E) be a graph, function $f: V\cup E \rightarrow \{1, 2, 3, ..., k\}$ is called vertex irregular total k-labeling in if every two different vertices in V have different weight. The weight of vertex x in V under function f is $wt(x) = f(x)+\Sigma_{xy\in E} f(xy)$. The minimum k for which a graph G has a the vertex irregular total k-labeling is called total vertex irregularity strength of G, denoted by TVs(G). Baca *et al*. (2007) obtained lower bound and upper bound of total vertex irregularity strength of a graph G such as the following theorem.

Theorem 1: Let G be a graph (p, q) with minimum degree δ and maximum degree Δ then (Baca *et al*., 2007):

$$\left\lceil \frac{p+\delta}{\Delta+1} \right\rceil \leq tvs(G) \leq p+\Delta-2\delta+1$$

MATERIALS AND METHODS

Determination of total vertex irregularity strength of all of graphs has not done completely. Until now, only several of class of graph have been determined its total vertex irregularity strength. Baca *et al*. (2007) observed about total vertex irregularity strength of cycles, paths, stars, complete graphs and prism graphs. By Baskoro *et al*. (2010) determined a new upper bound for the total vertex irregularity strength of graphs and found total vertex irregularity strength of forest. Nurdin also investigated the total vertex irregularity strength of trees. And then, Ahmad *et al*. (2011, 2012, 2014) obtained the total vertex irregularity strength of wheel related graphs, disjoint union of helm graphs and certain classes of unicyclic graph. By Al-Mushayt *et al*. (2013) also observed about total vertex irregularity strength of convex polytope graphs.

Baca *et al*. (2007) and Anholcer *et al*. (2009, 2010) obtained some results about the total vertex irregularity strength and the total edge irregularity strength of cycles and paths as stated in the following theorems.

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Theorem 2: Let P_m and C_n be a path and a cycle, respectively with $n\geq 1$ vertices (Baca *et al*., 2007). Then:

$$tes(P_m) = \left\lceil \frac{n+1}{3} \right\rceil$$

and:

$$tes(C_n) = \left\lceil \frac{n+2}{3} \right\rceil$$

Theorem 3: Let P_n be a path on n: vertices. Then:

$$tvs(P_n) = \left\lceil \frac{n+1}{3} \right\rceil$$

Corollary 4: Let be an n-regular graph on n vertices (Baca *et al*., 2007). Then:

$$\left\lceil \frac{n+r}{1+r} \right\rceil \leq tvs(G) \leq n\text{-}r+1$$

Hence, we have:

$$tvs(C_n) = tes(C_n) = \left\lceil \frac{n+2}{3} \right\rceil$$

RESULTS AND DISCUSSION

We will determine total vertex irregularity strength of comb product graph of P_m and C_n denoted by $tvs(P_m \triangleright C_n)$ for $m\geq 3$ and $n\geq 2$ such as the following theorem.

Theorem 3: For $m\geq 3$ and $n\geq 2$ then:

$$tvs(P_m \triangleright C_n) = \left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil$$

Proof : First, will be show $tvs(P_m \triangleright C_n) \geq \lceil (n\text{-}1)m+2/3 \rceil$. Note that the smallest degree of vertices of graph $P_m \triangleright C_n$ is 2 and sum of vertices whose degree 2 in $P_m \triangleright C_n$ is (n-1)m. To get optimal labeling, the weight of each vertices whose degree 2 are consecutive integer from 3, 4, 5, ..., (n-1)m+2. While the weight of graph $P_m \triangleright C_n$ whose degree 2 is the sum of 3 positive integers which is called label, that is a label of that vertex and two labels of edges which is associated with that vertex. Therefore, we get the largest minimum label which is used is $\lceil (n\text{-}1)(m+2)/3 \rceil$ and it isn't possible to have been smaller than $\lceil (n\text{-}1)m+2/3 \rceil$. So, we conclude that:

$$tvs(P_m \triangleright C_n) \geq \left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil$$

Next, we will prove that:

$$tvs(P_m \triangleright C_n) \leq \left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil$$

By showing there exist a vertex irregular total $\lceil (n\text{-}1)m+2/3 \rceil$-labeling of graph $P_m \triangleright C_n$. Let the set of vertices of graph is:

$$V = (P_m \triangleright C_n) = \{x_j, y_i^j \mid 1 \leq j \leq m \text{ dan } 1 \leq i \leq n-1\}$$

And the set of edges of graph $P_m \triangleright C_n$ is:

$$E(P_m \triangleright C_n) = \{x_j, y_i^j \mid 1 \leq j \leq m \text{ and } i = n\text{-}2,\ n\text{-}1\} \cup \{y_i^j y_{i+2}^j \mid 1 \leq i \leq n\text{-}3 \text{ and } 1 \leq j \leq m\} \cup \{y_1^j y_2^j \mid 1 \leq j \leq m\} \cup \{x_j x_{j+1} \mid 1 \leq i \leq j \leq m\text{-}2\} \cup \{x_m x_1\}$$

Construct a vertex irregular total $\lceil (n\text{-}1)m+2/3 \rceil$-labeling as stated in the following. Let:

$$r_j = \left[\frac{(n\text{-}1)j+2}{3} \right] \text{ untuk } 1 \leq j \leq m$$

$$\lambda(x_j) = \begin{cases} (n-1)m + j + 4 - 2\left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil - \left\lceil \frac{(n\text{-}1)j+2}{3} \right\rceil - \left\lceil \frac{(n\text{-}1)j+1}{3} \right\rceil; \text{if it is positive and } 1 \leq j \leq m\text{-}2 \\ 1; \text{ if } m(n\text{-}1)+j+4\text{-}2\left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil \text{-} \left\lceil \frac{(n\text{-}1)j+2}{3} \right\rceil - \left\lceil \frac{(n\text{-}1)j+1}{3} \right\rceil \text{ is nonpositive and } 1 \leq j \leq m\text{-}2 \\ (n\text{-}1)m+3\text{-}\left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil \text{-} \left\lceil \frac{(n\text{-}1)m\text{-}n+3}{3} \right\rceil - \left\lceil \frac{(n\text{-}1)m\text{-}n+2}{3} \right\rceil; \text{ if } j = m\text{-}1 \\ (n\text{-}1)m+4\text{-}\left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil \text{-} \left\lceil \frac{(n\text{-}1)m+1}{3} \right\rceil - \left\lceil \frac{(n\text{-}1)m+2}{3} \right\rceil; \text{ if } j = m \end{cases}$$

$$\lambda(y_i^j) = \begin{cases} 1; \text{ if } i = 1 \text{ and } j = 1 \\ \left\lceil \frac{2n\text{-}2+(j\text{-}3)(n\text{-}1)}{3} \right\rceil \text{ if } i = 1 \text{ and } 2 \leq j \leq m \\ \left\lceil \frac{i+n+(j\text{-}2)(n\text{-}1)}{3} \right\rceil \text{ if } 2 \leq i \leq n\text{-}1 \text{ and } 1 \leq j \leq m \end{cases}$$

$$\lambda(y_{n-2}^{\ j} x_j) = \left\lceil \frac{2n-1+(j-2)(n-1)}{3} \right\rceil$$

$$\lambda(y_{n-1}^{\ j} x_j) = \left\lceil \frac{2n+(j-2)(n-1)}{3} \right\rceil$$

$$\lambda(y_i^j y_{i+2}^{\ j}) = \left\lceil \frac{i+(n+1)+(j-2)(n-1)}{3} \right\rceil \text{ for } 1 \le i \le n\text{-}3 \text{ and } 1 \le j \le m$$

$$\lambda(y_1^j y_2^j) = \left\lceil \frac{2n+(j-3)(n-1)}{3} \right\rceil \text{ for } 1 \le j \le m$$

$$\lambda(x_j x_{j+1}) = r_m \text{ for } 1 \le j \le m\text{-}2$$

$$\lambda(x_m x_1) = r_m$$

Based on this labeling, we have the weight of the vertices of graph such as following:

$$\begin{aligned}
wt(y_1^1) &= \lambda(y_1^1)+\lambda(y_1^1 y_3^1)+\lambda(y_1^1 y_2^1) \\
&= 1+\left\lceil \frac{1+n+1+(-1)(n-1)}{3} \right\rceil+\left\lceil \frac{2n+(-2)(n-1)}{3} \right\rceil \\
&= 1+\left\lceil \frac{n+2-n+1}{3} \right\rceil+\left\lceil \frac{2n-2n+2}{3} \right\rceil \\
&= 1+\left\lceil \frac{3}{3} \right\rceil+\left\lceil \frac{2}{3} \right\rceil \\
&= 3
\end{aligned}$$

$$\begin{aligned}
wt(y_1^j) &= \lambda(y_1^j)+\lambda(y_1^j y_3^j)+\lambda(y_1^j y_2^j) \\
&= \left\lceil \frac{2n-2+(j-3)(n+1)}{3} \right\rceil+\left\lceil \frac{1+n+1+(j-2)(n-1)}{3} \right\rceil+\left\lceil \frac{2n+(j-3)(n-1)}{3} \right\rceil \\
&= \left\lceil \frac{2n-2+jn-j-3n+3}{3} \right\rceil+\left\lceil \frac{n+2+jn-j-2n+2}{3} \right\rceil+\left\lceil \frac{2n+jn-j-3n+3}{3} \right\rceil \\
&= \left\lceil \frac{-n+1+j(n-1)}{3} \right\rceil+\left\lceil \frac{-n+4+j(n-1)}{3} \right\rceil+\left\lceil \frac{-n+3+j(n-1)}{3} \right\rceil \\
&= \left\lceil \frac{-(n-1)+j(n-1)}{3} \right\rceil+\left\lceil \frac{-(n-1)+j(n-1)+3}{3} \right\rceil+\left\lceil \frac{-(n-1)+j(n-1)+2}{3} \right\rceil \\
&= \left\lceil \frac{(j-1)(n-1)}{3} \right\rceil+\left\lceil \frac{(j-1)(n-1)+3}{3} \right\rceil+\left\lceil \frac{(j-1)(n-1)+2}{3} \right\rceil \text{ for } 2 \le j \le m
\end{aligned}$$

$$\begin{aligned}
wt(y_2^j) &= \lambda(y_2^j)+\lambda(y_2^j y_4^j)+\lambda(y_1^j y_2^j) \\
&= \left\lceil \frac{2+n+(j-2)(n-1)}{3} \right\rceil+\left\lceil \frac{2+n+1+(j-2)(n-1)}{3} \right\rceil+\left\lceil \frac{2n+(j-3)(n-1)}{3} \right\rceil \\
&= \left\lceil \frac{2+n+jn-j-2n+2}{3} \right\rceil+\left\lceil \frac{2+n+1+jn-j-2n+2}{3} \right\rceil+\left\lceil \frac{2n+jn-j-2n+3}{3} \right\rceil \\
&= \left\lceil \frac{-n+4+j(n-1)}{3} \right\rceil+\left\lceil \frac{-n+5+j(n-1)}{3} \right\rceil+\left\lceil \frac{-n+3+j(n-1)}{3} \right\rceil \\
&= \left\lceil \frac{-(n-1)+j(n-1)+3}{3} \right\rceil+\left\lceil \frac{-(n-1)+j(n-1)+4}{3} \right\rceil+\left\lceil \frac{-(n-1)+j(n-1)+2}{3} \right\rceil \\
&= \left\lceil \frac{(n-1)(j-1)+3}{3} \right\rceil+\left\lceil \frac{(n-1)(j-1)+4}{3} \right\rceil+\left\lceil \frac{(n-1)(j-1)+2}{3} \right\rceil
\end{aligned}$$

$$\begin{aligned}
wt(y_i^j) &= \lambda(y_i^j)\lambda(y_{i-2}^j y_i^j)+\lambda(y_i^j y_{i+2}^j) \\
&= \left\lceil \frac{i+n+(j-2)(n-1)}{3} \right\rceil+\left\lceil \frac{i-2+(n+1)+(j-2)(n-1)}{3} \right\rceil+\left\lceil \frac{i+(n+1)+(j-2)(n-1)}{3} \right\rceil \\
&= \left\lceil \frac{i+n+nj-j-2n+2}{3} \right\rceil+\left\lceil \frac{i-2+n+1+nj-j-2n+2}{3} \right\rceil+\left\lceil \frac{i+n+1+nj-j-2n+2}{3} \right\rceil \\
&= \left\lceil \frac{i-n+2+j(n-1)}{3} \right\rceil+\left\lceil \frac{i-n+1+j(n-1)}{3} \right\rceil+\left\lceil \frac{i-n+3+j(n-1)}{3} \right\rceil \\
&= \left\lceil \frac{(j-1)(n-1)+i+1}{3} \right\rceil+\left\lceil \frac{(j-1)(n-1)+i}{3} \right\rceil+\left\lceil \frac{(j-1)(n-1)+i}{3} \right\rceil
\end{aligned}$$

for $1 \le j \le m$ and $3 \le i \le n\text{-}1$

$$\begin{aligned}
wt(x_m) &= \lambda(x_m)+\lambda(x_m x_1)+\lambda(y_{n-2}^m x_m)+\lambda(y_{n-1}^m x_m) \\
&= (n-1)m+4-\left\lceil \frac{(n-1)m+2}{3} \right\rceil-\left\lceil \frac{(n-1)m+1}{3} \right\rceil-\left\lceil \frac{(n-1)m+2}{3} \right\rceil+\left\lceil \frac{(n-1)m+2}{3} \right\rceil+\left\lceil \frac{2n-1+(m-2)(n-1)}{3} \right\rceil+\left\lceil \frac{2n+(m-2)(n-1)}{3} \right\rceil \\
&= (n-1m+4)-\left\lceil \frac{(n-1)m+2}{3} \right\rceil-\left\lceil \frac{(n-1)m+1}{3} \right\rceil+\left\lceil \frac{2n-1+m(n-1)-2n+2}{3} \right\rceil+\left\lceil \frac{2n+m(n-1)-2n+2}{3} \right\rceil \\
&= (n-1)m+4-\left\lceil \frac{(n-1)m+2}{3} \right\rceil-\left\lceil \frac{(n-1)m+1}{3} \right\rceil+\left\lceil \frac{(n-1)m+1}{3} \right\rceil+\left\lceil \frac{(n-1)m+2}{3} \right\rceil \\
&= (n-1)m+4
\end{aligned}$$

$$\begin{aligned}
wt(x_{m-1})&\lambda(x_{m-1})+\lambda(x_{m-2}x_{m-1})+\lambda(y_{n-1}^{m-1}x_{m-1})+(y_{n-2}^{m-1}x_{m-1}) \\
&= (n-1)m+3-\left\lceil \frac{(n-1)m+2}{3} \right\rceil-\left\lceil \frac{(n-1)m-n+3}{3} \right\rceil-\left\lceil \frac{(n-1)m-n+2}{3} \right\rceil+\left\lceil \frac{(n-1)m+2}{3} \right\rceil+\left\lceil \frac{2n+(m-2)(n-1)}{3} \right\rceil+\left\lceil \frac{2n-1+(m-2)(n-1)}{3} \right\rceil \\
&= (n-1)m+3-\left\lceil \frac{(n-1)m-n+3}{3} \right\rceil-\left\lceil \frac{(n-1)m-n+2}{3} \right\rceil+\left\lceil \frac{2n+m(n-1)-3n+3}{3} \right\rceil+\left\lceil \frac{2n-1+m(n-1)-3n+3}{3} \right\rceil \\
&= (n-1)m+3-\left\lceil \frac{(n-1)m-n+3}{3} \right\rceil-\left\lceil \frac{(n-1)m-n+2}{3} \right\rceil+\left\lceil \frac{m(n-1)-n+3}{3} \right\rceil+\left\lceil \frac{m(n-1)-n+2}{3} \right\rceil \\
&= (n-1)m+3
\end{aligned}$$

$$wt(x_1)=\lambda(x_1)+\lambda(x_1x_2)+\lambda(x_mx_1)+\lambda(y^1_{n-1}x_1)+\lambda(y^1_{n-2}x_1)$$

$$=(n-1)m+5-2\left\lceil\frac{(n-1)m+2}{3}\right\rceil-\left\lceil\frac{n+1}{3}\right\rceil-\left\lceil\frac{n}{3}\right\rceil+\left\lceil\frac{(n-1)m+2}{3}\right\rceil+\left\lceil\frac{(n-1)m+2}{3}\right\rceil+\left\lceil\frac{2n+(-1)(n-1)}{3}\right\rceil+\left\lceil\frac{2n-1+(-1)(n-1)}{3}\right\rceil$$

$$=(n-1)m+5-\left\lceil\frac{n+1}{3}\right\rceil-\left\lceil\frac{n}{3}\right\rceil+\left\lceil\frac{2n-n+1}{3}\right\rceil+\left\lceil\frac{2n-1-n+1}{3}\right\rceil$$

$$=(n-1)m+5-\left\lceil\frac{n+1}{3}\right\rceil-\left\lceil\frac{n}{3}\right\rceil+\left\lceil\frac{n+1}{3}\right\rceil+\left\lceil\frac{n}{3}\right\rceil$$

$$=(n-1)m+5$$

If $(n-1)m+j+4-2\left\lceil\frac{(n-1)m+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+1}{3}\right\rceil>0$ then:

$$wt(xj)=\lambda(x_j)+\lambda(x_{j-1}x_j)+\lambda(x_jx_j+1)+\lambda(y^j_{y-1}x_j)$$

$$=(n-1)m+j+4-2\left\lceil\frac{(n-1)m+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+1}{3}\right\rceil+\left\lceil\frac{(n-1)m+2}{3}\right\rceil+\left\lceil\frac{(n-1)m+2}{3}\right\rceil+\left\lceil\frac{2n+(j-2)(n-1)}{3}\right\rceil+\left\lceil\frac{2n-1+(j-2)(n-1)}{3}\right\rceil$$

$$=(n-1)m+j+4-\left\lceil\frac{(n-1)j+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+1}{3}\right\rceil+\left\lceil\frac{2n+(n-1)j-2n+2}{3}\right\rceil+\left\lceil\frac{2n-1+(n-1)j-2n+2}{3}\right\rceil$$

$$=(n-1)m+j+4-\left\lceil\frac{(n-1)j+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+1}{3}\right\rceil+\left\lceil\frac{(n-1)j+2}{3}\right\rceil+\left\lceil\frac{(n-1)j+1}{3}\right\rceil$$

$$=(n-1)m+j+4 \text{ for } 2\le j\le m-2$$

While if $(n-i)m+j+4-2\left\lceil\frac{(n-1)m+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+2}{3}\right\rceil-\left\lceil\frac{(n-1)j+1}{3}\right\rceil\le 0$ then:

$$wt(x_j)=\lambda(x_j)+\lambda(x_{j-1}x_j)+\lambda(x_jx_{j+1})+\lambda(y^j_{n-1}x_j)+\lambda(y_{n-2}{}^jx_j)$$

$$=1\left\lceil\frac{(n-1)m+2}{3}\right\rceil+\left\lceil\frac{(n-1)m+2}{3}\right\rceil+\left\lceil\frac{2n+(j-2)(n-1)}{3}\right\rceil+\left\lceil\frac{2n-1+(j-2)(n-1)}{3}\right\rceil$$

$$\ge(n-1)m+j+4 \text{ for } 2\le j\le m-2$$

Note that of function λ is mapping from $\{V(P_m\triangleright C_n)\cup E(P_m\cup C_n)\}$ into $\{1, 2, ..., \lceil(n-1)m+2/3\}\rceil$. The weight of the vertices of graph $P_m\triangleright C_n$ which is denoted by $wt(y_i^j)$ is consecutive positive integer from 3 until $(n-1)m+2$. While the weight of the vertices of graph $P_m\triangleright C_n$ which is denoted by $wt(x_j)$ is different positive integers starting from $(n-1)m+3$. This shows that λ is a vertex irregular total labeling. Therefore, we conclude that $TVs(P_m\triangleright C_n\le\lceil(n-1)m+2/3)\rceil$.

CONCLUSION

In this study, we find that:

$$tvs(P_m\triangleright C_n)=\left\lceil\frac{(n-1)m+2}{3}\right\rceil \text{ for } m\ge 3 \text{ and } n\ge 2$$

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