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Correlation Processing of Radar Signals with Multilevel Quantization

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Abstract: The object of this study is correlation processing of radar signals with multilevel quantization. It is proposed to solve the problem with the use of mathematical tools of the theory of probability. The study describes statistical characteristics of correlator input and output signals. The study takes into account random nature of both input signals of correlator. Correlator input signals product joint probability distribution density is presented. The formula for calculation of signal/noise ratio at correlator output is presented which accounts for quantization bit depth effect.

Key words: Radar, correlation processing, multilevel quantization, joint probability distribution density, signal/noise ratio, digital matched filtering

INTRODUCTION

Theory of radar signals detection involves multichannel space-time processing. Spatial processing of radar signals is performed by antenna array. Time processing of known radar signals is performed by matched filters. Time processing of radar signals with unknown structure involves the use of correlator. Correlator uses powerful direct radar signal as a reference signal. Target detection is possible after calculation of cross-correlation function of direct radar signal and radar signal reflected from target. This requires storing these signals. Realization of algorithms of multichannel space-time processing in digital form requires multilevel quantization of radar signals counts. Processing large volumes of multi-digit numbers requires the use of complex system of many high rate signal processors and broadband communications channels.

Small number of radar signal quantization levels counts will allow reducing the requirements for signal processors speed. An expression is necessary for assessment of the effect of the number of radar signals quantization levels counts on signal/noise ratio at correlator output.

The study describes the studies and developments which along with other developments are being carried out at the Department of informatics and computer technology of National Mineral Resources University "Mining" (Makhovikov and Pivovarova, 2015; Tcvetkov and Strizhenok, 2016).

EXISTING PROBLEMS IN THE THEORY OF DIGITAL MATCHED FILTERING

Existing algorithms for digital processing are a reflection of the traditional analog processing algorithms. Multilevel quantization with 10-12 levels, allows approaching to potentially attainable level of detection quality indicators. But herewith, the requirements for computational devices speed and communications channels bandwidth sharply increase. The processed signals are represented as vectors of discrete counts of processed signals, each count is considered to be multi-bit. The sampling frequency is selected based on width of the signal spectrum. The number of signal amplitude quantization levels is selected based on recommendations obtained from simulation modelling. When processing in time domain the instantaneous signals counts are used and when processing in frequency domain it is required to perform Fourier transforms on samples of instantaneous signal values. In these algorithms, the calculations are made with multi-bit signal samples. The algorithms do not account for effect of the number of quantization levels samples on the processing result. The methods of matched filters and signals transformations synthesis in multidimensional basis (Dudgeon and Mersereau, 1984) do not account for effect of signal samples bit capacities with which the transforms are done and random nature of reference signals.

At present, the theory of sign correlators is well developed (Chernjack, 1965; Kopilovich, 1966;

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Burunsuzjan, 1968). These studies consider various aspects of using binary quantization in traditional correlation processing. Use of binary quantization is an extreme case. The theory of binary quantization also cannot take account of the effect of quantization bit depth. Some studies (Trees, 2002) give calculation of a quantizer with optimal quantization levels providing minimum input signal reproduction error.

There is a methodology of signal/noise ratio calculation at the output of cross-correlation device (Tikhonov and Kharisov, 1991). In this study, it is assumed that additive mixtures of harmonic signal with Gaussian noise pass through narrow-band filters, then multiplied and fed to low frequency filter. The signal at the output of low frequency filter is considered which is expressed via one-dimensional characteristic function.

By this methodology, a number of cumbersome expressions is obtained allowing to calculate signal/noise ratio at correlator output at various variants of cross-correlation of noises in processing channels. The methodology does not allow taking account of bit depth of processed data and does not allow applying it to calculation of signal/noise ratio with arbitrary signal samples bit depth counts.

CALCULATION OF SIGNAL/NOISE RATIO AT CORRELATOR OUTPUT

As is known, the maximum of correlation processing device reaction corresponds to zero mismatch in time of reference signal s(t) and reflected signal η(t) and correlator reaction is described by correlation integral or convolution integral z[y(t)]:

$$z[y(t)] = \int_{-\infty}^{\infty} s(t)\eta(t)d(t) \tag{1}$$

Reference signal s(t) is the sum of direct radar signal and Gaussian noise ξ(t) with zero mathematical expectation and noise variance $\sigma^2\xi$. Radar signal, in general can be presented in form of harmonic oscillation with frequency ω_s, amplitude U_s and uniformly distributed initial phase φ_s:

$$s(t) = U_s\cos(\omega_s t + \varphi_s) + \xi(t) \tag{2}$$

Signal η(t) reflected from target will differ from reference signal in amplitude and phase of the harmonic oscillations. Noise ς(t) present in the channel will be statistically independent of noises ξ(t) of reference channel due to differences in radio links of signals propagation and non-identity of processing channels. Analytical record will be the same.

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$$\eta(t) = U_\eta\cos(\omega_\eta t + \varphi_\eta) + \varsigma(t) \tag{3}$$

Joint probability density $p_s(x)$ of the sum of harmonic signal with random initial phase and Gaussian noise in reference signal channel in view of their independence at some fixed point in time can be represented as product of one-dimensional probability densities (Tikhonov, 1982).

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$$p_s(x) = \frac{1}{\pi\sqrt{2\pi\sigma^2\xi}}\int_0^\pi \exp\left[-\frac{1}{2}\frac{\left(x - U_s\cos(\varphi)^2\right)}{\sigma^2\xi}\right]d\varphi \tag{4}$$

Joint probability density $P_\eta(y)$ of the sum of harmonic signal with random initial phase and Gaussian noise in reflected signal channel will differ from $P_s(x)$ by harmonic signal amplitude U_η, noise variance $\sigma^2\xi$ and phase difference $\Delta\varphi$:

$$p_\eta(y) = \frac{1}{\pi\sqrt{2\pi\sigma^2\xi}}\int_0^\pi \exp\left[-\frac{1}{2}\frac{\left(y - U_\eta\cos(\varphi + \Delta\varphi)^2\right)}{\sigma^2\xi}\right]d\varphi \tag{5}$$

Joint probability $p_s(x)$ density graph for two amplitude values $U_s = 0$ (red line) and $u_s = 1$ (blue line) are shown in Fig. 1. Noise variance is $\sigma^2\xi = 1$.

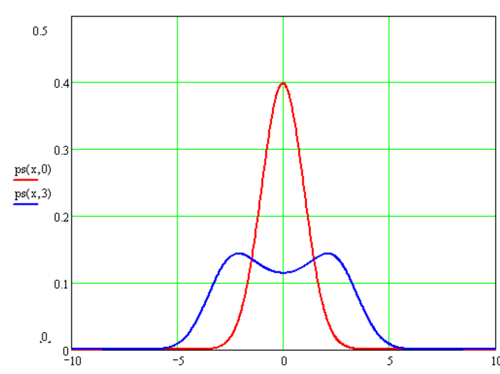

Fig. 1: Joint probability density

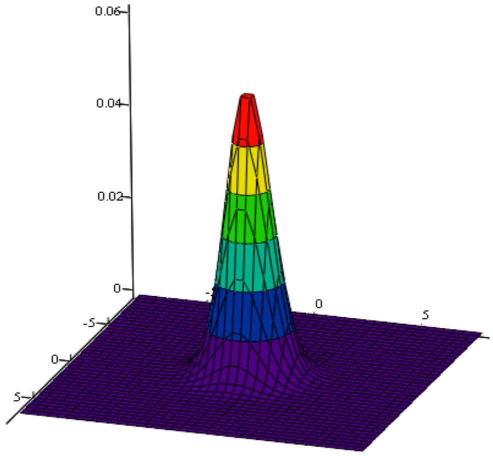

Fig. 2: Joint probability density $p_{s\eta}$ (x, y), U_s = 0, U_η = 0, $\sigma^2\xi$ = 1

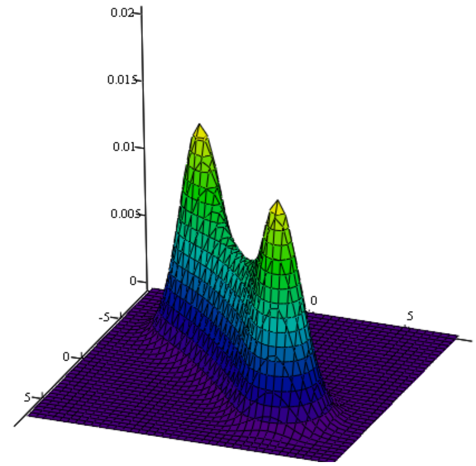

Fig. 3: Joint probability density $p_{s\eta}$ (x, y) at U_s = 4, $\sigma^2\xi$ = 1, U_η = 4,$\sigma^2\xi$ = 1

Direct radar signal and reflected from target radar signal are statistically independent. At the moment of complete coincidence in time of reference and reflected from target signals their initial phases will be random but the same ones and the phase difference $\Delta\varphi = 0$.

Expression $p_{s\eta}$ (x, y) of joint probability density of reference and reflected signals product is obtained on the basis of generalization of the known expression for density of harmonic signal and normal noise sum distribution (Tikhonov, 1982) and known rules of calculation of probability densities for two random values combination (Perov, 2003):

$$p_{s\eta}(x,y)=\frac{1}{2\pi^2\sqrt{\pi\sigma^2\xi}\,\sigma^2\xi}\int_0^\pi \exp\left[-\frac{1}{2}\frac{\left(x-U_\eta\cos(\varphi)^2\right)}{\sigma^2\xi}+\frac{\left(y-U_s\cos(\varphi)^2\right)}{\sigma^2\xi}\right]d\varphi \quad (6)$$

In the absence in additive mixture of signal and noise of harmonic signal the expression simplifies to the known expression of joint probability density of 2 independent normal noises with 0 mathematical expectations and unit noise variance. The graph $p_{s\eta}$ (x, y) at U_s = 0, $\sigma^2\xi$ = 1, U_η = 0,$\sigma^2\xi$ = 1 is shown in Fig. 2.

Upon detection of reflected signals the amplitudes of reverence and reflected signals are not equal to zero. For illustration purposes, the graph at $p_{s\eta}$ (x, y) at U_s = 4, $\sigma^2\xi$ = 1, U_η = 0,$\sigma^2\xi$ = 1 is shown in Fig. 3.

Transition from continuous value to discrete value allows, on the basis of expression $p_{s\eta}$ (x, y) for joint probability density of reference and reflected signals product, calculating signal/noise ratio at correlator output. There with, amplitudes of reference signal s(t) and reflected signal η(t) will be subjected to multilevel quantization with n levels and quantization level size $\Delta_i = n_i\text{-}n_{i-1}$. Signal sampling frequency is selected by Kotel'nikov theorem or Nyquist Shannon sampling theorem (Shannon, 1949).

The number of signals counts is generally = N. Signal/noise ratio at correlator output is equal to the ratio of mathematical expectation of maximum correlation integral to standard deviation of the noise at correlator output. Mathematical expectation of maximum correlation integral is calculated provided $U_\eta \neq 0$. Mathematical expectation $M_{s\eta}$ of maximum of correlation integral at correlator output takes the form Eq. 7:

$$M_{s\eta}(x,y)=N\sum_{i=1}^{n}\sum_{j=1}^{n}x_iy_i\int_{\Delta_i}\int_{\Delta_j}p_{s\eta}(x,y)dxdy \quad (7)$$

Noise variance $\sigma^2_{s\eta}$ (x, y), provided $U_s \neq 0$ at correlator output, takes the form Eq. 8:

$$\sigma^2_{s\eta}(x,y)=N\sum_{i=1}^{n}\sum_{j=1}^{n}(x_iy_i)^2\int_{\Delta_i}\int_{\Delta_j}p_{s\eta}(x,y)dxdy \quad (8)$$

Signal/noise ratio $q_{s\eta}$ at correlator output is equal to ratio of maximum correlation integral for $U_s \neq 0$, $U_\eta \neq 0$ to standard deviation of the noise at U_s = 0, $U_\eta \neq 0$ Eq. 9:

$$q_{s\eta}=\frac{M_{s\eta}(x,y)}{\sqrt{\sigma^2_{s\eta}(x,y)}}=\frac{\sqrt{N}\sum_{i=1}^{n}\sum_{j=1}^{n}x_iy_i\int_{\Delta_i}\int_{\Delta_j}p_{s\eta}(x,y)dxdy}{\sum_{i=1}^{n}\sum_{j=1}^{n}(x_iy_i)^2\int_{\Delta_i}\int_{\Delta_j}p_{s\eta}(x,y)dxdy}$$

CONCLUSION

The effect of quantization levels number on signal/noise ratio at correlator output is not accounted for in any of the algorithms considered. Quantization levels in the reviewed papers are optimized by the criterion of minimum reproduction error. Expression allows taking into account the effect of quantization levels number in signal/noise ratio at correlator output. This expression is applicable subject to representation of input actions as additive mixtures of simple harmonic oscillation and Gaussian noise. To calculate signal/noi ratio at correlator output it is necessary to:

- Calculate the joint probability density p_ς (x) of the sum of harmonic signal with random initial phase and Gaussian noise in reference signal channel
- Calculate the joint probability density p_η (y) of the sum of harmonic signal with random initial phase and Gaussian noise in reflected signal channel
- Calculate the joint probability density $p_{\varsigma\eta}$ (x, y) of reference and reflected signals product
- Calculate statistical characteristics of the signal at the output of correlation processing device with account of the signal quantization levels number in both channels

The expressions obtained allow getting dependences of signal/noise ratio on the number of quantization levels and the sizes of quantization levels. Such dependences will allow reducing the number of quantization levels of radar signals counts. The small amount of quantization levels will require less speed of signal processors and will result in simplification of digital signal processing system as a whole. Finding the optimal quantization levels is possible.

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