Communication and Diagnosis in Distributed Systems:
A Proposed Graph Network

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Abstract: The growing complexity of computer systems and the desire for distribution while keeping these systems as reliable as possible has led to a relation between fault-tolerance and graph theory with the intention of designing cost-effective fault-tolerant computer networks. This study presents a class of graph networks whose design is based on graph structure. This graph structure is referred to as R-graphs. Their construction technique and properties are presented and compared with other graph networks. The R-graphs which are regular, homogenous and posses a cyclic symmetry are shown to be easy to construct.

Key words: Graph networks, R-graphs, fault-tolerant networks, computer networks, large graphs

INTRODUCTION

Communication networks for computer systems cover a spectrum of architectures which extend from a network in which few computers are connected over long distances, up to the Very Large Scale Integration (VLSI) systems where a large number of processors are interconnected on a single chip. For a system that is distributed throughout one building, a Local Area Network (LAN) such as the Ethernet can be used. Many of the network architectures used in a multi-processor system are based on crossbar, multi-stage networks and multiple bus networks. Architectures such as star, ring and hypercube are considered in multi-computer systems.

Fault-tolerance which includes fault diagnosis is of critical importance in distributed systems especially if they are highly integrated. In these systems, the large number of interconnected elements increases the probability of faults that can occur. Communication networks of these systems are normally high by structured (e.g., binary tree, mesh, hypercube, etc.) and hence have fault-tolerance capabilities due to the existence of redundant paths from any source of a message to any destination. In facts, for a communication network to be capable of tolerating single faults, two paths between any pair of source and destination that have no links in common must exist.

With the ever growing complexity of computer systems a unique and highly desirable relation between fault tolerance and graph theory has been formed. The intention was to design cost-effective fault-tolerant computer networks. Thus, a number of fault tolerant communication architectures based on graph structures have been developed. These architectures are referred to as graph networks. Many graph networks are highly structured and posses attractive features such as low diameter, symmetry, simple routing, fault tolerance, etc. (Chen *et al.*, 1990; Bhuyan and Agrawal, 1984).

The notion of multi-foci fractal-graph networks which are used to characterize large dynamic telecommunications, computer and data networks. Networks considered in those studies are of varying degrees of connectivity and often irregular in appearance. The approach adopted focuses on the global structure and characteristics of large networks rather than on their microstructure was introduced by Jaggard *et al.* (1991).

Several researches are directed towards the study of the web as a graph such studies were fascinating and yields valuable insight into web algorithms for crawling, searching and community discovery and the sociological phenomena which characterize its evolution. In Cho, a design of crawl strategies on the web is presented. Analyzing the behavior of web algorithms that make use of link information can be found by Kleinberg (1999). In Meusel, however, a study of macroscopic structure of the web is presented. Experiments performed in this study showed that structure of the web is more intricate than suggested by earlier experiments.

In designing a fault tolerant communication network, two main categories of fault tolerant techniques may be implemented. The first is not directly related to the architecture of the system and its application does not involve modification in the network structure. The use of error correcting codes is an example of this category. In the second category which this study is more concerned about, fault tolerant techniques involve modifications in the architecture of the system. The implementation of multiple buses, adding extra communication links and adding extra switches are some examples of this category.

Network architecture can be expressed in terms of the network diameter and the degree of its processors. Network diameter is the maximum number of communication links (hops) between a source of a message and its destination along the shortest path while the degree of a processor is defined by the number of communication links connected to it. The design of a communication network is motivated by the minimization of both the network diameter (to reduce interprocessor distances) and the degree of processors (to provide a practical and cost effective network). These two issues are interrelated and they are considered whenever a graph network is designed. A graph network is normally of high connectivity (the connectivity of a graph G is the minimum number of vertices whose removal will disconnect G. It is also of high structure and capable of supporting routing and broadcasting strategies which are usually extendible to failure conditions.

In this study, a class of graph networks will be presented whose design is based on graph structures which we refer to as R-graphs.

THE R-GRAPHS

We consider the design of a class of graph networks that are both regular and homogenous. These graphs are a generalization of the $D_{\delta t}$ graphs proposed in the earliest pioneering work which has dealt with the connection assignment of diagnostic systems (Preparata *et al.*, 1977). A regular network is defined as the one with all its processors being of the same degree while the homogenous network is the one with all its processors being topologically identical. The underlying connection of the networks considered here is a single loop network R_0 which forms the cycle of n processors each of degree 2. The single loop system is 1-fault diagnosable, according to the PMC Model and its diagnosis problem has been studied by Preparata *et al.* (1977). To simplify the design, we will assume that all processors are of the same type. We further assume that R_0 is represented by a directed graph in which all processors are labeled. One way for labeling is to assume that the processors are numbered 0 through (n-1) in the direction of the graph. A directed link in R_0 which connects processors i to i+1 refers to the test assignment in which processor i has been assigned to test processor i+1 (all arithmetic is modulo n).

Although, the single loop network allows for a very simple routing, the network has a large diameter and it is not fault tolerant. In R-graphs, the diameter of the network can be reduced by an equal number of extra links to every

processor in R_0 which will result in increasing the network connectivity and hence its fault tolerance. The R-graphs are defined by the following equation:

$$R(n, h: k_1, k_2, ..., k_h) \tag{1}$$

Where:

n = Represents an arbitrary number of processors ($n \geq 3$)

h = Number of input(output) links in each processor. Output links represent test assignments of the processor

$k_1, ..., k_h$ = Integers represent the number of hops in R_0 between any processor in the network and the processors which are connected to it through its output links. For consistency, these integers always appear in ascending order

In order to form the underlying network R_0, the value of k_1 must be 1. Here, we emphasize case studies that we have developed for R-graphs to demonstrate the construction method and the properties of these graphs.

CASE STUDIES

Case study I: For n = 7, h = 2, k_1 = 1 and k_2 = 2, we have the graph network R(7, 2: 1, 2) which is shown in Fig. 1. The R-graphs are regular, homogenous and posses a cyclic symmetry. There is a number of distinct graph networks that can be formed for a given n and h, using Eq. 1. For any graph network there may exist a number of equivalent networks which are defined by the following eqaution:

$$R(n, h: k_1 \times m, k_2 \times m, ..., k_h \times m) \tag{2}$$
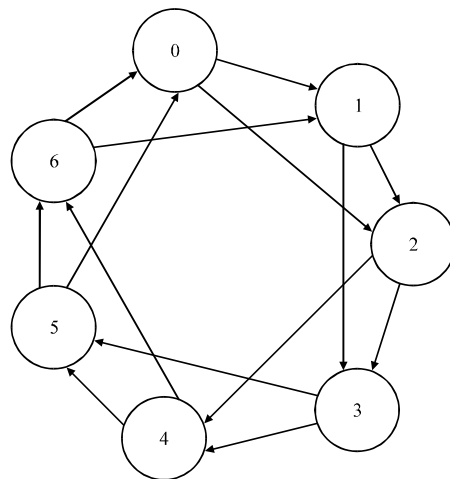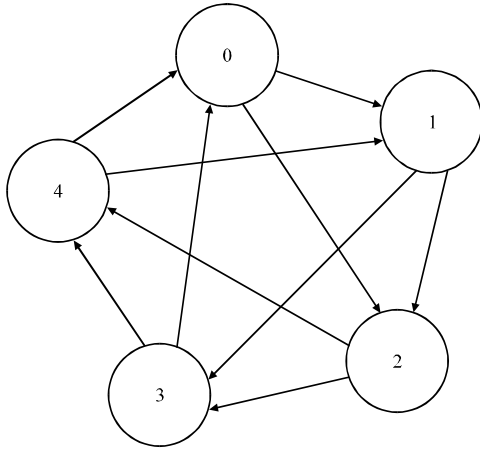

Fig. 1: The R(7, 2: 1, 2) graph network
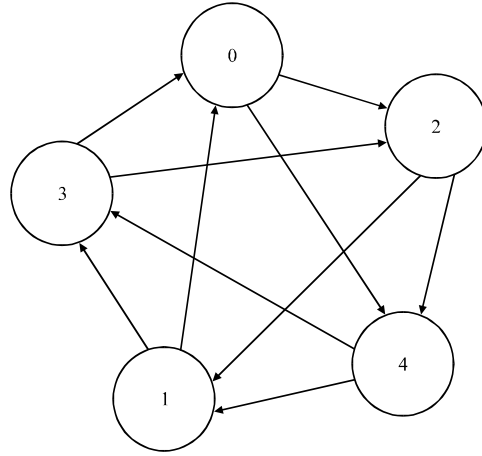
Fig. 2: The R(5, 2: 1, 2) graph network

Fig. 4: An equivalent R(5, 2: 1, 2) graph network

our work where they are vulnerable to single processor failures. It is therefore possible to conclude that only the graph network R(5, 2: 1, 2) is distinct. Graph networks that are generated from the R-graphs can be characterized by the following features:

- Both the regularity and homogeneity of processors in these networks are practically useful where they allow for the design of a distributed system that is composed of inexpensive replicated processors
- As with some other proposed graph networks (Brin and Page, 2012), the resulting architecture can be considered as logical rather than physical and hence, it may be used to represent task assignments. In fault diagnosis, task assignments are the scheduled tests of processors. Using graph networks, both the processors and their tasks can be represented as a graphical model which does not necessarily fit the physical architecture of the system
- The graph networks proposed here are easy to construct and they have a general form which can be extended to construct some other proposed graphs in an easier way as shown in the following case study
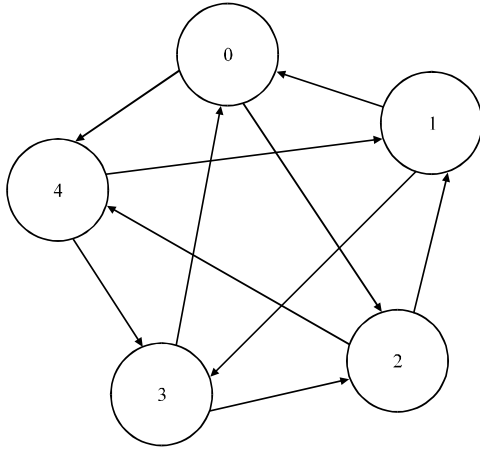

Fig. 3: The R(5, 2: 2, 4) graph network

In equation 2, m and n are relatively prime and all multiplications are mod n.

Case study II: For n = 5, h = 2, k_1 = 1 and k_2 = 2, we have the graph network R(5, 2: 1, 2) which is shown in Fig. 2 and formed by using Eq. 1.

If we choose m = 2, a graph network R(5, 2: 2, 4) will be formed which is shown in Fig. 3. The graph network shown in this figure is equivalent to that of Fig. 2. This equivalence can be made clearer if we consider that R_0 in Fig. 3 to consist of the directed cycle 0-2-4-1-3 instead of 0-4-3-2-1 as shown in Fig. 4.

In a similar way, the graph networks R(5, 2: 1, 3) and R(5, 2: 3, 4) can be shown to be equivalent to R(5, 2: 1, 2). The only two possible permutations that are left from n = 5 and h = 2 are R(5, 2: 1, 4) and R(5, 2: 2, 3) which are equivalent to each other. Moreover, in these networks each two adjacent processors are testing each other and for this reason such architecture will not be considered in

Case study III: The C-wrapped hexagonal architecture is a class of graph networks that has been studied in (Chen *et al.*, 1990) and implemented in an experimental distributed real-time system (Dolter *et al.*, 1991). The underlying connection of these networks is a hexagonal mesh.

In order to demonstrate the design procedure proposed by Bedrosian and Jaggard let d be the dimension of a hexagonal mesh and let n be the number of processors which is given by: n = 3d(d-1). Thus, for d = 3, n will be 19. Each processor i (i = 0, ..., n-1) has six neighbors which are given by:
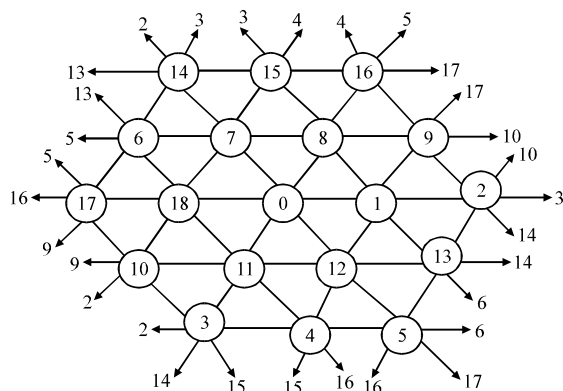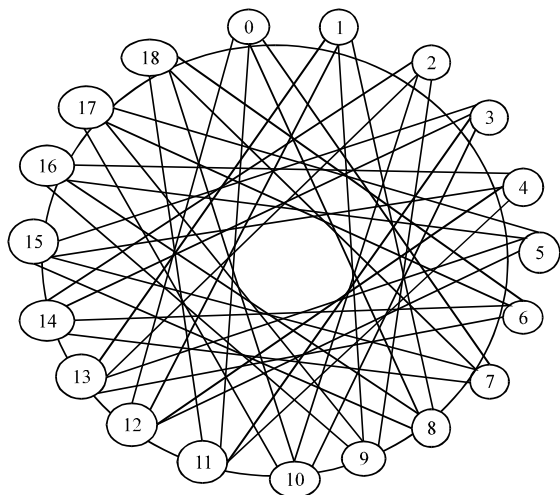
Fig. 5: A hexagonal mesh of dimension 3

Fig. 6: H(19, 3: 1, 7, 8) which is similar to C-wrapped hexagonal mesh of dimension 3

$$i + 1, i + 3d\text{-}1, i + 3d\text{-}2, i + 3d(d\text{-}1),$$
$$i + 3d^2\text{-}6d + 2, i + 3d^2\text{-}6d + 3$$

where all calculations are mod n. For every processor i, these values need to be found in order to implement a correct labeling and hence wrapping. The way in which a hexagonal mesh of dimension 3 is being labeled and wrapped is shown in Fig. 5.

By using the R-graphs and considering the resulting graph to be undirected, the wrapped hexagonal mesh of dimension d can be constructed using Eq. 2. The parameters of this eqaution will have the following values:

$$n = 3d(d\text{-}1), h = 3, k_1 = 1, k_2 = 3d\text{-}2 \text{ and } k_3 = 3d\text{-}1$$

Thus, for d = 3 a graph network R(19, 3: 1, 7, 8) is formed which is shown in Fig. 6. The network shown in

this figure is similar to the C-wrapped hexagonal mesh shown in Fig. 5. However, its connection procedure is shown to be much easier.

CONCLUSION

The spectrum of computer network architectures is wide. Some of these architectures are based on crossbar, multi-stage networks and multiple bus networks. Examples of more architectures are star, ring, hypercube, mesh and binary tree. In distributed systems, especially if they are highly integrated, fault-tolerance which includes fault diagnosis is of critical importance. If the communication networks of these systems need to be reliable and posses a fault-tolerant capability there should exist some redundant paths from any source of a message to any destination. The R-graphs which are a class of graph networks emerged due to a relation between fault-tolerance and graph theory are proposed in this study. They can be constructed according to a certain formula. Distinct number of R-graphs can be formed for a given number of nodes and links when applying this formula. The resulting graph networks are regular, homogenous and posses a cyclic symmetry. They therefore are assumed to be suitable to design a system composed of inexpensive replicated processes with certain fault diagnosis assignments. The construction technique and the properties of R-graphs are compared with other graph network architectures.

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