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Some Notes on the 3-Factor Analysis of 9×9 Sudoku

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Abstract: In this study, we consider a 9×9 post-filled Sudoku puzzle as an example of a three factorial design without blocking. Because of the rigidity of Sudoku, the all main effects and two factors interactions are all statistically insignificant. The three factor interaction is can be statistically significant or not. The use of column or row additions in the computation of the 3-factor interactions of a chosen Sudoku yields complementary results. If the rows addition gives an interaction effect that is statistically significant then the column result will be insignificant and vice versa. The treatment effects of the 3-factor effect of a post-filled Sudoku depend on whether the column or row sums are used in the computation of the interaction effect.

Key words: Sudoku, three factor design, interactions, main effects, significance

INTRODUCTION

Sudoku is a number puzzle. The main task of the player is to fill the 9 grids with the digits (1-9) for 9×9 Sudoku so that each column row and each of the nine sub-grids contains all the digits 1-9. The player is task with the responsibility of filling up the Sudoku in such a way that each digit will appear once in a row, column and sub-grid simultaneously. Sudoku is often presented to interested players with incomplete grids of which the player is expected to provide a unique solution by completely filling the grids with the digits (Semeniuk, 2005). Detailed scientific explanations of Sudoku can be found in the work of Delahaye (2006).

Many researchers have devised some methods of solving Sudoku. Some of them are: constraint programming (Crawford *et al*., 2009), computational complexity (Gabor and Woeginger, 2010) and Novel Hybrid Genetic algorithm (Deng and Li, 2013).

Sudoku can be seen in many mathematical literatures. Kumar *et al*. (2015) used Sudoku to demonstrate a three way blocking of an carefully planned experimental design. Sudoku is a Latin square (Emanouilidis, 2008) and a Gerechte designs which are a specialization of Latin squares (Bailey *et al*., 2008). Subramani and Ponnuswamy (2009) introduced Sudoku designs and Subramani extended it to the design of orthogonal (Graceo) Sudoku square designs. Also see Kuhl and Denley (2012) for the generalized Sudoku squares. In other to investigate the orthogonality properties of Latin squares (Dabbaghian and Wu, 2013), constructed a non-cyclic pan diagonal Sudoku. Because of the population and samples of Latin squares and Sudoku, Fontana (2014) developed an algorithm for the uniform sampling of them.

MATERIALS AND METHODS

Consider a 9×9 fully filled Sudoku as a three factor design with main factors A, B and C, 2-factors interactions AB, AC and BC and three factors interaction ABC.

A random samples of fully filled Sudoku is used to illustrate the instance of statistical significance or otherwise. Statistical significance means that the treatment effects are different.

RESULTS

The analysis of variance is employ to reveal the effects of the factors which are usually summarized in Table 1-7. A randomly chosen Sudoku gave the ANOVA summarized in Table 1.

For a post-filled Sudoku, the total sum of squares is 540 and the correcting factor is 2025. The 3-factor

Table 1: ANOVA table for a randomly chosen Sudoku

Effect	Degrees of freedom	Sum of squares	MSE	F-values	Decision at $\alpha = 0.05$
A	2	0	0	0	Insignificant
B	2	0	0	0	Insignificant
C	2	0	0	0	Insignificant
AB	4	0	0	0	Insignificant
AC	4	0	0	0	Insignificant
BC	4	0	0	0	Insignificant
ABC	8	105.333	13.1666	1.635	Insignificant
Error	54	434.667	8.0494		
Total	80	540			

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Table 2: ANOVA of Sudoku A using the columns sum in the computation of 3-factor effect

Effect	Degrees of freedom	Sum of squares	MSE	F-values	Decision at $\alpha = 0.05$
Others	18	0.000	0	0.000	Not significant
ABC	8	120.667	15.08	1.942	Not significant
Error	54	419.333	7.7654		
Total	80	540.000			

Table 3: ANOVA of Sudoku A using the rows sum in the computation of 3-factor effect

Effect	Degrees of freedom	Sum of squares	MSE	F-values	Decision at $\alpha = 0.05$
Others	18	0.000	0.000	0.000	Not significant
ABC	8	131.333	16.416	2.169	Significant
Error	54	408.667	7.568		
Total	80	540.000			

2	9	5	7	4	3	8	6	1
4	3	1	8	6	5	9	2	7
8	7	6	1	9	2	5	4	3
3	8	7	4	5	9	2	1	6
6	1	2	3	8	7	4	9	5
5	4	9	2	1	6	7	3	8
7	6	3	5	2	4	1	8	9
9	2	8	6	7	1	3	5	4
1	5	4	9	3	8	6	7	2

Fig. 1: Sudoku A

5	6	4	8	1	3	9	7	2
9	3	8	2	4	7	1	6	5
7	1	2	9	5	6	3	8	4
8	5	6	3	7	2	4	1	9
3	9	1	4	8	5	7	2	6
4	2	7	6	9	1	8	5	3
1	7	9	5	2	4	6	3	8
2	4	3	7	6	8	5	9	1
6	8	5	1	3	9	2	4	7

Fig. 2: Sudoku B

8	6	1	7	4	5	3	9	2
9	4	2	3	1	6	5	7	8
5	3	7	2	8	9	4	1	6
1	9	8	5	6	3	7	2	4
3	2	5	1	7	4	6	8	9
6	7	4	8	9	2	1	5	3
2	8	3	6	5	1	9	4	7
7	5	9	4	3	8	2	6	1
4	1	6	9	2	7	8	3	5

Fig. 3: Sudoku C

interaction effect may be statistically significance or not. Traditionally, in computing the sum of squares of three factor interaction ABC, the columns are summed in each of the grids of the Sudoku. But this research looked at using the rows in lieu of the columns. The Sudoku and their respective ANOVA tables are summarized in the three examples. Example 1 is given in Fig. 1.

The ANOVA of Sudoku A is summarized in Table 2-7 using the column sums and row sums in the computation of the three factors interaction effect. The tables are compressed since all the main effects and 2-factor effects are all non-significant. Example 2 is given in Fig. 2. Example 3 is given in Fig. 3.

Table 4: ANOVA of Sudoku B using the columns sum in the computation of 3-factor effect

Effect	Degrees of freedom	Sum of squares	MSE	F-values	Decision at $\alpha = 0.05$
Others	18	0.000	0	0.00	Insignificant
ABC	8	162.667	20.334	2.91	Significant
Error	54	377.333	6.9876		
Total	80	540.000			

Table 5: ANOVA of Sudoku A using the rows sum in the computation of 3-factor effect

Effect	Degrees of freedom	Sum of squares	MSE	F-values	Decision at $\alpha = 0.05$
Others	18	0.000	0	0.00	Insignificant
ABC	8	88.667	11.0833	1.32	Insignificant
Error	54	451.333	8.358		
Total	80	540.000			

Table 6: ANOVA of Sudoku B using the columns sum in the computation of 3-factor effect

Effect	Degrees of freedom	Sum of squares	MSE	F-values	Decision at $\alpha = 0.05$
Others	18	0	0	0	Insignificant
ABC	8	112	14	1.76636	Insignificant
Error	54	428	7.9259		
Total	80	540			

Table 7: ANOVA of Sudoku A using the rows sum in the computation of 3-factor effect

Effect	Degrees of freedom	Sum of squares	MSE	F-values	Decision at $\alpha = 0.05$
Others	18	0.00	0.000	0.000	Not significant
ABC	8	129.33	16.166	2.126	Significant
Error	54	410.67	7.605		
Total	80	540.00			

DISCUSSION

All the main effects and the 2-factor interaction effects are all statistically insignificant at any given level of confidence. Hence, their treatment effects are the same. The 3-factor interaction effects can be statistically significant or not. The total sum of squares and the correcting factor are constant for any given sample. The row and the column sums can be used in the computation of the 3-factor interaction effects but their results are different; if one is statistically significant, the other one will be statistically insignificant and vice versa. The effects of arrangement of numbers in each grid of the Sudoku are the same.

CONCLUSION

In a 3-factor analysis of a correctly solved 9×9 Sudoku, all the main effects and 2-factors interaction effects are all statistically insignificant for any given level of confidence. The 3-factor interaction effect can be significant or not depending or whether the columns or rows sum are used in the computation.

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REFERENCES

Bailey, R.A., P.J. Cameron and R. Connelly, 2008. Sudoku, gerechte designs, resolutions, affine space, spreads, reguli and hamming codes. Am. Math. Mon., 115: 383-404.

Crawford, B., C. Castro and E. Monfroy, 2009. Solving Sudoku With Constraint Programming. In: Cutting-Edge Research Topics on Multiple Criteria Decision Making, Shi, Y., S. Wang, Y. Peng, J. Li and Y. Zeng (Eds.)., Springer, Berlin, Germany, ISBN: 978-3-642-02297-5, pp: 345-348.

Dabbaghian, V. and T. Wu, 2013. Recursive construction of non-cyclic pandiagonal Latin squares. Discrete Math., 313: 2835-2840.

Delahaye, J.P., 2006. The science behind sudoku Sci. Am., 294: 80-87.

Deng, X.Q. and Y.D. Li, 2013. A novel hybrid genetic algorithm for solving sudoku puzzles. Optim. Lett., 7: 241-257.

Emanouilidis, E., 2008. Latin and cross Latin squares. Int. J. Math. Educ. Sci. Technol., 39: 697-700.

Fontana, R., 2014. Random Latin squares and sudoku designs generation. Electron. J. Stat., 8: 883-893.

Gabor, A.F. and G.J. Woeginger, 2010. How not to solve a sudoku. Oper. Res. Lett., 38: 582-584.

Kuhl, J.S. and T. Denley, 2012. A few remarks on avoiding partial Latin squares. Ars. Comb.,

Kumar, A., V. Varghese, E. Varghese and S. Jaggi, 2015. On the construction of designs with three-way blocking. Model Assisted Stat. Appl., 10: 43-52.

Semeniuk, I., 2005. Stuck on you. New Sci., 188: 45-47.

Subramani, J. and K.N. Ponnuswamy, 2009. Construction and analysis of sudoku designs. Model Assisted Stat. Appl., 4: 287-301.