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Homotopy Solution for Ratio-Dependent Predator-Prey System Problem Standard and Multistage Approach

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Abstract: In this study, an analytical expression for the solution of the ratio-dependent predator-prey system with constant effort harvesting by an adaptation of the Homotopy Perturbation Method (HPM) is presented. The HPM is treated as an algorithm for approximating the solution of the problem in a sequence of time intervals, i.e., HPM is converted into a Hybrid Numeric-Analytic Method. Residual error for the solution is presented.

Key words: Homotopy Perturbation Method, Multistage Homotopy Perturbation Method, ratio-dependent predator-prey, analytical solution, series solution

INTRODUCTION

Most modelling of biological problems are characterized by systems of Ordinary Differential Equations (ODEs). The prey is subjected to constant effort harvesting with r, a parameter that measures the effort being spent by a harvesting agency. The harvesting activity does not affect the predator population directly. It is obvious that the harvesting activity does reduce the predator population indirectly by reducing the availability of the prey to the predator. Adopting a simple logistic growth for prey population with e>0, b>0, c>0 and c>0 standing for the predator death rate, capturing rate and conversion rate, respectively, researchers formulate the problem as (Ghotbi *et al*., 2008):

$$\frac{dx(t)}{dt} = x(t)(1-x(t)) - \frac{bx(t)y(t)}{y(t)+x(t)} - rx(t), \quad x(t_0) = c_1 \tag{1}$$

$$\frac{dy(t)}{dt} = \frac{cx(t)y(t)}{y(t)+x(t)} - ey(t), \quad y(t_0) = c_2 \tag{2}$$

where, x(t) and y(t) represent the fractions of population densities for prey and predator at time, respectively. Equation 1 is to be solved according to biologically meaningful initial conditions and $x(t)\geq 0$, $y(t)\geq 0$.

Biazar and Montazari (2005) and Chowdhury *et al*. (2009) used the Adomian Decomposition Method (ADM) to handle the systems of prey-predator problem. Yusufoglu and Erbas (2008) and Rafei *et al*. (2007) employed the Variational Iteration Method (VIM) to compute an approximation to the solution of the system of non-linear differential equations governing the problem. Biazar *et al*. (2005) used the Power Series Method (PSM) to handle the systems. All the solutions above are in the form of convergent power series with polynomial base function.

In recent years, a great deal of attention has been devoted to study HPM which was first invented by He (2003) for solving a wide range of problems whose mathematical models yield differential equation or system of differential equations. HPM has successfully been applied to many situations. Chowdhury *et al*. (2009) present new modification of HPM by dividing the solution

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interval to finite number of subintervals. Alomari *et al.* (2009) applied the method for solving Schrodinger equation wich has complex solution.

SOLUTION PROCEDURE

Firstly, consider Eq. 1 subject to:

$$x(t^*) = c_1,\ y(t^*) = c_2 \tag{3}$$

Researchers note that when $t^* = 0$ we have the initial condition of Eq. 1, it is straightforward to choose:

$$x_0(t) = c_1,\ y_0(t) = c_2 \tag{4}$$

as the initial approximations of x(t) and y(t) and the linear operator should be:

$$L[\phi(t; q)] = \frac{\partial\phi(t; q)}{\partial t} \tag{5}$$

with the property:

$$L[A] = 0 \tag{6}$$

where, A is the integration constant which will be determined by the initial condition. If $q\in[0, 1]$ indicate the embedding parameter then the zeroth-order deformation problems are of the following form:

$$(1-q)L[\hat{x}(t; q) - x_0(t)] = qN_x[\hat{x}(t; q),\ \hat{y}(t; q)] \tag{7}$$

$$(1-q)L[\hat{y}(t; q) - y_0(t)] = qN_y[\hat{x}(t; q),\ \hat{y}(t; q)] \tag{8}$$

subject to the initial conditions:

$$\hat{x}(t^*; q) = c_1,\quad \hat{y}(t^*; q) = c_2 \tag{9}$$

in which researchers define the non-linear operators N_x and N_y as:

$$N_x[\hat{x}(t; q),\ \hat{y}(t; q)] = \frac{\partial\hat{x}(t; q)}{\partial t} - \hat{x}(t;\ q)(1-\hat{x}(t;\ q)) + \frac{b\hat{x}(t; q)\hat{y}(t; q)}{\hat{y}(t;\ q)+\hat{x}(t; q)} + r\hat{x}(t; q)$$

$$N_y[\hat{x}(t; q), \hat{y}(t; q)] = \frac{\partial\hat{y}(t; q)}{\partial t} - \frac{c\hat{x}(t;\ q)\hat{y}(t;\ q)}{\hat{y}(t;\ q)+\hat{x}(t;\ q)} + e\hat{y}(t; q)$$

For q = 0 and q = 1, the above zeroth-order deformation Eq. 7 and 8 have the solutions:

$$\hat{x}(t;\ 0) = x_0(t),\quad \hat{y}(t;\ 0) = y_0(t) \tag{10}$$

And:

$$\hat{x}(t;\ 1) = x(t),\quad \hat{y}(t;\ 1) = y(t) \tag{11}$$

when q increases from 0-1 then $\hat{x}(t; q)$ and $\hat{y}(t; q)$ vary from $x_0(t)$ and $y_0(t)$ to x(t) and y(t). Expanding $\hat{x}$ and $\hat{y}$ in Taylor series with respect to q, researchers have:

$$\begin{aligned}\hat{x}(t; q) &= x_0(t) + \sum_{m=1}^{\infty} x_m(t)q^m,\\ \hat{y}(t; q) &= y_0(t) + \sum_{m=1}^{\infty} y_m(t)q^m\end{aligned} \tag{12}$$

in which:

$$\begin{aligned}x_m(t) &= \frac{1}{m!}\frac{\partial^m\hat{x}(t;\ q)}{\partial q^m}\bigg|_{q=0},\\ y_m(t) &= \frac{1}{m!}\frac{\partial^m\hat{y}(t;\ q)}{\partial q^m}\bigg|_{q=0}\end{aligned} \tag{13}$$

Therefore, researchers have through Eq. 10 that:

$$\begin{aligned}x(t) &= x_0(t) + \sum_{m=1}^{¥} x_m(t)\\ y(t) &= y_0(t) + \sum_{m=1}^{¥} y_m(t)\end{aligned} \tag{14}$$

Define the vectors:

$$\begin{aligned}\vec{x}(t) &= \{x_0(t),\ x_1(t),...,x_n(t)\}\\ \vec{y}(t) &= \{y_0(t),\ y_1(t),...,y_n(t)\}\end{aligned} \tag{15}$$

Differentiating the zeroth-order Eq. 8 and 9, m times with respect to q then setting q = 0 and finally dividing by m!, researchers have the mth-order deformation equations:

$$L[x_m(t) - x_m x_{m-1}(t)] = R_{x,\ m}(\vec{x}(t),\ \vec{y}(t)) \tag{16}$$

$$L[y_m(t) - x_m y_{m-1}(t)] = R_{y,\ m}(\vec{x}(t), \vec{y}(t)) \tag{17}$$

with the following boundary conditions:

$$x_m(t^*) = 0,\quad y_m(t^*) = 0 \tag{18}$$

for all $m \geq 1$ where:

$$R_{x,m}(\vec{x}(t),\ \vec{y}(t))=\frac{1}{(m-1)!}\frac{\partial^{m-1}N_x[\hat{x}(t;\ q),\ \hat{y}(t;\ q)]}{\partial q^{m-1}}\bigg|_{q=0},$$
$$R_{y,m}(\vec{x}(t),\ \vec{y}(t))=\frac{1}{(m-1)!}\frac{\partial^{m-1}N_y[\hat{x}(t;\ q),\ \hat{y}(t;\ q)]}{\partial q^{m-1}}\bigg|_{q=0} \quad (19)$$

This way, it is easy to solve the linear non-homogeneous Eq. 18 and 19 at general initial conditions by using Maple, one after the other in the order m = 1, 2, 3, ... Thus, researchers successfully have:

$$x_1(t)=-\frac{c_1\left(-c_1+7c_2+10c_1^2+10c_1c_2\right)\left(t-t^*\right)}{10(c_1+c_2)},$$
$$y_1(t)=-\frac{c_2\left(3c_1+5c_2\right)\left(t-t^*\right)}{10(c_1+c_2)},$$
$$x_2(t)=\frac{1}{200\left(c_1+c_2\right)^3}c_1\left(c_1^3-30c_1^4+200c_1^5+19c_1^2c_2+19c_1c_2^2+49c_2^3+70c_1^3c_2+310c_1^2c_2^2+210c_1c_2^3+600c_1^4c_2+600c_1^3c_2^2+200c_1^2c_2^3\right)\left(t-t^*\right)^2,$$
$$y_2(t)=-\frac{1}{200\left(c_1+c_2\right)^3}c_2\left(-25c_2^3-9c_1^3-47c_1^2c_2-51c_1c_2^2+20c_1^3c_2+20c_1^2c_2^2\right)\left(t-t^*\right)^2$$
$$\vdots$$

By the same way, researchers can get the first fourth term to be as analytical approximate solution as:

$$x(t);\ \sum_{i=0}^{4}x_i(t),\quad y(t);\ \sum_{i=0}^{4}y_i(t)$$

terms. Now researchers divide the interval [0, T] to subintervals by time step Δt = 0.01. Then, researchers start from the initial conditions and researchers get the solution on the interval [0, 0.01). Further, researchers take c_1 = x (0.01) and c_2 = y(0.01) and t^* = 0.01, so researchers get the solution on the new interval [0.01, 0.02] and so on. Therefore, by choosing this initial approximation on the starting of each interval, the solution on the whole interval should be continuous. It is worth mentioning that if researchers take t^* = 0 and researchers fixed c_1 and c_2 then the solution will be the standard HPM solution which is not effective at large value of t.

ANALYSIS OF RESULTS

In this study, researchers compute the result using above algorithm for different cases which mention in the Table 1. Figure 1a-d presents the population fraction versus time for prey population fraction x(t) and predator population fraction y(t) for the fourth cases. Moreover, the residual error using standard HPM and the new algorithm is given in Fig. 2b and d. It is clear that the error within the range 10^{-12} which mean that is very small and it is not be possible in the standard HPM which give error 0.2 in small value of t as in Fig. 2a and c.

Table 1: Parameter values used for illustration purposes

Case	x(0)	y(0)	b	c	e	r
1	0.5	0.3	0.8	0.2	0.5	0.9
2	0.5	0.3	0.8	0.2	0.5	0.1
3	0.5	0.6	0.5	0.5	0.3	0.1
4	0.5	0.2	0.5	0.5	0.1	0.2

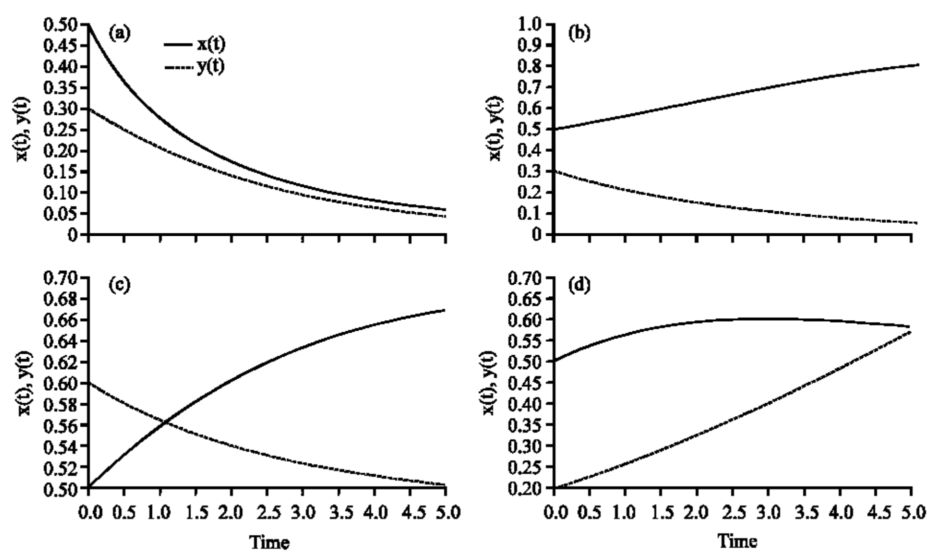

Fig. 1: Population fraction versus time; a) case 1; b) case 2; c) case 3 and d) case 4

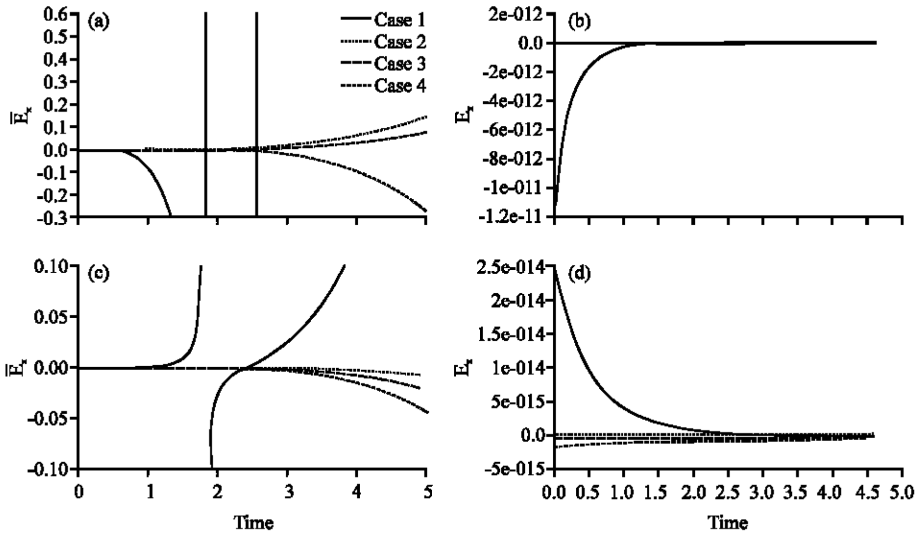

Fig. 2: Residual error for; a, c) HPM solution and b, d) MHPM solution

CONCLUSION

In this study, researchers interested to find the approximate analytic solution of the system of coupled nonlinear ODEs (1) by treated the HPM as an algorithm for approximating the solution of the problem in a sequence of time intervals. Residual error for the present solution is introduced.

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