Research Journal of Applied Sciences 3 (8): 529-538, 2008
ISSN: 1815-932X

Evaluation of Restraint Factor for Frames Braced Against Joint Translation

O.S. Abejide and M. Kachalla
Department of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria

Abstract: The lateral-torsion buckling modification factor, C_b, for Universal Columns in BS 5950 was investigated in order to determine its safety in frames braced against joint translation. These columns were evaluated based on the AISC specification for structural steel buildings. Results indicate that the modification factor C_b varied extensively with the end moment ratios and with associated safety values and column heights (1-7 m) of frames braced against joint translation. The C_b values that can be used depending on the steel section, end moment ratios and column heights are also predicted.

Key words: Universal columns, restraint factor and safety design criteria

INTRODUCTION

When an engineering structure is loaded in some way it will respond in a manner, which depends on the type and magnitude of the load and the strength and stiffness of the structure. Whether, the response is considered satisfactory depends on the requirements which must be satisfied. Such requirements might include safety of the structure against collapse, limitations on damage, deflections, or any of a range of other criteria. Each of these requirements may be termed a limit state. The violation of a limit state can be defined as the attainment of an undesirable condition for the structure (Melchers, 1987; Ditlevsen and Madsen, 1996).

The AISC (1999, 2005) and BS 5950-1 (2000) principles of safety and serviceability of steel structures for 31 universal column sections are examined. The AISC (2005) specification for the first time provides an integrated treatment of Allowable Stress Design (ASD) and Load and Resistance Factor Design (LRFD) and thus combines and replaces earlier specifications that treated the two design methods separately. AISC (2005) provisions reflect the latest advances in the state-of-the-art and they are intended to make the fundamental basis for stability analysis and design more apparent to the user. This new specification also, gives engineers the freedom to select or device their own methods within these constraints.

The study of structural reliability is concerned with the calculation and prediction of the probability of limit state violation for engineered structures at any stage during their life. In particular, the study of structural safety is concerned with the violation of the ultimate or safety limit states for the structure.

In probabilistic assessments, any uncertainty about a variable is explicitly taking into account. This is not the case in traditional ways of measuring safety, such as the factor of safety or load factor. These are deterministic measures in that the variables describing the structure and its strength are assumed to take on known (or conservative) values about which there is assumed to be no uncertainty. Precisely because of traditional and really quite central position in structural engineering, it is appropriate to review the deterministic safety measure prior to developing probabilistic safety measures.

Probabilistic structural analysis can be seen as an extension of deterministic structural analysis, which is the art of formulating a mathematical model within which the behavior of the structure is modeled using the parameters of load and resistances of the structure. Therefore, the purpose of a design is to get a low probability of failure, that is, a low probability of getting action values higher than resistances (Ellingwood *et al*., 1982). For example, herein, a low probability of applied or predicted buckling load exceeding buckling resistances based on restraint conditions and loading types is presented.

Hence, this study presents, a theoretical evaluation of the design parameters for universal columns when considered as random variables. It estimates the behavior of the braced columns with the intent of determining under what conditions they are safe. For the purpose of estimating the reliability of the structure, the First Order Reliability Method (FORM) compiled by Gollwitzer *et al*. (1988) is used and BS 5950-1 (2000) steel sections are analyzed. The design is based on the specifications of AISC (1999, 2005), while using hot-rolled or compact steel sections because of their generous availability.

Corresponding Author: O.S. Abejide, Department of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria

PROVISIONS FOR COLUMN DESIGNS

Columns in a steel building are often subjected to bending moments in addition to axial compressive forces. Even when beams are connected to the columns through simple connections, such as framing angles, they exert bending moments on the columns due to eccentrically applied support reactions. Columns in moment-resisting frames, of course, are subjected to considerable bending. Members acted on simultaneously by compressive axial forces and bending moments are referred to as beam-columns (SCI, 2005).

Columns design will normally require that attention be given to whichever of the following checks are relevant for the particular application; overall flexural buckling; local buckling; buckling of component parts; tension or torsional flexural buckling.

Under the assumption that a frame is braced against joint translation, the conditions of overall flexural buckling, local buckling and buckling of components are negligible in design (ICC, 2003). As such the design is concerned basically with the check for lateral-torsion buckling of the structure at the columns.

The AISC (1999) LRFD specifications for steel buildings gives an equation for doubly symmetrical I-shapes and channels with $l_b > l_r$ as:

$$M_{cr} = C_b \frac{\pi}{l_b}\sqrt{EI_y GJ + \left[\frac{\pi E}{l_b}\right]^2 I_y C_w} \quad (1)$$

Alternatively,

$$M_{cr} = \frac{C_b S_x X_1 \sqrt{2}}{l_b / r_y}\sqrt{1 + \frac{X_1^2 X_2^2}{2\left(l_b / r_y\right)^2}} \quad (2)$$

For the same conditions, the AISC (2005) specification modifies the equation to:

$$M_n = F_{cr}.S_x \quad (3)$$

Where:

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{l_b}{r_{ts}}\right)^2}\sqrt{1 + \frac{0.078 J_c}{S_x h_o}\left(\frac{l_b}{r_{ts}}\right)^2} \quad (4)$$

This Eq. 5 is used in the check for lateral-torsional buckling of doubly symmetrical I-shaped members and channels bent about their major axis.

$$l_r = 1.95 r_{ts}\frac{E}{0.7f_y}\sqrt{\frac{J_c}{S_x h_o}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7f_y}{E}.\frac{S_x h_o}{J_c}\right)^2}} \quad (5)$$

$$r_{ts}^2 = \frac{I_y h_o}{2S_x} \quad (6)$$

For doubly symmetric I-shapes:

$$C = 1.0 \quad (7)$$

From above, Eq. 3 becomes

$$M_n = C_b \pi^2 E.\frac{I_y h_o}{2l_b^2}\sqrt{1 + \frac{0.156 J l_b^2}{I_y h_o}} \quad (8)$$

The limit state of lateral-torsional buckling is thus,

$$M_n \le M_p \quad (9)$$

$$M_p = F_y Z_x \quad (10)$$

$$\lambda = k\frac{l_b}{r_y}$$

For the joint conditions k = 1.0 (Kuchenbecker *et al*., 2004).

C_b and C_m: The nominal flexural strength of the column, M_n, can be modified by the lateral-torsional buckling modification factor, C_b, in Eq. 8. This can be done using an equation that relates C_b with the coefficient C_m when no lateral translation of the frame occurs. This is given as:

$$C_m = 1/C_b \quad (11)$$

To find the value of C_m, the ASD code divides beam-columns into 3 categories namely:

- Compression members in frames subjected to side-sway or joint translation (for example, member AB in the moment-resisting frame of Fig. 1a)
- Compression members in frames braced against side-sway or joint translation and not subjected to transverse loading between their ends (for example, member AB in the braced frame of Fig. 1b)
- Compression members in frames braced against side-sway and subjected to transverse loading (for example member CD in the braced frame of Fig. 1b)

For frames braced against joint translation and not subjected to transverse loading between their ends.

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} \quad (12)$$

where, M_1/M_2 is the ratio of the smaller moment to the larger moment at the end of the un-braced length in the plane of bending. The ratio M_1/M_2 is positive when the end moments M_1 and M_2 are in the same direction (reverse curvature) and negative, if otherwise (i.e., single curvature). A member in single curvature in general has larger lateral displacements than a corresponding member in double curvature and consequently is subjected to larger moments and larger bending stresses.

In general, for any braced frame modifying the moment ratio M_1/M_2 results in a modification of C_m and C_b and consequently, the flexural torsion strength of the columns. It is on this basis that the restraint factor for frames braced against joint translation is evaluated for optimum values, so as to determine its safety conditions for ratios of small to large moments; for example, from -1.0 to 1.0. The steel sections as provided in BS 5950-1 (2000) and shown in Table 1 are used in order to evaluate their performance up to a braced column height of 7.0 m.

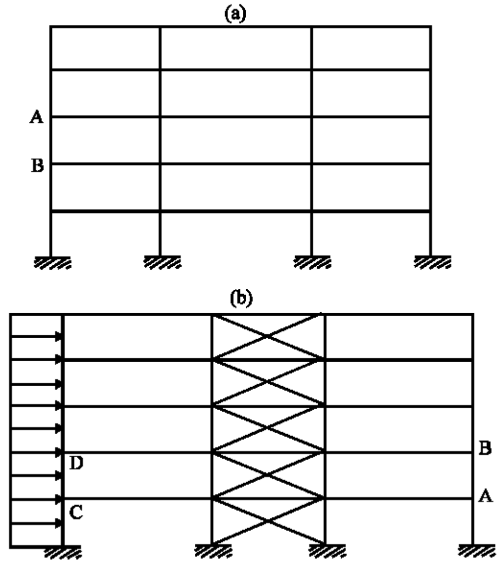

Fig. 1: Frames with and without sideway (a) un-braced frame (b) braced frame

Table 1: Universal columns dimensions and properties, BS 5950-1 (2000)

Section destination: Dimension (mm^2)	Section destination: Mass/m ($kg\ m^{-1}$)	I_y(cm^4) 2nd moment of area	J (cm^4) torsion constant	h_o (mm) distance between flange centroids	Z_x (cm^3) plastic modulus
356×406	634	98100	13700.00	397.60	14200
356×406	551	82700	9240.00	388.10	12100
356×406	467	67800	5810.00	378.60	10000
356×406	393	55400	3550.00	369.80	8220
356×406	340	46900	2340.00	363.50	7000
356×406	287	38700	1440.00	357.10	5810
356×406	235	31000	812.00	350.80	4690
356×368	202	23700	558.00	347.60	3970
356×368	177	20500	381.00	344.44	3460
356×368	153	17600	251.00	341.30	2970
356×368	129	14600	153.00	338.10	2480
305×305	283	24600	2030.00	321.20	5110
305×305	240	20300	1270.00	314.80	4250
305×305	198	16300	734.00	308.50	3440
305×305	158	12600	378.00	302.10	2680
305×305	137	10700	249.00	298.80	2300
305×305	118	9060	161.00	295.80	1960
305×305	97	7310	91.20	292.50	1590
254×254	167	9870	626.00	257.40	2420
254×254	132	7530	319.00	251.00	1870
254×254	107	5930	172.00	246.20	1480
254×254	89	4860	102.00	243.00	1220
254×254	73	3910	57.60	239.90	992
203×203	86	3130	137.00	201.70	977
203×203	71	3540	80.20	198.50	799
203×203	60	2070	47.20	195.40	656
203×203	52	1780	31.80	193.70	567
203×203	46	1550	22.20	192.20	497
152×152	37	706	19.20	150.30	309
152×152	30	560	10.50	148.20	248
152×152	23	400	4.63	145.60	182

MATERIALS AND METHODS

We know that for a structurally safe structure:

$$R = 1 - P_f \tag{13}$$

where:

R = Reliability

P_f = Probability of failure

Thus, if X_S is the applied load and X_R the strength or resistance of the structure, then the structure will fail when $x_R \leq x_S$. Note that X_S and X_R are non-negative independent random variables with probability density functions $f_R(x_R)$ and $f_S(x_S)$, respectively. The probability of failure, P_f is given as:

$$P_f = P[X_R \leq X_S] = \iint_{x_R \leq x_S} f_R(x_R)\, f_S(x_S)\, dx_R\, dx_S$$
$$= \int_0^\infty F_R(x_S) f_S(x_S) dx_S = \int_0^\infty [1 - F_S(x_R)] f_R(x_R) dx_R$$

Therefore,

$$P_f = 1 - \int_0^\infty F_S(x_R)\, f_R(x_R) dx_R \tag{14}$$

The limit state function of X_R and X_S can be expressed as:

$$g(X_R, X_S) = X_R - X_S$$

and the limit state surface is given as:

$$g = x_R - x_S = 0$$

since $g \leq 0$ defines the failure event. But we know that

$$P_f = F_g(0) \tag{15}$$

where, $F_g(.)$ is the Cumulative Density Function (CDF) of the safety margin $g = g(X_R, X_S)$, when both X_R and X_S are normal, then $N(X^0_R, \sigma_{XR})$; $N(X^0_S, \sigma_{XS})$. Hence, the probability of failure will be obtained as:

$$P_f = \Phi\left(-\frac{g^0}{\sigma_g}\right) = \Phi(-\beta) \tag{16}$$

where,

$\Phi(.)$ = The standard normal CDF and

$$g^0 = E[g] = X^0_R - X^0_S$$
$$\sigma^2_g = \sigma^2_{X_R} - \sigma^2_{X_S}$$

The reliability index, β, is thus given as:

$$\beta = \frac{g^0}{\sigma_g} \tag{17}$$

Therefore, if the resistance of a column is expressly influenced by the modification factor, C_b then, Eq. 8 gives the capacity or resistance of the column. The maximum load that may be applied to the column is a function of the sectional and material properties. Thus, the limit state function for frames braced against joint translation is given as:

$$g(M_n, M_p) = M_n - M_p \tag{18}$$

Equation 9 gives the limit state conditions for steel frames as expressed in AISC (2005). This limit state condition is evaluated in the face of varying values of C_b as site conditions may predict. The design values may be different from site conditions. The essence of the variable C_b is to determine the ultimate moment capacity of the column in prevailing site conditions using other design parameters of the columns.

RESULTS AND DISCUSSION

The stochastic models generated using the data in Table 1 are analyzed using the FORM to give values of safety index, β and probability of failure, P_f for the Universal column sections in BS 5950-1 (2000). The lateral torsion buckling modification factor C_b, was varied for all sections for values of length between 1 and 7 m at an increment of end moment ratio (M_1/M_2) values of 0.1 between -1.0 and 1.0.

For example, from Fig. 2, the reliability of the section (152×152 mm ×23 kg m^{-1}) increased as the value of the end moment ratio M_1/M_2 and modification factor increased. This is shown by a rising curve when the

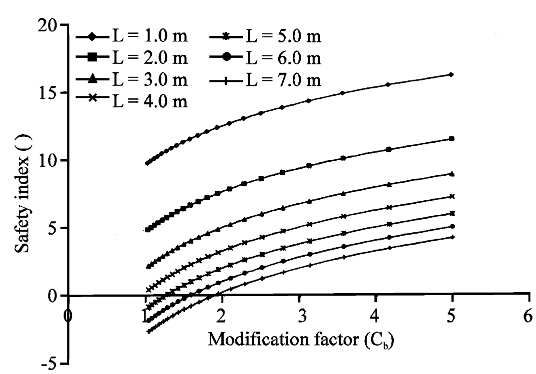

Fig. 2: Safety levels for 152×152 mm×23 kg m^{-1} UC

Table 2: Allowable end moment ratio

Span (m)	1.0	2.0	3.0	4.0	5.0	6.0	7.0
End moment ratio M_1/M_2	-1.0-1.0	-1.0-1.0	-1.0-1.0	-0.2-1.0	0.3-1.0	0.5-1.0	0.7-1.0

Table 3: Failure moment ratio with respect to slenderness ratio in parenthesis

	Span (m)						
Sections (kg m^{-1})	1	2	3	4	5	6	7
152×152×23	_	_	_	-1.0 to -0.3 (0.925)	-1.0 to 0.2 (0.740)	-1.0 to 0.4 (0.616)	-1.0 to 0.6 (0.528)
152×152×30	_	_	_	-1.0 to -0.6 (0.957)	-1.0 to -0.1 (0.766)	-1.0 to 0.2 (0.638)	-1.0 to 0.4 (0.547)
152×152×37	_	_	_	-1.0 (0.967)	-1.0 to -0.4 (0.774)	-1.0 to 0.0 (0.645)	-1.0 to 0.4 (0.522)
203×203×46	_	_	_	_	-1.0 to -0.7 (1.026)	-1.0 to -0.2 (0.855)	-1.0 to 0.1 (0.732)
203×203×52	_	_	_	_	-1.0 to -0.8 (1.036)	-1.0 to -0.4 (0.863)	-1.0 to -0.1 (0.740)
203×203×60	_	_	_	_	_	-1.0 to -0.6 (0.866)	-1.0 to -0.2 (0.742)
203×203×71	_	_	_	_	_	-1.0 to -0.9 (0.833)	-1.0 to -0.6 (0.757)
203×203×86	_	_	_	_	_	_	-1.0 to -0.9 (0.762)
254×254×73	_	_	_	_	_	-1.0 to -0.8 (0.890)	-1.0 to -0.4 (0.925)
254×254×89	_	_	_	_	_	_	-1.0 to -0.8 (0.935)
254×254×107	NF	NF	NF	NF	NF	NF	NF
254×254×132	NF	NF	NF	NF	NF	NF	NF
305×305×97	NF	NF	NF	NF	NF	NF	NF
305×305×118	NF	NF	NF	NF	NF	NF	NF
305×305×137	NF	NF	NF	NF	NF	NF	NF
305×305×158	NF	NF	NF	NF	NF	NF	NF
305×305×198	NF	NF	NF	NF	NF	NF	NF
305×305×240	NF	NF	NF	NF	NF	NF	NF
305×305×283	NF	NF	NF	NF	NF	NF	NF
354×254×107	NF	NF	NF	NF	NF	NF	NF
354×254×167	NF	NF	NF	NF	NF	NF	NF
356×368×129	NF	NF	NF	NF	NF	NF	NF
356×368×153	NF	NF	NF	NF	NF	NF	NF
356×368×177	NF	NF	NF	NF	NF	NF	NF
356×368×202	NF	NF	NF	NF	NF	NF	NF
356×406×235	NF	NF	NF	NF	NF	NF	NF
356×406×287	NF	NF	NF	NF	NF	NF	NF
356×406×340	NF	NF	NF	NF	NF	NF	NF
356×406×393	NF	NF	NF	NF	NF	NF	NF
356×406×467	NF	NF	NF	NF	NF	NF	NF
356×406×551	NF	NF	NF	NF	NF	NF	NF
356×406×634	NF	NF	NF	NF	NF	NF	NF

NF: Not Failing

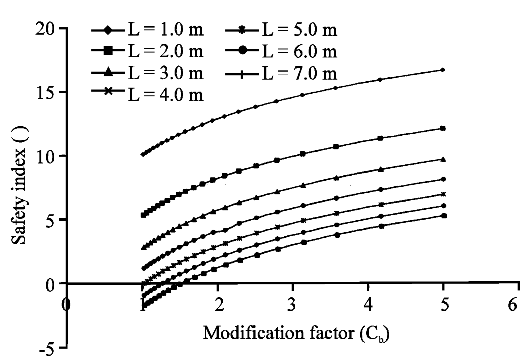

Fig. 3: Safety levels for 152×152 mm ×30 kg m^{-1} UC

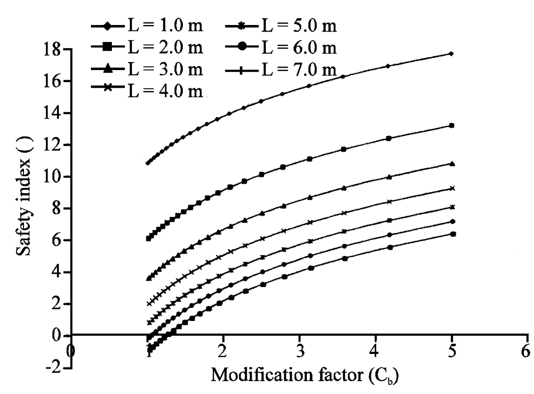

Fig. 4: Safety levels for 152×152 mm×37 kg m^{-1} UC

length is constant. As the length is varied between 1 and 7 m, a decrease in the reliability index is shown at each constant value of C_b. This is represented by similar curves of smaller amplitude as the length increases.

For the above study, the structure is safe for spans of 1-3 m for all values of C_b. When span is 4 m, the study will experience failure at M_1/M_2 values between -1.0 and -0.3. After this range, the section will not be safe for other increments of M_1/M_2 and consequently C .$_b$ The first failure criterion is the optimum design point.

When column height is 5 m, the study will experience failure at M_1/M_2 values between -1.0 and 0.2. After this range, the section may appear safe for other increments of M_1/M_2 and consequently C_b. When span is 6 m, the study fails at M_1/M_2 values between -1.0 and 0.4 and after this range; the study appears safe for further increments of M_1/M_2. When span is 7 m, the study fails at M_1/M_2 values between -1.0 to 0.6 and after this range any further increment in the value of M_1/M_2 though appearing safe is not realistic. This indicates that there is an increase in the range of values of M_1/M_2 and consequently C_b for which failure is induced as the height increases from 4-7 m. It then suggests that the range of end moment ratio M_1/M_2

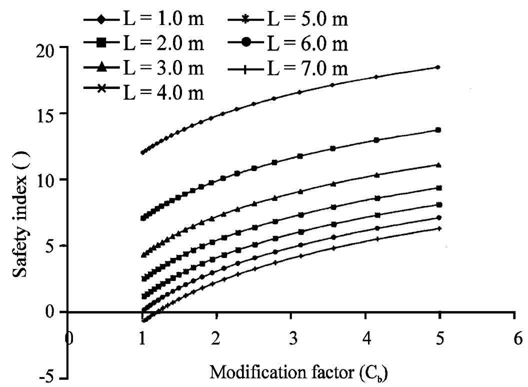

Fig. 5: Safety levels for 203×203 mm ×46 kg m^{-1} UC

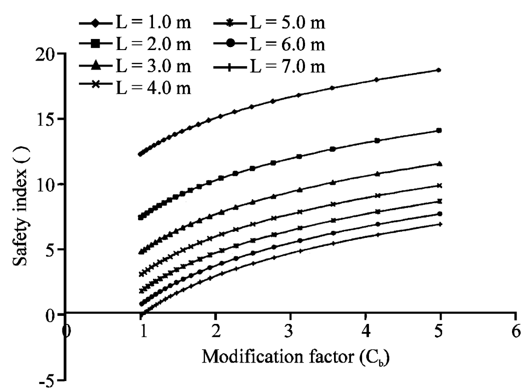

Fig. 6: Safety levels for 203×203 mm ×52 kg m^{-1} UC

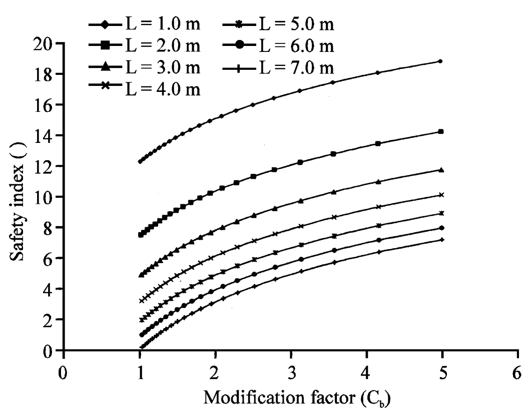

Fig. 7: Safety levels for 203×203 mm ×60 kg m^{-1} UC

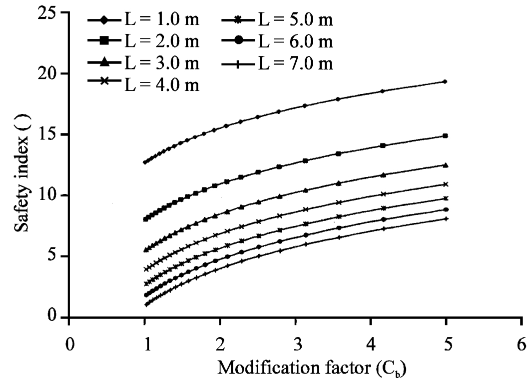

Fig. 8: Safety levels for 203×203 mm ×71 kg m^{-1} UC

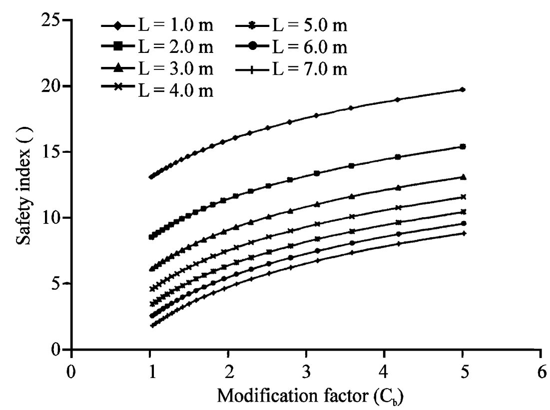

Fig. 9: Safety levels for 203×203 mm ×86 kg m^{-1} UC

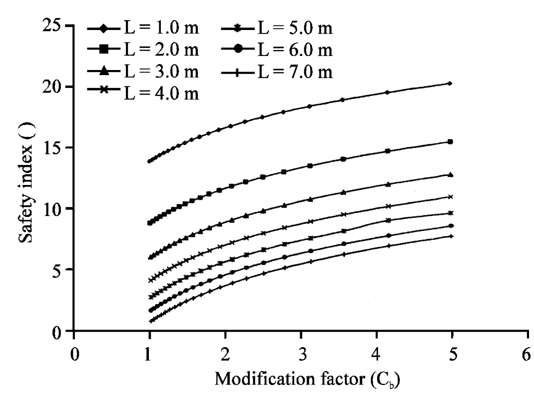

Fig. 10: Safety levels for 254×254 mm ×73 kg m^{-1} UC

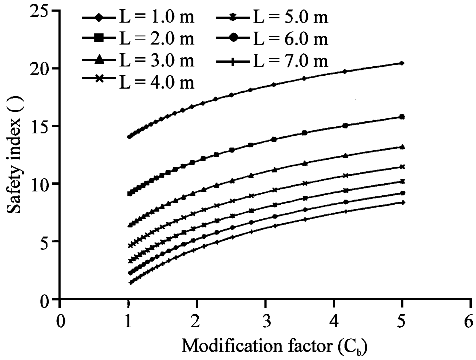

Fig. 11: Safety levels for 254×254 mm ×89 kg m^{-1} UC

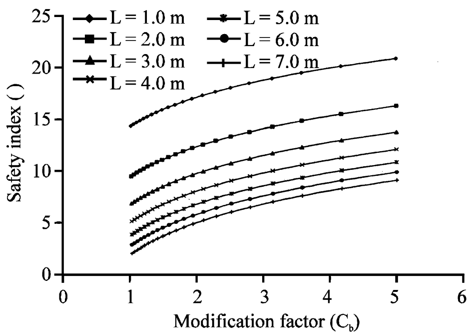

Fig. 12: Safety levels for 254×254 mm ×107 kg m^{-1} UC

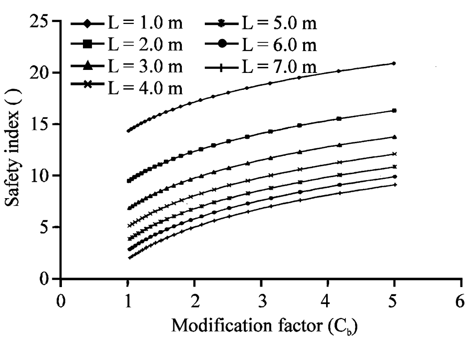

Fig. 13: Safety levels for 254×254 mm ×132 kg m^{-1} UC

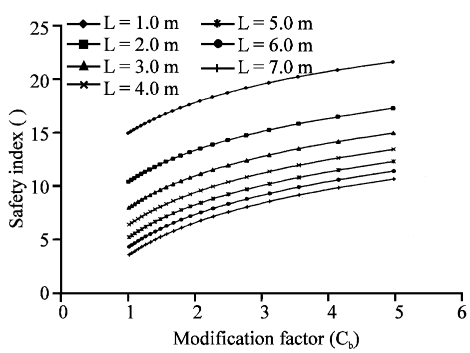

Fig. 14: Safety levels for 254×254 mm ×167 kg m^{-1} UC

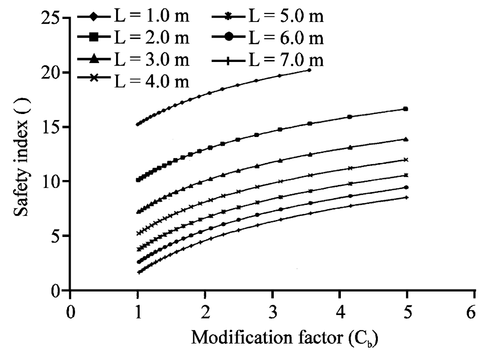

Fig. 15: Safety levels for 305×305 mm ×97 kg m^{-1} UC

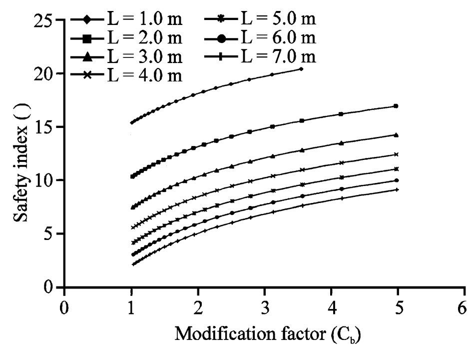

Fig. 16: Safety levels for 305×305 mm ×118 kg m^{-1} UC

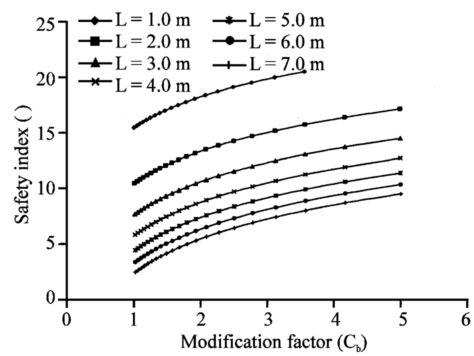

Fig. 17: Safety levels for 305×305 mm ×137 kg m^{-1} UC

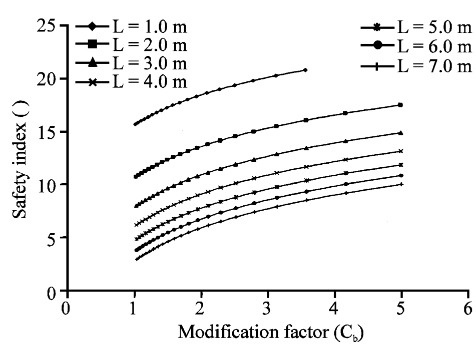

Fig. 18: Safety levels for 305×305 mm ×158 kg m^{-1} UC

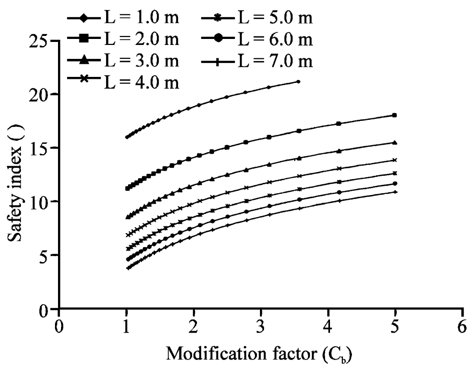

Fig. 19: Safety levels for 305×305 mm ×198 kg m^{-1} UC

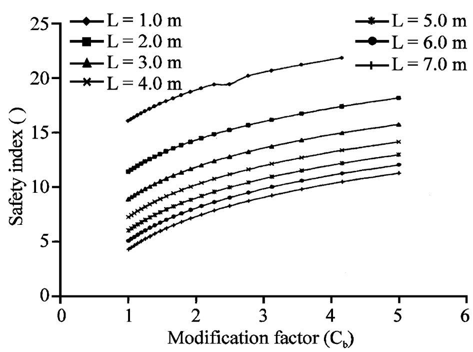

Fig. 20: Safety levels for 305×305 mm ×240 kg m^{-1} UC

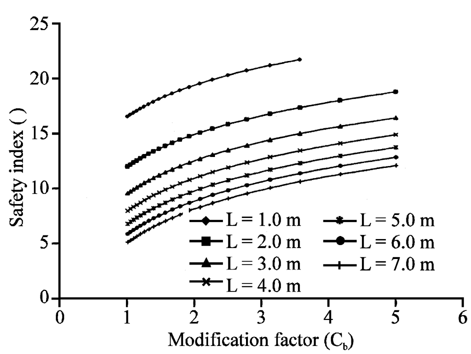

Fig. 21: Safety levels for 305×305 mm ×283 kg m^{-1} UC

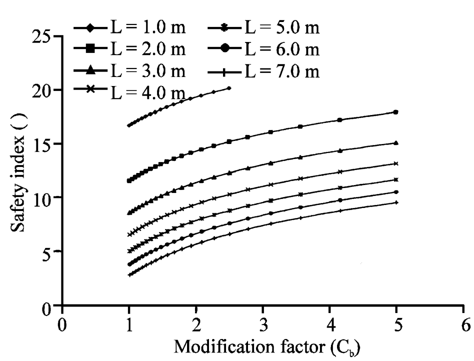

Fig. 22: Safety levels for 356×368 mm ×129 kg m^{-1} UC

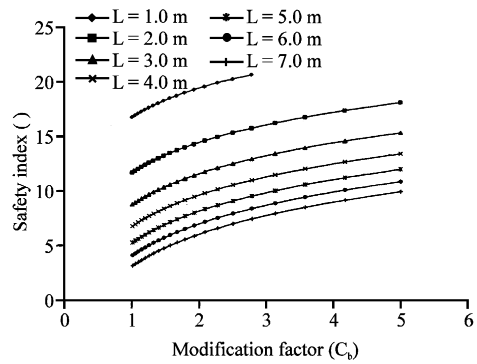

Fig. 23: Safety levels for 356×368 mm ×153 kg m^{-1} UC

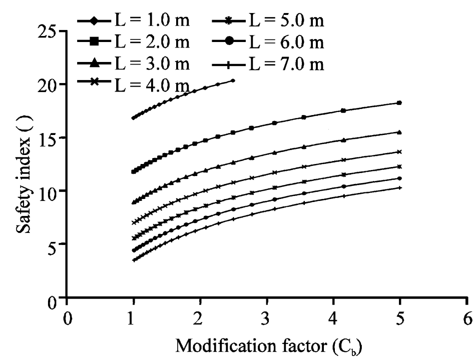

Fig. 24: Safety levels for 356×368 mm ×177 kg m^{-1} UC

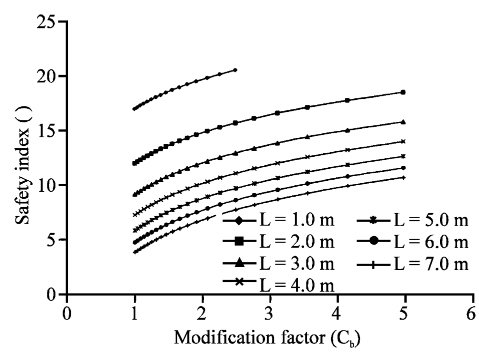

Fig. 25: Safety levels for 356×368 mm ×202 kg m^{-1} UC

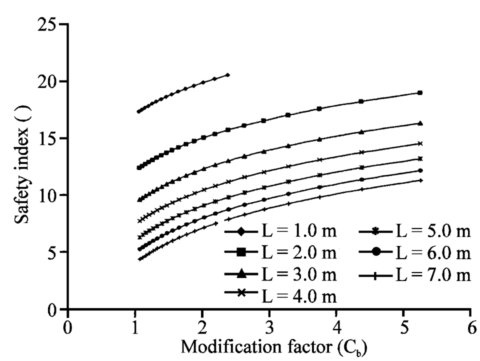

Fig. 26: Safety levels for 356×406 mm ×235 kg m^{-1} UC

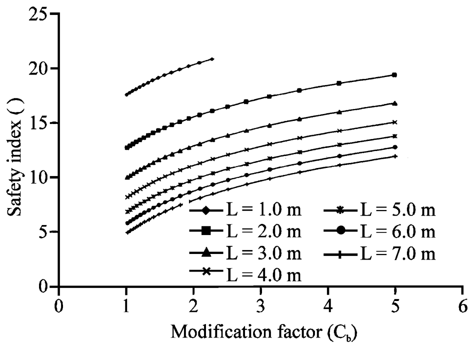

Fig. 27: Safety levels for 356×406 mm ×287 kg m^{-1} UC

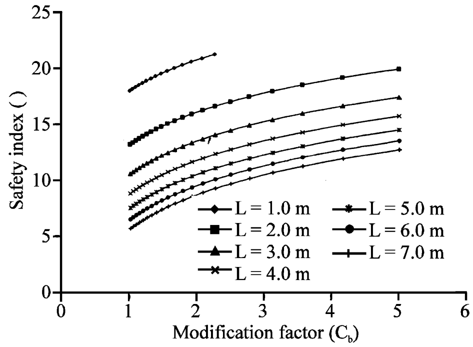

Fig. 28: Safety levels for 356×406 mm ×340 kg m^{-1} UC

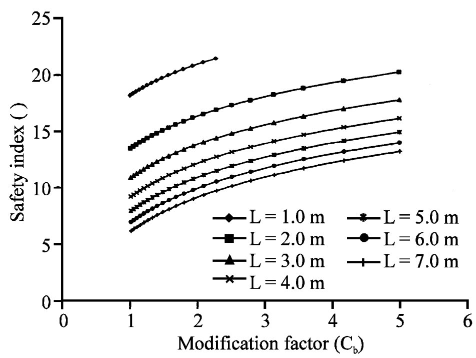

Fig. 29: Safety levels for 356×406 mm ×393 kg m^{-1} UC

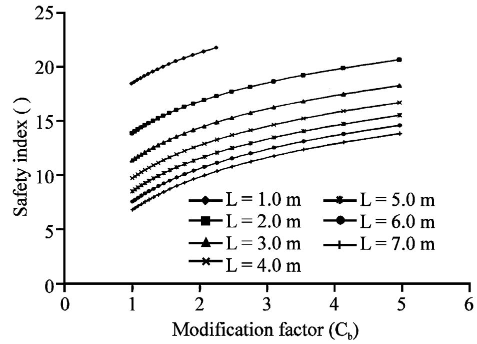

Fig. 30: Safety levels for 356×406 mm ×467 kg m^{-1} UC

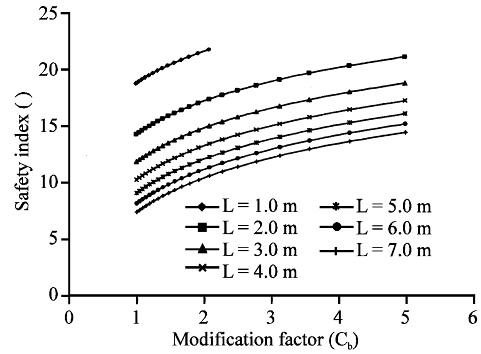

Fig. 31: Safety levels for 356×406 mm ×551 kg m^{-1} UC

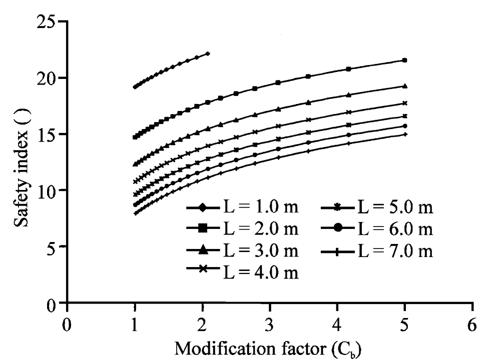

Fig. 32: Safety levels for 356×406 mm ×634 kg m^{-1} UC

for frames braced against joint translation for section 152×152 mm ×23 kg m^{-1} UC to be chosen on site during construction varies with clear column heights as shown in Table 2 and 3.

This concept applies to other column sections in Fig. 2-11 where an increase in height between 1 and 7 m induces failure as modification factor C_b is varied. However, from Fig. 12-32, it is observed that an increase in span between 1 and 7 m induces no failure as modification factor C_b is varied.

CONCLUSION

A theoretical evaluation of the modification factor for frames braced against joint translation is carried out. From the foregoing, it can be seen that for all sections of frames braced against joint translation, the modification factor, C_b and safety index β, can be predicted when faced with challenges on site as results indicate.

Also, certain universal column sections will not fail between the heights investigated regardless of direction of end moments, moment ratio M_1/M_2 and modification factor C_b. However, some of these columns will fail between spans of 4 and 7 m, even when they appear safe as the end moment ratio M_1/M_2 and modification factor C_b increases after the first failure point. This generally means that if these studies, which appear safe after the first failure criteria need be used, then the C_b values obtained from end moment ratio should be limited by providing adequate bracings at required intervals along the column heights.

Furthermore, it was observed that members in single curvature (end moments M_1 and M_2 in opposing directions) have higher tendencies of failure than those in double curvature (when end moments M_1 and M_2 are in the same direction).

Notations:

J	=	Torsional constant
S_x	=	Elastic section modulus about major axis
E	=	Modulus of elasticity of steel
G	=	Shear modulus of elasticity of steel
F_{cr}	=	Buckling stress
I_y	=	Moment of inertia about y-axis
M_n	=	Nominal flexural strength
C_w	=	Warping constant
P_f	=	Probability of failure
F_y	=	Minimum yield stress of type of steel used
β	=	Safety index
π	=	Pie
C_m	=	Coefficient for no lateral translation of frame
h_o	=	Distance between flange centroids
C_b	=	Lateral-torsion buckling modification factor
$G(x)$	=	Performance function
M_p	=	Plastic bending moment
M_{cr}	=	Moment capacity
l_b	=	Distance between points braced against lateral displacement of the compression flange
R	=	Reliability
r_{ts}	=	Effective radius of gyration
l_r	=	Limiting laterally unbraced length
Z_x	=	Plastic section modulus about major axis
r_y	=	Radius of gyration about y-axis
k	=	Restraint factor

REFERENCES

AISC, 1999. Load and resistance factor design specification for structural steel buildings. American institute of steel construction, Chicago, Illinois, USA. ANSI/AISC 360-05. www.aisc.org/1999spec.

AISC, 2005. Specification for structural steel buildings. American institute of steel construction, Inc., Chicago, Illinois, USA. www.aisc.org/2005spec.

BS 5950-1, 2000. Structural use of steel work in buildings. Part 2: Code of practice for design-rolled and welded sections. British Standards Institution, London. ISBN: 0-580-33239-X.

Ditlevsen, O. and H.O. Madsen, 1996. Structural Reliability Methods. John Wiley and Sons Limited, West Sussex, England, pp: 1-162. ISBN: 0-471-96086-1,TA656. D5713.

Ellingwood, B., J. MacGregor, T.V. Galambos and C.A. Cornell, 1982. Probability based design load criteria in load factors and load combination. J. Structural Division. ASCE., 108 (ST4): 978-995.

Gollwitzer, S., T. Abdo and R. Rackwitz, 1988. First Order Reliability Method (FORM) Manual. RCP GMBH, Munich, Germany, pp: 1-47.

ICC Evaluation Service, Inc., 2003. Acceptance Criteria for Buckling-Restrained Braced Frame Sub-Assemblages. AC238, pp: 1-6.

Kuchenbecker, G.H., D.W. White and A.E. Surovek-Maleck, 2004. Simplified design of building frames using first-order analysis and K = 1.0. Proceedings. SSRC Ann. Technol. Sessions, pp: 1-20.

Melchers, R.E., 1987. Structural Reliability: Analysis and Prediction. 1st Edn. Ellis Horwood Limited, West Sussex, England, pp: 1-322. ISBN: 0-85312-930-4, TA656.5.

SCI, 2005. Steel Designers Manual. 6th Edn. In: Davison, B. and G.W. Owens (Eds.). Blackwell Science Limited, Oxford, UK. ISBN: 13:978-14051-3412-5.