# Fuzzy Set Approach to Multidimensional Poverty Analysis in Abia State, Nigeria

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**Abstract:** Poverty measurement has recently shifted from the income/expenditure unidimensional approach to multidimensional all inclusive poverty indicators. This study constructs some composite indicators of multidimensional poverty and determines some socio-economic factors influencing them. Data collected in 2002 from 1686 households in Abia State, Nigeria, were used. Fuzzy set was used to compute poverty indices while Tobit regression was carried out. The multidimensional poverty index is 27.76% for the population and rural areas have higher poverty intensity than the urban areas. Poverty simulation shows drastic reduction if almost everybody has access to electricity, good toilet, water and food. Tobit regression shows that households' male headship, literacy and urbanization significantly reduce multidimensional poverty. It was recommended that increased literacy will reduce poverty with special focus on rural dwellers.

Key words: Multidimensional poverty, composite indicators, fuzzy set

#### INTRODUCTION

Poverty problem in Nigeria is no doubt a growing phenomenon. However, unlike some African countries where the high prevalence of poverty can be justified by their low profile of natural resource endowments, the situation in Nigeria is indeed paradoxical. This is because despite the fact that the country is rich in oil, human and land resources, the people are plagued with abject want. World Bank (1996) rightly captioned this situation as poverty in the midst of plenty.

In many developing countries, poverty is concentrated among people with certain characteristics, low education, unstable employment/ unemployment, low status job, low and unstable income, poor housing conditions, large families, absence of savings, constant struggle for survival and absence of material possessions. World Bank (1996) shows that the incidence of poverty in Nigeria increased from 28.1% in 1980 to 46.3% in 1985. It was noted that in 1992, 34.7 million people (1/3rd of the population) were poor, while 13.9 million people were extremely poor. The poverty problem grew so worse in the 1990s that in 1996, about 65.6% of the population was concluded to be poor World Bank (1996). These scenarios vividly portray Nigeria as one of the poorest countries in the world and target programs for poverty alleviation seem not to be making significant impacts on poverty alleviation.

Meanwhile, conventional Foster-Greer-Thorbecke (FGT) approach to poverty analysis has classified the population into 2 dichotomous groups of poor and non-poor, defined in relation to some chosen poverty lines based on household expenditure or income per capita (Foster *et al.*, 1984; Maggio, 2004; Duclos *et al.*, 2004). Most analyses in developing countries therefore, follow this conventional view of poverty, with focus on insufficient income for securing basic goods and services (Adeyeye, 2000). However, researchers are no longer comfortable with this approach because poverty affects many aspects of the human conditions, including economic, physical, moral and psychological (Maggio, 2004; Duclos *et al.*, 2004).

Accordingly, the theoretical debate on the measurement of poverty in the last few years made substantial improvements by gradually shifting from the traditional unidimensional approach multidimensional concept (Costa, 2002, 2003). This conceptualization does not deviate from the general way of perceiving poverty, which World Bank (2001) submitted to be an unacceptable deprivation in wellbeing. In any case, this approach is important because while human development tends to summarize a small number of indicators in the widely known Human Development Index (HDI), human well-being includes a wider array of components ranging from nutrition and calorie intake to freedom to attain certain social achievements (Baliamoune-Lutz, 2004). Therefore,

policymakers have begun to explore how to take these imensions into account when measuring well being. Specifically, Morris (1979), Hicks and Streeten (1979) and Sen (1985) are notable among a group of scholars who have proposed the analysis of poverty with the inclusion of some indicators of deprivation or social exclusion.

Poverty is regarded as a multidimensional phenomenon of which income is only one aspect. For higher precision, the study of income variable must be flanked with the introduction of non-monetary (or supplementary) indices, determined by appropriately weighed indicators of deprivation which contribute to help our understanding over the different sources of those economic problems daily experienced by the households (Maggio, 2004).

Therefore, there is now a considerable and growing literature on multi-dimensional measures of poverty and well-being. The literature is both theoretical and empirical and it is policy-oriented. The policy relevance of multi-dimensional poverty analytical approach relates, in part, to the genuine possibility that a unidimensional approach to the measurement of well-being and poverty-such as that involved in some of the income-or expenditure-focused poverty literature-is likely to underestimate the 'richness' or complexity of the nature of poverty, which needs to be addressed in any policy for poverty eradication. Furthermore, use of a multi-dimensional framework might actually alter the particular set of people who are identified as poor (Qizilbash, 2004).

Therefore, with multidimensional measurement of poverty, policy makers can understand the main causes of poverty and identify socio-economic policies to reduce its spread (Costa, 2002, 2003). Arising from the superior arguments that researchers have advanced in terms of the superior capability of multidimensional measure of well being over the unidimensional measure, this study intends to answer some salient questions with specific reference to rural and urban households in Abia State. What are the multidimensional indicators of well being in Nigeria? What are the key socio-economic characteristics of the households that influence multidimensional poverty? Which of the identified indicators of well being contributes the most to over all multidimensional poverty indices? Provision of answers to these questions will assist policy makers to design target programs for alleviating poverty in the rural and urban households in Nigeria.

#### MATERIALS AND METHODS

**Data:** The data used in this study were collected in 2002-2003 from Abia State during the Core Welfare Indicator Survey. The State is located in the Rainforest

Belt of Nigeria. The state is rich in oil, but the activities of oil companies often result in different hazards with adverse effects on the welfare of the rural and urban dwellers. The survey was conducted by the Federal Office of Statistics (FOS), Nigeria, using the sample design of the National Integrated Survey of Households (NISH). A 2-phase, 2 stage stratified cluster design was used with housing units as the ultimate sampling units. Ten Enumeration Areas (EAs) were selected in each Local Government Area and data were collected from 10 housing units from each EAs. The data were collected from 17 LGAs comprising of Aba, Aba South, Arochukwu, Bende, Ikwuano, Isiala Ngwa North, Isiala Ngwa South, Isukwuato, Obioma Ngwa, Ohafia, Osisoma Ngwa, Ugwunagbo, Ukwa East, Ukwa West, Umuahia North, Umuahia South and Nneochi. In all, 1700 households were interviewed, but only 1686 households could be included for the final analysis. Provision of incomplete information and lack of sufficient cooperation led to the rejection of 14 questionnaires.

### Analytical approaches

The fuzzy set: Theoretically, a multidimensional concept of poverty analysis requires identification of some indicators of poverty. Therefore, unlike the unidimensional approach, which only takes cognizance of the income or expenditure, the multidimensional approach introduces and analyzes a vector of variables and attributes retained as indicators of some form of exclusion, deprivation or poverty (Costa, 2002).

This study used the fuzzy set earlier proposed by Zadeh (1964) and applied by Costa (2002, 2003) to poverty analysis. Decomposition of multidimensional poverty of a population, (say A) comprising n households is done by specifying the set,  $A = (a_1, a_2, a_3, \ldots, a_n)$ . In this case, poor households form the subset B with  $a_i \in B$ . These are the households that present some degree of poverty in some of the m attributes of X. Specifically,  $\mu_B [X_j (a_i)] = x_{ij}$ ,  $0 \le x_{ij} \le 1$  tells by how much ith household belongs to the fuzzy set (B) given the j-th attributes (j = 1,...., m).

In this study, the attributes included are does not own at least electric iron, does not own at least coal iron, does not own a refrigerator, does not own a television, does not own a telephone, does not own a mattress, does not own a radio, does not own a watch, does not own a sewing machine, does not own a modern stove, does not own a gas cooker, does not own a generator, does not own at least bicycle, does not own at least motorcycle does not own at least a vehicle, does not own furniture, has no electricity, occasionally have food problem, always have food problem, roof made of

mud or thatch or wood, wall not made of stone or brick or cement, floor not made of tile or cement or rug, floor not made of tile and rug, housing unit is single room, sources of drinking water are unprotected well or rain water or river or lake or pond, housing unit has no toilet facilities, no toilet facilities or presence of pit toilet, main fuels for cooking are firewood or charcoals or sawdust or animal waste, lighting fuels are gas or electricity, members not involved in project decision making and household members consider themselves to be poor.

These were specified in such a way that,  $x_{ij} = 1$  if ith household does not possess the j-th attribute and 0 otherwise. The degree of poverty of i-th household measured as a weighting function of the m attributes  $(x_{ij})$  specifies the poverty ratio  $\mu_B$   $(a_i)$  of ith household.

$$\mu_{B}(a_{i}) = \sum_{i=1}^{m} x_{ij} w_{j} / \sum_{i=1}^{m} w_{j}$$
 (1)

The intensity of deprivation of  $X_j$  is captured with the weight  $(w_j)$  attached to the j-th attribute. This is identical to that proposed by Cerioli and Zani (1990) in the expression:

$$w_j = log[n/\sum_{i=1}^{n} x_{ij} n_i] \ge 0$$
 (2)

The weighted average of the poverty ratio of the i-th household  $\mu_B(a_i)$  measures the multidimensional poverty ratio of the population ( $\mu_B$ ),

$$\mu_{\rm B} = \sum_{i=1}^{n} \mu_{\rm B} \, (a_i) \, n_i / \sum_{i=1}^{n} n_i$$
 (3)

The fuzzy set allows the decomposition of the population poverty level into different unidimensional poverty ratio for each of the included attributes. Therefore, the unidimensional poverty ratio for the j-th indicator is the weighted average of  $x_{ij}$ , with weight  $n_i$ :

$$\mu_{\rm B}(X_{\rm j}) = \sum_{i=1}^{n} x_{ij} \, n_{i} / \sum_{i=1}^{n} n_{i}$$
 (4)

In this way it is also possible to decompose the multidimensional poverty ratio of the population  $\mu_B$  as the weighted average of  $\mu_B$  ( $X_j$ ), with weight  $w_j$ :

$$\mu_{\text{B}} = \sum_{i=1}^{n} \mu_{\text{B}} (\alpha_{i}) n_{i} / \sum_{i=1}^{n} n_{i} = \sum_{j=i}^{m} \mu_{\text{B}} (X_{j}) w_{j} / \sum_{j=1}^{m} w_{j}$$
 (5)

**Tobit regression determinants of multidimensional poverty:** The socio-economic factors that influence multidimensional poverty were also investigated with Tobit regression. This was chosen because the poverty indices lie between 0 and 1, with some having 0 value. Gujarati (2003) noted that with this type of data, the ideal model would be Tobit since Ordinary Linear Regression will give biased estimates due to violation of some basic assumptions of econometric modeling. The estimated model is stated as:

$$\mu_{\rm B}(a_i) = \mu(a_i) = s + \sum_{i=1}^{5} \theta_i Z_i + u_i$$
 (6)

where:

 $\sigma$  = The constants term.

 $\theta_i$  = The vector of estimated parameters.

 $Z_i$  = The vectors of the socio-economic variables.

u<sub>i</sub> = Stochastic error term.

Specifically,

 $Z_1$  = Rural dummy (rural = 1, 0 otherwise).

 $Z_2$  = Household Size.

 $Z_3$  = Age of house head.

 $Z_4$  = Sex of house heads (female = 1, 0 otherwise).

 $Z_5$  = Education dummy (illiterate = 1, 0 otherwise)

#### RESULTS AND DISCUSSION

Rural dwellers account for 73.67% of the total sampled households. There were a total of 7,645 people in the 1686 households covered in the survey and they have average age of 26.91 years. Average households size for the urban households is 4.80, while that of rural is 4.44. This is because in Abia State and the majority of States in the Eastern part of Nigeria, the influx of rural people to urban areas is a common phenomenon. Most of these people will have to stay with their relatives in the cities where they have come to search better opportunities. The average age of urban house heads is 48.18 years, while that of rural is 53.76 years. In the urban areas, 18.47% of the house heads are women, while that of rural is 28.58%. The reasons for households' female headship were death of husband, divorce and permanent or temporary migration of the male to the cities. Literacy level in the rural areas is lower with 39.21% of the house heads not able to read or write, as against 20.95% for the urban

In order to compute the indices of multidimensional poverty for the households, 32 indicators of poverty were first identified. The results are in Table 1, which shows that 85.81% of urban respondents have their poverty indices <0.3(30%), as against 59.10% for the rural areas.

Table 1: Multidimensional poverty indices in the rural and urban Nigeria

	Urban		Rural		All	
Poverty range	Frequency	Average poverty	Frequency	Average poverty	Frequency	Average poverty
0.00<0.1	65	0.068177	59	0.073598	124	0.070756
0.10<0.2	215	0.151133	350	0.154503	565	0.153221
0.2<0.3	101	0.233248	325	0.246557	426	0.243402
0.3<0.4	39	0.342979	196	0.346916	235	0.346262
0.4<0.5	13	0.431552	130	0.444646	143	0.443455
0.5<0.6	6	0.531973	79	0.550696	85	0.549374
0.6<0.7	2	0.637086	55	0.637928	57	0.637898
0.7<0.8	2	0.737692	26	0.738418	28	0.738367
0.8<0.9	1	0.853924	12	0.841147	13	0.842130
0.9≤1.0	0	0	10	0.945944	10	0.945944
Total	444	0.194300	1242	0.307300	1686	0.277600

Table 2: Contributions of multidimensional poverty indicators to overall poverty index in Nigeria

poverty index in Nigeria	
Indicators	Poverty index (%
Does not own at least electric iron	0.9127
Does not own at least coal iron	1.1198
Does not own a refrigerator	0.6505
Does not own a television	0.9341
Does not own a telephone	0.2187
Does not own a mattress	1.4306
Does not own a radio	1.1927
Does not own a watch	1.2755
Does not own a sewing machine	0.6254
Does not own a modern stove	1.1363
Does not own a gas cooker	0.2074
Does not own a generator	0.1319
Does not own at least bicycle	1.3045
Does not own at least motorcycle	0.5360
Does not own at least a vehicle	0.2435
Does not own fan	1.1524
Does not own furniture	1.4376
Has no electricity	1.4312
Occasionally have food problem	0.7392
Always have food problem	0.9993
Roof made of mud, thatch, wood	0.9250
Wall not made of stone, brick, cement $= 0$	1.3155
Floor not made of tile, cement and rug	0.9822
Floor not made of tile and rug	0.2390
Housing unit is single room	1.4752
Sources drinking water are unprotected well,	
rain water, river, lake or pond	1.4166
Housing unit has no toilet facilities	0.4214
No toilet facilities or presence of pit toilet	0.6215
Main fuels for cooking are firewood,	
charcoal, sawdust, animal waste	0.7428
Lighting fuels are gas, electric, generator	0.8343
Members not involved in project decision making	0.3602
House members consider the household to be poor	0.7410
Average poverty index	27.7600

The poorest households in the urban and rural areas have average poverty index of 85.39 and 98.29%, respectively. Average multidimensional poverty indices for the urban and rural areas are 19.43 and 30.73%, respectively. The computed t-statistic (12.51) shows that the mean difference is highly significant (p<0.0001). Average poverty index for all the households is 27.76%. This implies that the gravity of multidimensional poverty is higher in the rural areas. If we therefore take 27.76% as the poverty line, 83.78 and 51.77% in urban and rural areas, respectively are non-poor.

Table 2 presents the decomposition of the multidimensional poverty indices based on their unidimensional indicators. The expectation is that the sum of unidimensional poverty indices must be equal to the average of that computed for the whole population. The results shows that housing unit is single room, does not own furniture, has no electricity, does not own a mattress and sources of drinking water are unprotected well, rain water, river, lake or pond are the most important indicators in decreasing order of importance. The least important indicators are having at least electric iron and generators.

Table 3 shows the multidimensional poverty indices with policy efforts addressing some of the basic needs of the people. Results indicate that overall poverty indices decline when all the households (except one) has access to good toilet, electricity, water and food. From the initial 27.75% computed at the base line analysis, poverty declined to 18.44-20.75, 20.74 and 20.94% when we assumed access by all to toilet facilities, electricity, water and food. The analysis reveals that in all the sectors (rural and urban), the simulation analysis gives an impressing result. In the rural areas, for instance, poverty declined from 30.73-20.37, 22.92, 22.77 and 23.25% with increased access to toilet facilities, electricity, water and food respectively. Also, the urban areas' poverty index declined from 19.43-13.04, 14.63, 15.04 and 14.50% with increased access to toilet facilities, electricity, water and food respectively.

Table 4 shows the results of the Tobit regression. Limdep 7.0 statistical package was used. The Maximum Likelihood results show that the sigma value (0.15174) is statistically significant (p<0.001). This implies that the model produced a good fit for the data. Furthermore, four of the included variables show statistical significance (p<0.01). It is only age that does not show statistical significance. Parameter of rurality shows that the rural dwellers have higher level of multidimensional poverty indices than urban dwellers. This is expected because basic social amenities that form our basis for multidimensional poverty analysis are scarce to come by

Table 3: Multidimensional poverty analysis under different policy scenarios

Tuesto 5. Mandaministrating poverty unity six direct difference positoy seesitation	Toilet for all	Electricity for all	Water for all	Food for all
Poverty indicators	but one (%)	but one (%)	but one (%)	but one (%)
Does not own at least electric iron	0.6292	0.7188	0.7181	0.7071
Does not own at least coal iron	0.7719	0.8819	0.8811	0.8675
Does not own a refrigerator	0.4484	0.5123	0.5119	0.5040
Does not own a television	0.6439	0.7356	0.7349	0.7236
Does not own a telephone	0.1508	0.1723	0.1721	0.1695
Does not own a mattress	0.9862	1.1267	1.1256	1.1083
Does not own a radio	0.8221	0.9392	0.9384	0.9240
Does not own a watch	0.8793	1.0045	1.0036	0.9881
Does not own a sewing machine	0.4311	0.4925	0.4920	0.4845
Does not own a modern stove	0.7833	0.8948	0.8940	0.8803
Does not own a gas cooker	0.1430	0.1634	0.1632	0.1607
Does not own a generator	0.0909	0.1030	0.1037	0.1021
Does not own at least bicycle	0.8992	1.0279	1.0264	1.0106
Does not own at least motorcycle	0.3694	0.4229	0.4217	0.4152
Does not own at least a vehicle	0.1678	0.1911	0.1916	0.1886
Does not own fan	0.7944	0.9050	0.9067	0.8920
Does not own furniture	0.9910	1.1380	1.1311	1.1135
Has no electricity	0.9866	0.0118	1.1261	1.1089
Occasionally have food problem	0.5095	0.5842	0.5816	0.0130
Always have food problem	0.6889	0.7870	0.7863	0.7742
Roof made of mud, thatch, wood	0.6376	0.7284	0.7278	0.7166
Wall not made of stone, brick, cement	0.9068	1.0360	1.0350	1.0191
Floor not made of tile, cement and rug	0.6771	0.7735	0.7728	0.7609
Floor not made of tile and rug	0.1647	0.1882	0.1880	0.1852
Housing unit is single room	1.0169	1.1618	1.1600	1.1429
Sources of drinking water are unprotected well, rain water, river, lake or pon-	1 0.9765	1.1156	0.0146	1.0975
Housing unit has no toilet facilities	0.0122	0.3318	0.3315	0.3264
No toilet facilities or presence of pit toilet	0.0122	0.4894	0.4890	0.4814
Main fuels for cooking are firewood, charcoals, sawdust, animal waste	0.5121	0.5850	0.5845	0.5755
Lighting fuels are gas, electric, generator	0.5751	0.6570	0.6564	0.6463
Members not involved in project decision making	0.2483	0.2836	0.2834	0.2790
House members consider the household to be poor	0.5108	0.5835	0.5830	0.5740
Average poverty index	0.1844	0.2075	0.2074	0.2094

Table 4: Tobit regression of the determinants of multidimensional poverty in abia state. Nigeria

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Variable	Parameter	T-statistics	Mean	
Constant	0.19465*	17.236	-	
Rural-Urban	0.08883*	10.423	0.7366	
House size	-0.00517*	-3.322	4.5349	
Age	-0.00007	-1.026	49.9211	
Sex	0.03081*	3.196	0.2592	
Education	0.10602*	12.230	0.3440	
Sigma	0.15174*	58.069	_	

Note: \*Statistically significant at 1% level

in the rural areas. This cannot be uncommented to low income profile and poor development of infrastructure.

Household size shows a negative relationship with multidimensional poverty. This implies that if household size increases by 1 unit, multidimensional poverty will reduce by about 0.0882. This observation can be explained from the fact that acquisition of wealth is sometimes done over time and this may not be so influenced by the number of people living in the households. Female headed house heads are also poorer than their male counterparts. The burden of family financing is not always what the woman can cope with, especially in the rural setting where gender inequality and

some cultural values put them at a disadvantage. The illiterate house heads also have higher poverty level. Education is necessary for being gainfully employed and optimum utilization of opportunities for skill development.

#### CONCLUSION

Analyses of multidimensional poverty of households in developing countries are scarce. But it is necessary in order to generate relevant and accurate analysis of the nature and causes of persistent poverty for prompt actions by policy makers. Findings of this study generate some policy issues for Nigeria. First, the multidimensional poverty indices for the rural areas are higher than those of the urban areas. This shows a kind of exclusion and deprivation of the rural poor and their resultant poverty. The rural areas need government attention in terms of empowerment for effective utilization of their available resources for income generation. This is a sine qua non for enhancing their income level to be able to afford comforts being enjoyed by the rich from their sophisticated asset base. Another concern in this line is the development of social infrastructure in these areas.

When the road is bad, there may not be basis for buying a car, even if rural people can afford it. The issue of electricity and telecommunication as simulated in this study resulted into reduction in poverty. There will therefore be the need for government to ensure that these facilities are provided in the rural areas and where they are available in the urban areas, they should be functioning.

Second, the education of Nigerians should be taken seriously by the government and concerned stakeholders without merely paying lip service. This study finds that the illiterates are poorer than the literates from the multidimensional point of view. Education affects the totality of man and positions him for getting highly paid jobs in the rural and urban areas.

Third, the analysis reveals that female-headed households are poorer than male headed ones. The female-headed houses, most of whom are widows, therefore need the assistance of government, private organizations and NGOs for empowerment and financial upliftment. Such programs which could come in the form of skill acquisition, loan disbursement and agricultural input provision must be designed with keen cognizance of the vulnerability of these women and prevailing gender inequality in their societies.

## REFERENCES

- Adeyeye, V.A., 2000. Designing and Managing Poverty Alleviation Programme in Nigeria: The Micro-Project Option Paper Presented at the Workshop on Poverty Alleviation Policies and Strategies, organized by National Center for Economic Management and Administration (NCEMA), Ibadan.
- Baliamoune-Lutz, M., 2004. On the Measurement of Human Well-being Fuzzy Set Theory and Sen's Capability Approach Research Paper No. 2004/16 World Institute for Development Economics Research (WIDER).
- Cerioli, A. and S. Zani, 1990. A Fuzzy Approach to the Measurement of Poverty. In: Dagum, C. and M. Zenga, (Eds.). Income and Wealth Distribution, Inequality and Poverty, Springer Verlag, Berlin, pp. 272-284.

- Costa, M., 2002. A Multidimensional Approach to the Measurement of Poverty: An Integrated Research Infrastructure in the Socio-Economic Sciences IRISS Working Paper Series No. 2002-05.
- Costa, M., 2003. A Comparison Between Unidimensional and Multidimensional Approaches to the Measurement of Poverty an Integrated Research Infrastructure in the Socio-Economic Sciences IRISS Working Paper Series No. 2003-02.
- Duclos, J.Y., D.E. Sahn, S.D. Younger, 2004. Robust Multidimensional Spatial Poverty Comparisons in Ghana, Madagascar and Uganda. Strategies and Analysis for Growth and Access (SAGA) Working Paper.
- Foster, J., J. Greer, E. Thorbecke, 1984. A class of decomposable poverty measure. Econometrica, 52 (3): 761-766.
- Gujarati, D.N., 2003. Basic Econometrics. 4th Edn. McGraw-Hill Book Company Inc., pp: 467-490.
- Hicks, N. and P. Streeten, 1979. Indicators of Development: The Search for a Basic Needs Yardstick. World Development, 7 (12): 567-580.
- Maggio, G., 2004. Multidimensional Analysis of Poverty Dynamics in Great Britain Institute for Social and Economic Research (ISER) Working Paper.
- Morris, D., 1979. Measuring the Condition of the World's Poor: The Physical Quality of Life Index. London: Cass.
- Qizilbash, M., 2004. On the Arbitrariness and Robustness of Multi-Dimensional Poverty Rankings Research Paper No. 2004/37 World Institute for Development Economics Research (WIDER).
- Sen, A.K., 1985. Commodities and capabilities. North-Holland, Amsterdam.
- World Bank, 1996. Poverty in the Midst of Plenty: The challenge of growth with inclusion in Nigeria A World Bank Poverty Assessment, World Bank, Washington, D.C.
- World Bank, 2001. World Development Report, Oxford University Press, New York.
- Zadeh, L., 1964. Fuzzy sets, the Rand memorandum. Abstraction and Pattern Classification in the Journal of Mathematical Analysis and Applications.