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On the Existence and Uniqueness of a Two-Step Arrhenius Reaction with Srong Viscous Dissipation Term

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Abstract: This study studies the effect of activation energies and strong viscous dissipation term on two-step Arrhenius combustion reactions to give further insight into the theory of combustion under physical reasonable assumptions. We extend the non-uniformity of vessels discussed in Olanrewaju. It is shown, as in one step reaction that in a non-uniform vessel, maximum temperature occurs toward the end of the tube. On the other hand, in a uniform tube maximum temperature occurs at the centre. Also maximum temperature for diverging or converging channel is greater than that of a uniform vessel. The graph shows effect of viscous dissipation term on the temperature of the system.

Key words: Arrhenius reaction, strong viscous, combustion reaction, uniform vessel, viscous dissipation

INTRODUCTION

In some cases the one step reaction is sufficient to explain some reaction processes even when there are intermediary steps.

However, in the cases of cool flames, Williams (1985) one step reaction has been found to be insufficient to explain all the features observed in the cool flames phenomenon, one therefore needs at least 2 steps. Also a cigarette like burning requires more than one step according to koriko and Ayeni (2001). Similarly, combustion taking place within k-fluid (Shubrin, 2001) is treated as a two-step irreversible chemical reaction of methane oxidation as follows:

Step 1: $CH_4 + 1.5(O_2 + 3.76N_2) = CO + 2H_{20} + 5.64N_2$

Step 2: $CO + 0.5(2_2 + 3.76N_2) = CO_2 + 1.88N_2$

Olanrewaju (2005) examined solutions of two step reactions with variable thermal conductivity. He considered not only the generalized temperature dependences of reactions rate, but he also proposed suitable approximation of the kinetic reactions in the limit of large/small activation energy. He also discussed the effects of non-uniform vessel .Numerical investigations were carried out on the explosion and ignition processes with two steps mechanism as well as the effect of variational thermophysical properties. The method of high activation energy asymptotic is used in the analysis. The time-to-ignition was illustrated graphically.

Moreover , Gbolagade and Makinde (2005) examined the thermal ignition in a strongly exothermic reaction of a variable viscosity combustible material flowing through a channel with isothermal walls under Arrhenius kinetic, neglecting the consumption of the materials. Analytical solutions are constructed for the governing nonlinear boundary value problem using perturbation technique together with a special type of Hermite-pade approximant and important properties of the temperature field including bifurcations and thermal criticality are discussed.

In this study, we extend the word of Olanrewaju (2002) to determine the effects of activation energies and strong viscous dissipation term on 2-step reactions.

MATHEMATICAL FORMULATION

In this study, we consider the steady state energy equation of the form

$$\rho c_p\, u \frac{dT}{dx} = k\frac{d^2T}{dx^2} + Q_1 A_1 x^a T^n e^{-E_1/RT} + Q_2 A_2 Y^b T^m e^{-E_2/RT} + \mu\left(\frac{\partial u}{\partial x}\right)^2 \qquad (1)$$

with the continuity equation for a converging vessel of the form

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$$\rho A u = \text{constant} = c \tag{2}$$

We let

$$A = A_0\, e^{-wx} \tag{3}$$

Where A_0 and w are constant and A is the cross-section area. And

k	=	Tthermal conductivity
R	=	Universal gas constant
T	=	Temperature
ρ	=	Density
c_p	=	Specific heat at constant pressure
A_i	=	Pre-exponential factors ,i = 1, 2
x	=	Mass fraction of species A
Y	=	Mass fraction od species B
a	=	Reaction order of species A
b	=	Reaction order of species B
x	=	Space variable
Q_i	=	Heat release/unit mass, i = 1, 2
μ	=	Viscosity
$\mu\left(\frac{\partial u}{\partial x}\right)^2$	=	Viscous dissipation term
n, m	=	Numerical exponents of temperature
u	=	Velocity

METHOD OF SOLUTION

We take the advantages of the suitable dimensionless variables

$$\left.\beta = \frac{RT_0}{E_1},\ \theta = \frac{T - T_0}{\beta T_0},\ u' = \frac{u}{v_0}\ r = E_2/E_1\right\} \tag{4}$$

Putting (4) into (1) with (2) and (3), (1) becomes

$$e^{wx}\frac{d\theta}{dx} = \lambda\frac{d^2\theta}{dx^2} + \delta_1\left(1+\beta\theta\right)^n e^{\theta/1+\beta\theta} + \delta_2\left(1+\beta\theta\right)^m e^{r\theta/1+\beta\theta} + \gamma\, e^{2wx} \tag{5}$$

under the conditions

$$\theta(-1) = \theta(1) = 0 \tag{6}$$

Where

$$\lambda = \frac{k\beta T_0 A_0}{c_p \rho u' v_0}$$

$$\delta_1 = \frac{Q_1 A_1 A_0 X^a}{c_p \rho u' v_0 T_0 \beta}\ e^{-E_1/RT_0}$$

$$\delta_2 = \frac{Q_2 A_2 A_0 Y^b}{c_p \rho u' v_0 T_0 \beta}\ e^{-rE_1/RT_0}$$

$$\gamma = \left(\frac{wc}{\rho A_0}\right)^2$$

EXISTENCE AND UNIQUENESS OF SOLUTION

Theorem 1 (Olanrewaju, 2002): Let D denote the region (n (n+1)).

Dimensional space,one dimension for t and n dimensions for vector x] $|t-t_0| \le a$, $\|x-x_0\| < b$.
If

$$\begin{cases} x_1^1 = f_1\left(x_1, x_2, \ldots, xn, t\right),\ x_1\left(t_0\right) = x_{10} \\ x_2^1 = f_2\left(x_1, x_2, \ldots, xn, t\right),\ x_2\left(t_0\right) = x_{20} \\ . \\ . \\ . \\ x_n^1 = f_n\left(x_1, x_2, \ldots, xn, t\right),\ x_n\left(t_0\right) = x_{n0} \end{cases} \tag{7}$$

Then, the system of Eq. (4) has a unique solution of

$$\frac{\partial f_i}{\partial x_j},\quad i, j=1,2,\ldots,n$$

are continuous in D.

(H1): $\Gamma > 0,\ 0 \le x_1 \le 1,\ \ 0 \le x_2 \le b$ and $c \le x_3 \le C$, where b, c and Care positive constants.

Theorem 2: IF (H1) holds then problem (1) has a unique solution satisfying (2).

Proof: We let

$$y^1 = \begin{pmatrix} x_1^1 \\ x_2^1 \\ x_3^1 \end{pmatrix} = \begin{pmatrix} g_1\left(x_1, x_2, x_3\right) \\ g_2\left(x_1, x_2, x_3\right) \\ g_3\left(x_1, x_2, x_3\right) \end{pmatrix} \tag{8}$$

Clearly $\frac{\partial f_i}{\partial x_j}$ is bounded for i = 1, 2, 3.

Thus, g_i, i = 1, 2, 3 are lipschitz continuous. Hence there exists a unique solutions of Eq. 1 and 2.

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