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Modification of Darcy's Law for Turbulent Flow in Saturated Porous Media

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Abstract: Darcy's law is an essential equation in determining the permeability of porous media, which is vital tool in seepage and drainage control in soils. However, whenever the aggregate sizes of the porous media and hydraulic gradient are large, the flow in drains will be semi-turbulent to turbulent. This research aims at modifying the true Darcy's permeability determined under small hydraulic gradients that ensure laminar or nearly laminar flow to allow for reduced efficiency caused by turbulence at greater hydraulic gradient and provide a relation between the maximum gradient for laminar flow and porosity of the porous media. A modeled experiment was set up using sand samples of different porosities from riverbed as porous media, which were packed in a vertical transparent cylinder tube of diameter 1.85×10^{-2}m and relative permeabilities were determined for gradient ranging from 1.15-15.00. The result of the experiment shows that relative permeability increases with increasing porosity but decreases with increasing gradient. The maximum (or limiting) hydraulic gradient for laminar flow decreases with increasing porosity. Also, the vertical fluid flow in any porous medium is laminar or nearly laminar as much as hydraulic gradient is less than or equal to 1.04.

Key words: Hydraulic conductivity, permeability, laminar flow, turbulence flow, porosity

INTRODUCTION

The permeability of a porous medium is its capacity for transmitting a fluid under the influence of a hydraulic gradient (Sherwani, 1978). When flow is laminar, the velocity of flow increases in direct proportion to the hydraulic gradient. When flow is semi-turbulent to turbulent, the seepage velocity increases at smaller rate than gradient (Cedergreen, 1976). For laminar flow, ration of volume flux rate to gradient is a constant regardless of hydraulic gradient, but for semi-turbulent to turbulent flow ratio of volume flux rate to gradient diminishes with increasing hydraulic gradient (Cedergreen, 1976).

Turbulent flow may occur in cavernous limestone ad volcanic rocks, where the passage way may be large, or in coarse gravels, particular in the vicinity of a discharging wells (Sherwani, 1978). When water is flowing in highly permeable materials, the condition can vary from laminar or nearly laminar flow at very small gradients to semi-turbulent or nearly turbulent as the gradient become larger (Cedergreen, 1976).

Darcy's law is applicable only to laminar flow (Sower and Sower, 1970). A variety of practical solution to seepage and drainage problem is developed with flow nets and validity of Darcy's law and the assumption that flow is laminar, not turbulent. In coarse natural formations and coarse open-graded aggregate in drainage system it is likely that flow will sometimes be semi-turbulent to turbulent (Cedergreen, 1976). This research provides a method for adapting Darcy's law to this kind of flow.

Theory: It has been found experimentally that Darcy's law can be expressed as

$$q = c_1 i + c_2 \tag{1}$$

where,

- q = Volume flux rate (m/s);
- i = Hydraulic gradient ;
- c_1 = Hydraulic conductivity (m/s);
- c_2 = Intercept along the hydraulic gradient and
- c_1/c_2 = D, deviation from Darcy's law

However, from the generalized form of Darcy's law which is written as

$$V_s = -\frac{K}{\mu}\left(\frac{dp}{ds} - pg\frac{dz}{ds}\right) \tag{2}$$

where,

- s = Distance in direction of flow, always positive.
- V_s = Volume flux across a unit area of the porous medium in unit time along the flow paths

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z = Vertical coordinate considered downward
ρ = Density of the fluid
k = Permeability of the medium
dp/ds = Pressure gradient along s at the point to what V_s refers and
dz/ds = Sinθ where θ is between s and the horizontal V_s

For case of vertical downward flow when driving head is h

h = head constant
L = Length of the sample (porous medium)

θ = 90° (which is the highest gradient) then,
dz/ds = Sin 90° = 1 and by integrating $\frac{dp}{ds}$ in Eq. 2,

$$\frac{dp}{ds} = -\frac{\rho g h}{L}$$

By substituting for $\frac{dz}{ds}$ and $\frac{dp}{ds}$ in Eq. 2, we have

$$V_s = -\frac{k}{\mu}\left(-\frac{\rho g h}{L} - \rho g\right)$$

$$V_s = \frac{k}{\mu}\left(\frac{\rho g h}{L} + \rho g\right)$$

$$V_s = \frac{k\rho g}{\mu}\left(\frac{h}{L} + 1\right) \quad (3)$$

$$V_s = q = \frac{Q}{A} \quad (4)$$

Where,
Q = Volume flux rate of fluid through sand ($m^3\,s^{-1}$)
A = Cross-sectional area of the sand (m^3)

Therefore, Eq. 3 can be written as;

$$q = \frac{k\rho g}{\mu}\left(\frac{h}{L} + 1\right) \quad (5)$$

The permeability, k of a porous medium is related to hydraulic conductivity, K by Hubert King relation

$$K = \frac{k\rho g}{\mu} \quad \text{(Domenico and Schwartz, 2000)} \quad (6)$$

Where,
μ = Viscosity (Nsm^{-2})
ρ = Density (kgm^{-3}) and
g = Acceleration due to gravity (ms^{-2})

By using Eq. 6 in 5, Eq. 5 can be written as,

$$q = K\left(\frac{h}{L} + 1\right) \quad (7)$$

Where,
q = Volume flux rate ($m\,s^{-1}$)
K = Hydraulic conductivity ($m\,s^{-1}$) and
i = $\left(\frac{h}{L} + 1\right)$ is hydraulic gradient.

Darcy's permeability obtained at a small hydraulic gradient are refers to "true permeability" k_0 because the values of permeabilities determined from relation q/i are constant irrespective of the hydraulic gradient, while 'effective' permeability k' are obtained at larger gradient when the values of permeabilities are no more constant (Corey, 1977).

True Darcy's permeability and effective permeability are related by the expression

$$k' = k_0\left(\frac{i_0}{i'}\right) = \frac{q}{i} = q\left(\frac{h}{L} + 1\right)^{-1} \quad (8)$$

Where,
k_0 = The true Darcy's permeability obtain at a small hydraulic gradient.
i_0 = Smallest gradient which produces laminar flow
k' = Larger gradient which produces turbulent flow
n = An exponent that varies with the gradient.
Values of n can be computed from (8) as follows

$$\log\left(\frac{k'}{k_o}\right) = n\log\left(\frac{i_o}{i'}\right)$$

$$n = \log\left(\frac{k'}{k_o}\right) \Big/ n\log\left(\frac{i_o}{i'}\right) \quad (9)$$

MATERIALS AND METHODS

Sand sample were collected from the riverbed of 2 different rivers within the University of Ibadan. Sizeable quantities of these samples were washed and rinsed in order to remove organic particles and unwanted grains. Thereafter, the sand samples were sun dried and later placed in an oven for 1 h at temperature of 120°C.

After the samples were allowed to cool down, the stony particles ad pebbles were removed. Five different sieves were used to sieve the available sand samples in order to obtain samples of different grain size. The porosity ϕ of each size was determined by volumetric approach.

The permeability test, which is the process by which the ability of a porous medium to allow the passage of fluid through it was carried out in the laboratory, using the constant head method. This was done for hydraulic gradient ranging from 1.15±0.05 to 15.00±0.05. These include very low gradients that produce laminar flow and larger gradient that produces various degree of turbulence. The results of these experiments were used to prepare plots of relative values of permeabilities q/i = k' (effective permeability) versus hydraulic gradient.

RESULTS AND DISCUSSION

Table 1 shows the value of volume flux rate, q obtained at different hydraulic gradients for sample A - E. This was obtained by determining the volume of water discharged across a unit cross-sectional area per unit time. Equation 4 was used to compute this, where *A* is the cross-sectional area of the cylindrical tube used.

Table 2 shows the relative permeability *k'* at each hydraulic gradient y using Eq. 6 and 7. Each values of volume flux obtained at Table 1 are divided by their respective hydraulic gradient to obtain relative conductivity, which was later converted to relative permeability by using Eq. 6. It is observed that relative permeability decreases with increasing hydraulic gradient but increases with increasing porosity.

The values of exponent *n* were obtained by using Eq. 9. The values of n are zero for hydraulic gradients with constant values of relative permeability when compared with lowest hydraulic gradient, which gives the 'true permeability'. Those with other values apart from zero show the variation from lowest hydraulic gradient, 1.15 and then presented in Table 3.

The region of hydraulic gradients where laminar flow exist gives correction factor to be 1 because the value of n is zero in equation $c = (i^o/i')^n$ and for region beyond where $n \neq 0$, the geometric mean value of n's at different hydraulic gradient for a porous medium with the same porosity could be taken to compute their respective correction factor. It should be noted here that i_o is the hydraulic gradient at the lowest hydraulic gradient, (1.15) and i' is the hydraulic gradient that is been considered. The results were presented in Table 4 and it was observed

Table 1: Volume flux rate at different hydraulic gradient

Hydraulic gradient I	*A* $q\times10^{-4}$ (m s^{-1})	*B* $q\times10^{-4}$ (m s^{-1})	*C* $q\times10^{-4}$ (m s^{-1})	*D* $q\times10^{-4}$ (m s^{-1})	*E* $q\times10^{-4}$ (m s^{-1})
1.15	0.529	1.196	1.334	1.909	3.738
1.25	0.575	1.300	1.450	2.050	4.038
1.50	0.690	1.560	1.710	2.445	4.830
1.88	0.863	1.931	2.100	3.019	6.000
2.50	1.125	2.525	2.725	3.975	7.450
3.00	1.290	3.000	3.240	4.710	8.250
3.75	1.538	2.675	3.975	5.850	10.163
5.00	1.900	4.850	5.250	7.750	13.400
7.50	2.625	7.125	7.725	11.475	19.950
15.00	4.950	13.950	15.150	22.500	39.750

Table 2: Relative values of permeability $k' \times 10^{-4}$ (m^2) at different hydraulic gradients

	Hydraulic gradient									
Porosity	1.15	1.25	1.50	1.88	2.50	3.00	3.75	5.00	7.50	15.00
0.250	0.47	0.47	0.47	0.47	0.46	0.44	0.42	0.39	0.36	0.34
0.333	1.06	1.06	1.06	1.05	1.03	1.02	1.00	0.99	0.97	0.95
0.364	1.18	1.18	1.16	1.14	1.11	1.10	1.08	1.07	1.05	1.03
0.400	1.69	1.67	1.66	1.63	1.62	1.60	1.59	1.58	1.56	1.53
0.420	3.32	3.30	3.28	3.26	3.04	2.81	2.76	2.73	2.71	2.70

Table 3: Relative values of exponent *n*

	Hydraulic gradient									
Porosity	1.15	1.25	1.50	1.88	2.50	3.00	3.75	5.00	7.50	15.00
0.250	0	0	0	0	0.028	0.069	0.095	0.127	0.142	0.137
0.333	0	0	0	0.019	0.037	0.040	0.049	0.047	0.046	0.043
0.364	0	0	0.064	0.071	0.079	0.073	0.071	0.067	0.062	0.053
0.400	0	0.143	0.067	0.074	0.055	0.053	0.052	0.046	0.043	0.039
0.420	0	0.073	0.073	0.013	0.115	0.174	0.156	0.133	0.108	0.081

Table 4: Correction factor *c* at various hydraulic gradients

	Hydraulic gradient									
Porosity	1.15	1.25	1.50	1.88	2.50	3.00	3.75	5.00	7.50	15.00
0.250	1	1	1	1	0.934	0.919	0.901	0.879	0.848	0.798
0.333	1	1	1	0.981	0.970	0.936	0.955	0.944	0.930	0.905
0.364	1	1	0.982	0.967	0.949	0.937	0923	0.905	0.880	0.840
0.400	1	0.995	0.985	0.972	0.955	0.945	0.933	0.917	0.895	0.859
0.420	1	0.993	0.978	0.961	0.938	0.924	0.908	0.887	0.858	0.810

Table 5: Values for maximum gradient for laminar flow for different porosity

Sample	Porosity	Maximum gradient (i_m)
A	0.250	1.88
B	0.333	1.55
C	0.364	1.25
D	0.400	1.15
E	0.420	1.15

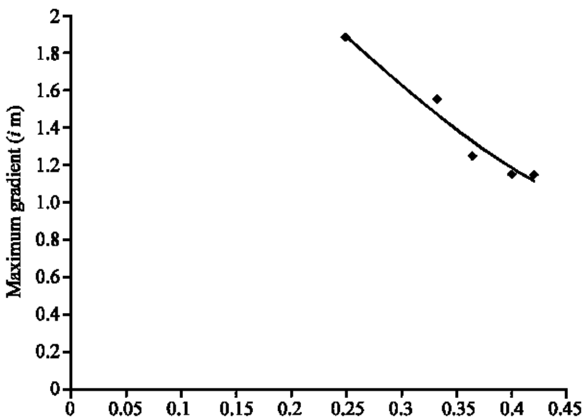

Fig. 1: Graph of maximum gradient (i_m) against porosity

that beyond the region where correction factor is 1, the correction factor decreases with increasing hydraulic gradient.

The region where correction factor is 1 shows the range of hydraulic gradients that produces laminar flow. From Table 4 it shows that maximum hydraulic gradient for laminar flow are 1.88, 1.50 and 1.25 for samples A, B and C, respectively and 1.15 for both sample C and D. These was clearly presented in Table 5 which shows that limiting value of hydraulic gradient for laminar flow decreases with increasing porosity with all other parameter kept constant.

Figure 1 is the plot of maximum hydraulic gradient for laminar flow i_m against porosity, which shows that maximum hydraulic gradient, is related to porosity of porous media with polynomial equation of order 2. The equation is $i_m = 6.0008\phi^2 - 8.6326\phi + 3.6759$ with correlation coefficient of 0.96. It is now very obvious that for vertical flow there must exist a hydraulic gradients above which turbulent flow set in. if the porosity of a porous medium is known this hydraulic gradient can be determined from the equation above.

Also from this equation, the maximum hydraulic gradient for laminar flow vis-à-vis Darcian flow for the most porous medium can be determined to be 1.04. This is true because the maximum value of porosity is 1 and therefore gives the limit value of hydraulic gradient for vertical laminar flow.

CONCLUSION

The result of the experiment shows that relative permeability increases with increasing porosity but decreases with increasing gradient. Also, the maximum (or limiting) hydraulic gradient for laminar flow decreases with increasing porosity.

The plot of maximum hydraulic gradient against porosity reveals that the vertical fluid flow in any porous medium is laminar or nearly laminar as much as hydraulic gradient is less than or equal to 1.04. This indicates that beyond this value, turbulence flow will surely set in.

It is also observed that correction factor c decreases with increasing hydraulic gradient, which should be expected because turbulence increases with hydraulic gradient.

Finally, a general expression (or equation) that take care of both laminar and turbulent flow of fluid in porous media can be taken to be volume flux rate, q is equal to the product of 'true' Darcy permeability obtained at very small gradient, k_o, hydraulic gradient i and correction factor, c which varies with hydraulic gradient (i.e. $q = k_o ic$).

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