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A New Generalized Mathematical Model for the Study of Diabetes Mellitus

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Abstract: We present a new generalized Mathematical model for the study of diabetes mellitus. The model takes into account all glucose intake and insulin injected (administered) as a function of the molecular weight of carbohydrate and protein intake, respectively. We also used the model to monitor the blood plasma glucose level in non-diabetic and suspected diabetic subjects. The results obtained were compared with results in literature graphically.

Key words: Generalized, malhematical model, diabetes mellitus, insulin, glucose intake

INTRODUCTION

Diabetes mellitus is a group of metabolic diseases characterized by hyperglycemia resulting from defects in insulin secretion, insulin action or both. The chronic hyperglycemia of diabetes is associated with long term damage, dysfunction and failure of various organs, especially the eyes, kidneys, nerves, hearts and blood vessels.

Diabetes is a condition in which the body cannot properly use the energy from food that has been eaten. Foods when eaten get broken down into a form of sugar called glucose; glucose is the body's main fuel. This sugar (glucose) enters the blood and goes to the cells where it is used as energy.

When carbohydrates are consumed the body turns them into glucose, which provides sufficient energy for every day tasks and physical activities.

The monomer for carbohydrates is the monosaccharide which is commonly known as the simple sugar. All monosaccharide have the formula $(CH_2O)_n$. glucose is a hexose's (6-carbon) sugar. Its molecular formula is $(CH_2O)_6$ or $C_6H_{12}O_6$ (ADA, 2004; Ganong, 1999).

Alais and Linden (1999) classified protein into group base on their molecular weight. Davis (1962) gives a differential model of diabetes mellitus which he used to consider the effect of the subsequent levels of plasma insulin. Mbah (1998) developed Davis model's and considered the effect of the basal glucose level to the subsequent levels of plasma glucose and insulin. This study revisits Mbah model and generalizes the model.

MATHEMATICAL FORMULATION

Based on the above we wish to present a new generalized mathematical model for the study of diabetes mellitus which will take into account all glucose in take and insulin injected (administered) as a function of the molecular weight of carbohydrate and protein intake, respectively. The model shall be used for monitoring the blood plasma glucose level in a diabetic subject.

According to Mbah (1998) the mathematical model used for the study of insulin dependant diabetes mellitus subject is given as

$$\frac{dx}{dt} = a_1 q e^{-k(t-t_o)} - a_2 xy - a_3 x \quad (1)$$

$$\frac{dy}{dt} = b_1 x - b_2 y + b_3(\gamma t + \delta) \quad (2)$$

$$x(to) = x_o, y(t_0) = y_o ,$$

Where x and y to represent glucose and insulin, respectively, a_2, a_3, b_1 and b_2 are all constants associated with the mechanism which was determined experimentally. In Eq. 1, a_2 measures the plasma membrane permeability of the glucose a_3 measures the body cell utilization of the plasma glucose. Likewise in Eq. 2, b_1 measures the pancreatic response to glucose stimulation while b_2 measures the level of insulin degradation. The first expression in Eq. 1 denotes the quantity of glucose intake i.e., $a_1 q e^{-k(t-t_0)}$ And the last expression in Eq. 2 i.e., $b_3(\gamma t+\delta)$ represents the quantity of insulin injected into the patients. Q is the quantity of initial glucose

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intake, k is the delay parameter associated with time taken to absorb this quantity of initial glucose intake in the blood we note that $a_1$was not defined similarly b_3, γ and δ are not defined.

Following Mbah (1998) we present a new generalized mathematical model for the study of diabetes mellitus as follow

$$\frac{dc}{dt} = -a_1cp - (a_2 + a_4)c + a_3mwce^{-q(t-t_o)} \quad (3)$$

$$\frac{dp}{dt} = b_1c - b_2p + b_3mwpe^{-r(t-t_o)} \quad (4)$$

$$c(t_o) = c_o, p(t_o) = p_o$$

In this new mathematical model, c and p represent carbohydrate (glucose) and protein (insulin), respectively. In Eq. 3 a_1 measures the plasma membrane permeability of the carbohydrate into the cell in the presence of protein while a_2 measures the body cell utilization of the plasma carbohydrate in this model and a_3 is the concentration of glucose in carbohydrate foods intake. We denote the intensity of exercise with a_4 and this measures the level of conversion of glycogen to glucose during exercise.

Also in Eq. 4, b_1 measures the pancreatic response to carbohydrate stimulation while b_2 measures the rate of protein degradation and b_3 is the concentration of insulin (Protein) intake.

We use $mwce^{-q(t-t_0)}$ to represent the quantity of glucose intake, mwc is the molecular weight of the initial carbohydrate intake; q is the delay parameter associated with time taken to absorb this quantity of carbohydrate in the blood and a_3 measures the concentration of the carbohydrate intake. Also, we $mwpe^{-r(t-t_0)}$ use to represent the quantity of protein intake, where mwp is the molecular weight of the initial protein injected. r is the delay parameter associated with time taken to absorb this quantity of initial protein intake.

METHOD OF SOLUTION

Consider the model equation again

$$\frac{dc}{dt} = -a_1cp - (a_2 + a_4)c + a_3mwce^{-q(t-t_0)} \qquad C\,(_{to}) = c_o$$

$$\frac{dp}{dt} = b_1c - b_2p + b_3mwpe^{-r(t-t_0)} \qquad P\,(_{to}) = p_o$$

We now solve the system of equation numerically.

Let

$$f\,(t,c,p) = -a_1cp - (a_2 + a_4)c + a_3mwce^{-a(t-t_0)}$$

And

$$g\,(t,c,p) = b_1c - b_2p + b_3mwpe^{-r(t-t_0)}$$

$$c_{i+1} = c_i + \frac{h}{6}(f_1, +2(f_2 + f_3) + f_4)$$

$$p_{i+1} = p_i + \frac{h}{6}(g_1 + 2(g_2 + g_3) + g_4)$$

Where

$$f_1 = f(t_i, c_i, p_i)$$

$$g_1 = g(t_i, c_i, p_i)$$

$$f_2 = f(t_i + \frac{h}{2}, c_i + \frac{h}{2}f_1, p_i + \frac{h}{2}g_1)$$

Table 1: Comparison of Mbah and the new model

Time	MBM	ADM
0	92.5	92.5
0.1	108.4519	109.7582
0.2	118.5354	120.6918
0.3	124.6428	127.3404
0.4	128.0514	131.0808
0.5	129.6273	132.8462
0.6	129.96	133.2716
0.7	129.4517	132.7908
0.8	128.3781	131.7007
0.9	126.9288	130.2056
1	125.2345	128.4466
1.1	123.3858	126.5212
1.2	121.4459	124.4973
1.3	119.4584	122.422
1.4	117.454	120.3284
1.5	115.454	118.2394
1.6	113.4733	116.1707
1.7	111.5219	114.1333
1.8	109.6068	112.1346
1.9	107.7326	110.1794
2	105.9019	108.2706
2.1	104.1165	106.4101
2.2	102.3771	104.5984
2.3	100.6837	102.8358
2.4	99.036	101.1216
2.5	97.4333	99.4552
2.6	95.8745	97.8355
2.7	94.3587	96.2612
2.8	92.8846	94.7312
2.9	91.4509	93.2439
3	90.0564	91.798
3.1	88.6997	90.3921
3.2	87.3795	89.0248
3.3	86.0946	87.6946
3.4	84.8436	86.4003
3.5	83.6254	85.1404
3.6	82.4387	83.9138
3.7	81.2825	82.7191
3.8	80.1555	81.5552
3.9	79.0567	80.421
4	77.9851	79.3153

Table 1: Continued

Time	MBM	ADM
0	150	150
0.1	165.7877	167.0942
0.2	175.7546	177.9148
0.3	181.8143	184.5255
0.4	185.2475	188.3069
0.5	186.9132	190.1849
0.6	187.3884	190.7814
0.7	187.062	190.5152
0.8	186.1966	189.6689
0.9	184.97	188.4342
1	183.5032	186.9411
1.1	181.8791	185.2786
1.2	180.1543	183.5076
1.3	178.368	181.6701
1.4	176.5473	179.7954
1.5	174.7111	177.9035
1.6	172.8727	176.0086
1.7	171.0414	174.1208
1.8	169.2236	172.2469
1.9	167.424	170.3918
2	165.6457	168.5588
2.1	163.891	166.7504
2.2	162.1612	164.9681
2.3	160.4573	163.2128
2.4	158.7799	161.4852
2.5	157.1292	159.7854
2.6	155.5052	158.1136
2.7	153.908	156.4697
2.8	152.3372	154.8533
2.9	150.7925	153.2642
3	149.2736	151.702
3.1	147.78	150.1662
3.2	146.3112	148.6564
3.3	144.8669	147.1719
3.4	143.4465	145.7124
3.5	142.0495	144.2773
3.6	140.6754	142.866
3.7	139.3238	141.478
3.8	137.994	140.1129
3.9	136.6857	138.77
4	135.3983	137.4489

$$
\begin{aligned}
g_2 &= g(t_i + \frac{h}{2}, c_i + \frac{h}{2} f_1, p_i + \frac{h}{2} g_1) \\
f_3 &= f(t_i + \frac{h}{2}, c_i + \frac{h}{2} f_2, p_i + \frac{h}{2} g_2) \\
g_3 &= f(t_i + \frac{h}{2}, c_i + \frac{h}{2} f_2, p_i + \frac{h}{2} g_2) \\
f_4 &= f(t_i + h, c_i + hf_3, p_i + hg_3) \\
g_4 &= f(t_i + h, c_i + hf_3, p_i + hg_3)
\end{aligned} \tag{5}
$$

Then, we obtain the results in Table 1.

DISCUSSION

A new generalized mathematical model used for the study of diabetes mellitus was considered. This model takes into account the quantity of glucose intake and insulin injected as a function of molecular weight of carbohydrates and protein, respectively. The model was solved and the computational results obtained were compared with the former model i.e., Mbah model. This comparison were showed graphically in Fig. 1 and 2 for a normal and suspected diabetes patient, respectively. We observed that there is no much different between the results obtained for Mbah model and this new model.

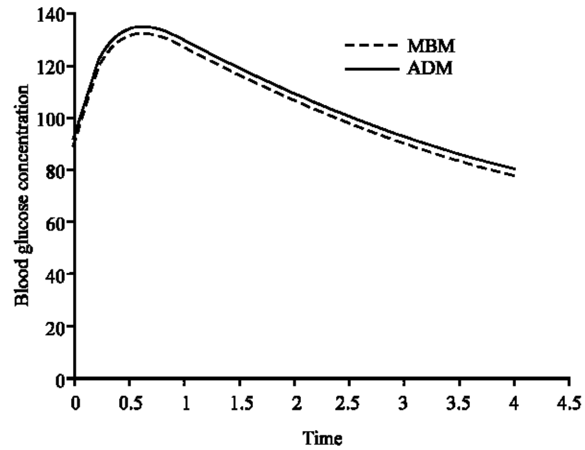

Fig. 1: Graph of glucose concentration against time for MBM and ADM $a_2 = 0.05$, $a_3 = 0.03$, $b_1 = 0.05$, $b_2 = 2$

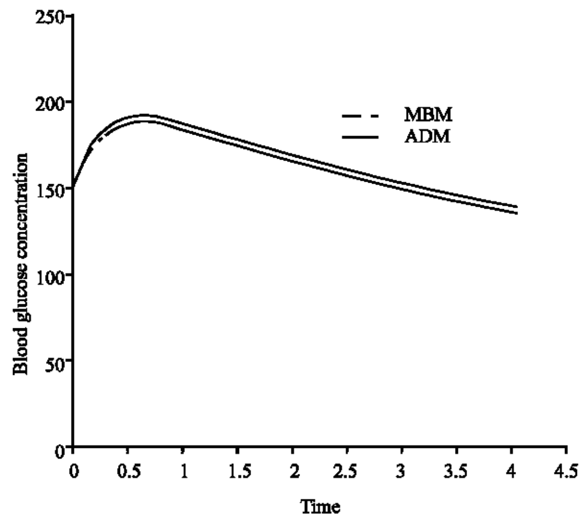

Fig. 2: Graph of glucose concentration against time for MBM and ADM $a_2 = 0.03$, $a_3 = 0.03$, $a_4 = 0$, $b_1 = 0.03$, $b_2 = 2$

RECOMMENDATION

We therefore, recommend this new mathematical model for the study of diabetes mellitus since the molecular weight of carbohydrate (glucose) and protein (insulin) can easily be calculate.

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