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Relationship Between the Impact of Exogenous Interventions on Arma and Garch Predictions

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Abstract: A common problem encountered in high frequency financial time series is the occurrence of extreme observations, or significant spikes in volatility, with subsequent influence on model specification, parameter estimation and future predictions in ARMA and GARCH models. We present estimated biases of Garch(1,1) model coefficients by unrecognized exogenous interventions at unknown dates with particular attention to additive level outliers. We further determine their approximate and simulated influences on estimated predictions through the inflation of innovation variance estimates. Our conclusion maintains that the severity of bias depends on the distance of an outlier from the prediction origin.

Key words: Additive outliers, ARMA model, exogenous interventions, GARCH model, prediction error, time series

INTRODUCTION

Recent research has shown that outliers may have unexpectedly adverse effects on time series and volatility modeling. It is possible that detected nonlinearity is really due to a few, or indeed just one, outliers, or that the presence of nonlinearity will not be detected because of outliers, or that estimation results change considerably due to only a few outliers. Particularly, the modeling of volatility has been an active area of research in finance and has been largely motivated by the importance of risk considerations in economic and financial markets. Estimates of volatility are used widely for a variety of reasons, including modeling the premium in forward and future markets, portfolio selection, asset management, pricing primary and derivative assets, valuation of warrants and options, designing optimal hedging strategies for options and future markets, evaluating risk spill-overs across markets, measuring announcement effects in event studies and examining asymmetries and leverage effects.

Engle (1982) first captured the time-varying nature of volatility with the Autoregressive Conditional Heteroscedasticity (ARCH (p)) model. Bolleserlev (1986) generalized the ARCH model to GARCH (p, q) and this has proved to be the single most popular time-varying volatility modeling in practice. Garch has the ability to accommodate 2 key stylized facts of volatility in financial data, the persistence of volatility and volatility clusters, leptokurtic data and mathematical and computational simplicity. Many theoretical results, including the statistical properties of the models and the asymptotic properties of several estimation methods, are now available and these provide a solid foundation for applications of the various models (Li *et al.*, 1999) for a survey, directed towards practitioners, of recent important theoretical results for GARCH models). Franses and Ghijsels (1999) suggested steps of correcting the data for outliers when using GARCH models to forecast volatility. In our work, particular attention is on additive outliers being the most prevalent (Kleiner *et al.*, 1979).

We determine the estimated biases (in level) of additive outliers on GARCH parameters and their subsequent influence on Garch (1,1) predictions. Our approach is inspired by Hotta and Tsay (1998) who distinguished between additive outliers that only affect the level, but leave the variance unaffected and those that also affect the conditional variance. They designated the former, Additive Level Outliers (ALO) and the later, Additive Volatility Outliers (AVO).

In the analysis of business, economic and environmental data, it is often of interest to determine the effects of exogenous interventions such as a change in fiscal policy or the implementation of a certain pollution control measures that occurred at some known time points. In performing the analysis, one must take into proper account the dynamic characteristics of the intervention and the dynamic structure or serial

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dependence of the observations. We consider the additive structure for the time series $\{Z_t\}$, with

$$Z_t = y_t + e_t, \quad (1)$$

Where y_t represents the effect of the intervention and e_t represents the noise term.

INFLUENCE OF ADDITIVE OUTLIERS ON ARIMA AND GARCH INNOVATIONS

In detecting process, we treat outliers as non-repetitive exogenous interventions at unknown time points and these interventions only affect the level of the series. We adopt the additive formulation in (1) where now y_t represents the effect of outliers on the series Z_t and e_t is the noise term following an ARIMA (p, d, q) model. An Additive Outlier (AO) only affects the series at t_0:

$$Z_{t=} \frac{\theta(B)}{\phi(B)} a_t + \alpha\xi(t, t_o) \quad (2)$$

Where $\xi(t,t_o) = \{1 \text{ if } t = t_o \text{ and } 0 \text{ otherwise}\}$ and $\phi(B)$ may contain difference operators. Following Hillmer *et al.* (1983) but using slightly different notation, let u_t be the innovations estimated from the model without taking account of outliers.

For an invertible model, multiply (3) by $\phi(B)/\theta(B) = \pi(B)$. Then

$$u_t = \alpha\ \pi(B)\ \xi(t,t_o) + a_t \quad (3)$$

Using ordinary least squares, the impact α of the intervention and its variance are:

$$\hat{\alpha} = (u_{to} + \pi_1 u_{to} + 1 + \ldots + \pi_{n-to} u_n) / \text{wn-to}, \quad (4)$$

$$\text{var}\ (\hat{\alpha}) = \sigma_a^{\ 2} / w_n - t_o$$

Where $w_t = 1 + \pi_1^{\ 2} \ldots + \pi_t^{\ 2}$.

These formulae reflect the influence of an AO on all innovations from t_o onwards.

Now, considering a situation where a market correction does not influence volatility (such as institutional change, or a rogue trade), then a GARCH (1,1) model with an additive outlier affecting only the series is defined as

$$y_t - x_t^{'}\xi - \beta d_t = \varepsilon_t,\ \varepsilon_t |\ \psi_{t-1} \sim N\ (0, h_t)$$

$$h_t = a_1 + a_2\varepsilon_{t-1}^{\ 2} + a_3 h_{t-1},\ \ t = 1, 2, \ldots, T \quad (5)$$

Where ψ_{t-1} is the filtration up to time *t*, x_t may contain only some constant terms, d_t equals one when t = s and zero otherwise. In (5) the outlier does not influence volatility being the lagged disturbances that enter the conditional variance (Doormik and Ooms, 2005). Equation 5 is a standard GARCH model with a dummy variable as regressor. Then, the direct impact, ω, of a neglected outlier (on the sequence of variance innovations) is obtained from the detection procedure of Franses and Dijk (2000) for GARCH (1,1) models using the 'variance innovation'

$$v_t = \varepsilon_t^{\ 2} - h_t, \quad (5a)$$

as

$$\omega = 2\ \beta\varepsilon_t + \beta^2, \quad (5b)$$

Where β is a dummy coefficient.

In ARIMA series, outliers are known to directly affect the estimate of the innovation variance and consequently transfer the effect on the width of the prediction intervals, as they are proportional to the standard deviation of the innovations. The next study considers how the effect is transferred to the GARCH model.

A PREDICTIVE RELATIONSHIP BETWEEN ARMA AND GARCH MODELS

Let us assume that the coefficients of the ARIMA models are known and suppose that the outlier at time T has been ignored. Then the l-step-ahead prediction of the future observation Z_{T+m+l} from forecast origin T + m is given by

$$Z_{T+m}(l) = \pi_1\ (l)\ Z_{T+m} + \pi_2\ (l)\ Z_{T+m-1} + \pi_3(l) Z_{T+m-2} + \quad (6)$$
$$= \sum_{j\geq 0} \pi^{(l)}_{\ j+1} Z_{T+m-j}$$

The forecast weights in this linear combination,

$$\pi_j^{(l)} = \pi_{j+l-1} + \sum^{l-1} \pi_h \pi_j^{(l-h)}, j=1, 2, \quad (7)$$

are calculated from the $\pi_j^{(l)} = \pi_j$ weights in

$$1 - \sum_{i\geq 1} \pi_j B_j = \phi(B)(1-B)^d / \theta(B) \quad (8)$$

Box and Jenkins (1976). For invertible process these weights approach zero fairly rapidly.

The forecast weights $\pi_j^{(l)}$ determine the extent to which an unrecognized outlier of magnitude δ affects the forecast. If the additive outlier at time T occurred m $\geq$ 0 periods prior to the forecast origin, we can write the l-step-ahead forecast error as

$$Z_{T+m+l}-Z_{T+m}{}^{(1)} = e_{T+m}{}^{(l)}-\delta\pi_{m+1},{}^{(l)} \quad (9)$$

where $e_{T+m}(l) = a_{T+m+1}+\psi_1 a_{T+m+l-1}+\psi_l+\iota_{l-1}a_{T+m+l}$ and ι_j, j = 0, 1, ..., are the coefficients of B_j in $\Sigma_{j\geq 0}\iota_j B^j = \theta(B) / [\phi(B)(1-B)^d]$, respectively, Hillmer (1984). The mean square of the l-step-ahead forecast error in (9) is

$$\text{MSPE}(l; m, \delta)\,\sigma^2 = \Sigma\psi^{2(l-1)}{}_{j,j=0} + \delta^2 [\pi_{m+t}]^2,{}^{(l)} \quad (10)$$

and the relative increase in this mean square that is due to the additive outlier at time T is

$$\triangle\text{MSPE}(l; m, \delta) = (\delta/\sigma)^2 [\pi_{m+l}]^{2\,(l)}\Sigma\psi^2_{j,j=0} \quad (11)$$

For illustration we study the increases in the mean square forecast error for autoregressive and integrated first order moving average processes.

For models without moving average components the forecast weights in Eq. 7 simplify to $\pi_j = 0$ for $j>p^{(l)}+d$. Only outliers that occur during the most recent p+d observations (that is when $m\leq p+d-1$) affect the forecasts. In the first order autoregressive process, for example, $\pi_1^{(l)}$ = and $\pi_1^{(l)} = 0$ for j>1. Additive outliers that occur prior to the forecast origin (m>0) leave the forecasts unaffected. If the outlier occurs at the forecast origin (m = 0), the relative increase in the mean square of the l-step-ahead forecast error is given by

$$\triangle\text{MSPE}(l; m = 0, \delta) = (\delta/\sigma)^2 \phi^{2l} / (1+\phi^2 + \ldots + \phi^{2(l-1)}),$$
$$= (\delta / \sigma)^2 \phi^{2l} (1 - \phi^2) / (1 - \phi^{2l}) \quad (12)$$

As expected the effect of an outlier at the last available observation is disastrous as the most recent observation gets a large weight in the forecast function.

For the ARIMA (0,1,1) process, $y_t = y_{t-1}+a_t - \theta a_{t-1}$, the π-weights are given by $\pi^{(l)}_{m+1} = (1-\theta)\theta^m$ for all l and ι_j = $(1-\theta)$ for all j>0 ($\psi_0 = 1$). The relative increase in the mean square of the l-step-ahead forecast error that is due to the undetected outlier at time T is

$$\triangle\text{MSPE}(l; m = \delta) = (\delta/\sigma)^2 (1 - \theta)^2 \theta^{2m} /(l\text{-}1)(1+\theta)^2] \quad (13)$$

Table 1 and 2 show these increases for various values of l (forecast lead time), θ (moving average parameter), m (time between outlier occurrence and forecast origin) and δ (magnitude of outlier). If the outlier occurs 5 or more periods prior to the forecast origin ($m\geq 5$) the effects of an unrecognized outlier are not too worrisome. For example, for a moving average parameter θ between 0.7 and 0.9 (which corresponds to a smoothing constant between 0.1 and 0.3 in simple exponential smoothing) the percent increases due to a δ = 5σ outlier m = 5 periods prior to the forecast origin are at most 8.7%; for m = 10, they are not more than 3%.

Table 1: Influence of an additive outlier on the estimates of GARCH (1, 1) process with estimated AR coefficient in the ARIMA (0, 1, 1) process

	E(ϕ)			E(θ)	
$\phi = a_1 + a_3$	$\delta = 3\sigma$	$\delta = 5\sigma$	$\theta = a_3$	$\delta = 3\sigma$	$\delta = 5\sigma$
0.1	0.091	0.075	0.1	0.091	0.075
0.3	0.275	0.231	0.3	0.275	0.231
0.5	0.466	0.405	0.5	0.466	0.405
0.7	0.688	0.610	0.7	0.688	0.610
0.9	0.884	0.857	0.9	0.884	0.857

Table 2: Influence of an additive outlier on the estimates of GARCH (1, 1) process with estimated MA coefficient in the ARIMA (0, 1, 1) process

	E(ϕ)			E(θ)	
$\phi = a_1 + a_3$	$\delta = 3\sigma$	$\delta = 5\sigma$	$\theta = a_3$	$\delta = 3\sigma$	$\delta = 5\sigma$
0.1	0.165	0.247	0.1	0.165	0.247
0.3	0.344	0.403	0.3	0.344	0.403
0.5	0.528	0.566	0.5	0.528	0.566
0.7	0.715	0.736	0.7	0.715	0.736
0.9	0.904	0.911	0.9	0.904	0.911

In summary, the analysis so far reveals that additive outliers around the forecast origin can have a disastrous effect on the forecast performance. Thus, the influence of an AO that occurs well before the forecast origin, however, is usually discounted rapidly.

Now considering the ARMA (1,1) model of the form;

$$y_t^2 = a_1+ (a_2+a_3)\, y_{t-1}^2-a_3 v_{t-1}+v_t \quad (14)$$

Applying the transformation $v_t = y^2 - h_t$, where v_t is by construction a martingale function with $E(V_t) = 0$ and $Var(v_t) = \sigma^2$

The GARCH (1,1) equivalent is

$$h_t = a_1 + a_2 y_{t-1}^2+a_3 h_{t-1,} \quad (15)$$

or we have on substitution for v_t,

$$y_t^2-v_t = a_1+a_2 y_{t-1}^2+a_3 (y_{t-1}^2-v_{t-1})$$
$$\Rightarrow y_t^2 = a_1 +a_2 y_{t-1}^2 +a_3 (y_{t-1}^2-v_{t-1}) + v_t$$
$$\Rightarrow y_t^2 = a_1 + (a_2 + a_3)\, y_{t-1}^2 - a_3 v_{t-1}+v_t \quad (16)$$

To obtain the parameter estimates of ARMA (1,1), we have that

$$\hat{\Phi} = a_1 + a_3 \quad (17)$$

while

$$\hat{\theta} = a_3 \quad (18)$$

$$\Rightarrow \phi = a_1 + \theta \text{ or } a_1 = \phi - \theta \quad (19)$$

Multiplying (16) by y_t^2, y_{t-1}^2 and taking expectations, we have

$$E(y_t^2 y_t^2) = E\{a_1 y_t^2 + (a_2 + a_3) y_t^2 y_{t-1}^2 - a_3 y_t^2 v_{t-1} + v_t y_t^2\}$$
$$\Rightarrow Y_o = a_1 E(y_t^2) + (a_2 + a_3) Y_o - a_3 E(y_t^2 v_{t-1}) + E(v_t y_t^2) \quad (20)$$

$$E(y_t^2 y_{t-1}^2) = E\{a_1 y_{t-1}^2 + (a_2 a_3) y_{t-1}^2 y_{t-1}^4 - a_3 y_{t-1}^2 v_{t-1} + v_t y_{t-1}^2\}$$
$$\Rightarrow Y_1 = a_1 E(y_{t-1}^2) + (a_2 + a_3) Y_1 - a_3 E(y_{t-1}^2 v_{t-1}) + E(v_t y_{t-1}^2) \quad (21)$$

To obtain $E(y_t^2 v_{t-1})$ in (21), we multiply (16) by v_{t-1} and take expectations,

$$E(y_t^2 v_{t-1}) = E\{a_1 + (a_2 + a_3) y_{t-1}^2 v_{t-1} - a_3 v_{t-1}^2 + v_t v_{t-1}$$
$$= (a_2 + a_3)\sigma_v^2 - a_3 \sigma_v^2 = a_2 \sigma_v^2 \quad (22)$$

Substituting (22) into (20), we have

$$Y_0 = a_1 E(y_t^2) + (a_2 + a_3) Y_o - a_2 a_3 \sigma_v^2 + \sigma_v^2$$
$$\Rightarrow Y_0[1 - (a_2 + a_3)] = a_1 E(y_t^2) + \sigma_v^2(1 - a_2 a_3) \quad (23)$$

Similarly, substituting (22) into (21) yields

$$Y_1[1 - (a_2 + a_3)] = a_1 E(y_{t-1}^2) + \sigma_v^2(1 - a_2 a_3) \quad (24)$$

On solving (23) and (24) simultaneously, we have

$Y_o = a_1/(1 - a_2 - a_3)^2 + (1 - 2a_2a_3 - a_3^2)\sigma_v^2/(1 - (a_2 + a_3)^2$
Thus, $Var(y_t^2) = Y_o = (1 - 2a_2a_3 - a_3^2)\sigma_v^2/(1 - (a_2 + a_3)$
And, $Y_1 = a_1^2/(1 - a_2 - a_3)^2 + (a_2 - a_2^2a_3 - a_2a_3^2)\sigma_v^2/(1 - (a_2 + a_3)^2$
$\Rightarrow Cov(y_t^2 y_{t-1}^2) = V_1 = (a_2 - a_2^2a_3 - a_2a_3^2)\sigma_v^2/(1 - (a_2 + a_3)^2$

Generalizing, we have that

$Y_2 = a_1^2/(1 - a_2 - a_3)^2 + (a_2 + a_3)Y_1$
$Y_3 = a_1^2/(1 - a_2 - a_3)^2 + (a_2 + a_3)^2 Y_1$
.
.
.
$Y_k = a_1^2/(1 - a_2 - a_3)^2 + (a_2 + a_3)^{k-1} Y_1$

Where $V_2 = (a_2 + a_3)Y_1$
$V_3 = (a_2 + a_3)^2 Y_1$
.
.
.
$V_k = (a_2 + a_3)^{k-1} Y_1$

Thus the autocorrelation functions are

$\rho_1 = (a_2 - a_2^2a_3 - a_2a_3^2)/(1 - 2a_2a_3 - a_3^2)$
$\rho_2 = (a_2 + a_3)^2(a_2 - a_2^2a_3 - a_2a_3^2)/(1 - 2a_2a_3 - a_3^2)$
.
.
.
$\rho_k = (a_2 + a_3)^{k-1}\rho_1$

Now, suppose the observations are generated from

$$z_t = x_t + e_t, \quad (25)$$

Where $x_t = y_t^2$ follows an ARMA (1,1) model in (14) with serially uncorrelated v_t and e_t is an independent sequence of variables, independent of the sequence of x_t. The variable e_t has distribution H, given by

$$H = (1 - \alpha)\delta_0 + \alpha G,$$

Where δ_0 is the distribution that assigns probability 1 to the origin and G is the arbitrary distribution. Thus, with probability 1-α, the AR(1) process x_t itself is observed and with probability α the observation is the AR (1) process x_t plus an error with distribution G. Further insights into the effects of AOs to the ARCH model can be seen as follows: Let

$$z_t = x_t + e_t$$
$$x_t = a_1 + a_2 x_{t-1} + v_t$$
$$e_t \sim (1 - \alpha)\delta_0 + \alpha G$$

Making the autoregressive transformation of z_t we have

$$z_t - a_2 z_{t-1} = x_t - a_2 x_{t-1} + e_t - a_2 e_{t-1} \quad (26)$$

Here we observe that the sum of the 2 uncorrelated moving average process on the RHS of (26) is MA (1). Thus (26) represents an ARMA (1,1) process implying that an AR (1) model with AOs becomes an ARMA (1,1) model in (26). Similarly, the ARCH (1) model with AOs becomes GARCH (1,1). Hence, GARCH (1,1) model in (15) is able to capture AOs.

Thus, the characteristic features and predictive influences of an undetected outlier in ARMA (p, q) model can be transformed to GARCH (p, q) model.

BIASES OF GARCH (1,1) MODEL PARAMETERS BY ADDITIVE OUTLIERS

To determine the estimated biases of the coefficients by additive outliers, we take note of the fact that the

Table 3: Percentage increases in mean square predictor error due to a $\delta = 5\sigma$ additive outlier in the GARCH(1,1) process. Moving average coefficient is estimated in ARIMA(0,1,1) process

Forecast lead time l	Outlier five steps prior to forecast origin m = 5 origion			Outlier ten steps prior to forecast m = 10		
l→	1	2	3	1	2	3
a_3= 0.1	2.3	1.3	0.9	2.3	1.3	0.9
a_3= 0.3	1.4	0.9	0.7	1.3	0.8	0.6
a_3= 0.5	2.2	1.8	1.5	0.7	0.5	0.4
a_3= 0.7	8.4	7.7	7.1	0.7	0.6	0.6
a_3= 0.9	7.9	7.8	7.7	3.1	3.1	3.1

estimated AR parameter will be the same as the estimated MA parameter under the AO hypothesis of the null distribution (Eq. 26).

The least squares estimate of the first order autoregressive process is given by ϕ in $Z_t = \phi Z_{t-1} + a_t$, t = 2, . . ., n is given by

$$\hat{\phi} = \frac{\sum_{t=2}^{n} Z_t Z_{t-i}}{\sum_{t=2}^{n} z^2_{t-1}} \qquad (27)$$

The additive outlier at time T = n - m implies that $Z = (Z_1, Z_2, \ldots, Z_n)'$ follows a multivariate normal distribution with mean vector and covariance matrix $\Gamma = \{\gamma_{|i-j|} = \sigma^2\phi^{|i-j|}/(1-\phi^2);\ 1 = i, j = n\}$. Here η is a column vector of zeros, with the exception of δ in the T = (n-m) th position.

Since $E(\Sigma^n_{t=2} Z_t Z_{t-1}) = (n-1)\gamma_1$, $E(\Sigma^n_{t=2} Z^2_{t-1}) = (n-1)\gamma_0 + \delta^2$ if m>0 and $E(\Sigma^n_{t=2} Z_t^2 Z_{t-1}) = (n-1)\gamma_0$ If m = 0, we find as a very crude first order approximation (which, nevertheless, is not too bad if n is reasonably large)

$$E(\hat{\phi}) \cong \frac{(n-1)\gamma_1}{(n-1)\gamma_0 + \delta^2} = \frac{\phi}{1 + \frac{(\delta/\sigma^2)(1-\sigma^2)}{n-1}} \cong \phi - \frac{1}{n-1}\phi(1-\phi^2)(\delta/\sigma)^2 \text{ if } m > o, \cong \frac{(n-1)\gamma_1}{(n-1)\gamma_0} = \phi \text{ if } m = 0 \qquad (28)$$

For outliers that occur prior to the forecast origin, the bias in the least squares estimate amounts to $-(n-1)^{-1}\phi(1-\phi^2)(\delta/\sigma)^2$. For example, a 3σ outlier among n = 100 observation in an AR(1) process with $\phi = 0.5$ leads to a bias of -0.034; for a 5σ outlier the bias amounts to -0.095; (Table 3). The least squares estimate is unaffected if the outlier occurs at the last observation.

Using results about the moments of quadratic forms of normal random variables and ignoring terms of order smaller than $(n-1)^{-1}$, one can show that for m > 0.

$$E(\phi') \cong \phi - \frac{2\phi}{n-1} - \frac{1}{n-1}\phi(1-\phi^2)(\delta/\sigma)^2 \qquad (29)$$

For our GARCH (1,1) model, the bias estimate is

$$\{-(n-1)^{-1}(a_1 + a_3)(1-(a_1+a_3)^2)(\delta/\epsilon_t^2 - h_t)^2\}, \qquad (30)$$

(using Eq. 5a, 17 and 18).

In considering the ARIMA (0,1,1) process to investigate the effect of an additive outlier at time T = n–m on the estimate of the moving average coefficient, we first calculate the effect of the outlier on the lag one autocorrelation r_1 of the first differences, $D_t = Z_t - Z_{t-1}$, t = 1, 2, . . ., n. These differences follow a first order moving average process with mean vector η and covariance structure $\gamma_0 = \sigma^2(1+\theta^2)$, $\gamma_1 = -\theta\sigma^2$, $\gamma_j = 0$ for j>1. The coefficients in η are zero, with the exception of $\eta_T = \delta$ and $\eta_{T+1} = -\delta$. Let us assume that *m* > 1, which means that the outlier has not occurred during the last two observations. Then

$E(\Sigma^n_{t=2} D_t D_{t-1}) = (n-1)\gamma_1 - \delta^2$, $E(\Sigma^n_{t=2} D^2_{t-1}) = (n-1)\gamma_0 + 2\delta^2$ and

$$E(r_1) \cong \frac{(n-1)\gamma_1 - \delta_2}{(n-1)\gamma_0 + 2\delta^2} = \frac{\frac{-a_3}{1+a_3} - \frac{(\delta/\varepsilon_t^2 - h_t)^2}{(n-1)(1+a_3^2)}}{1 + \frac{2(\delta/\varepsilon_t^2 - h_t)^2}{(n-1)(1+a_3^2)}} \qquad (31)$$

The moving average parameter a_3 and the lag one autocorrelation ρ_1 of the first differences of an ARIMA (0,1,1) process are related as $\rho_1 = -a_3/(1+a_3^2)$. Moment estimates of a_3 can be obtained by replacing ρ_1 in the above equation by r_1 and solving the resulting quadratic equation for the invertible solution a_3. (If the absolute value of r_1 is larger than 0.5, a_3 is set equal to 1 or-1). Equation 31 shows us how the outlier affects the expectation of r_1. Accordingly, a very crude approximation of $E(a_3) = a_{3*}$, can be obtained by solving the quadratic equation $E(r_1) = -a_{3*}/(1 + a_{3*}^2)$. The invertible solution (that is, the one solution that is between -1 and +1)

$$a_{3*} = -\frac{1}{2E(r_1)} \pm \sqrt{\frac{1}{4[E(r_1)]^2} - 1} \qquad (32)$$

allows us to quantify the effect of an additive level outlier on the moving average estimate in the ARIMA(0,1,1) process thereby transforming same to the GARCH parameters. For example, a 5σ outlier among n = 100 observations in the GARCH(1,1) process with $a_3 = 0.5$ leads to $E(r_1) = -0.429$ and $E(a_3) = 0.566$; Table 3.

Since the basic version of the least squares model (used for estimating ARMA parameters) assumes that the expected value of all error terms, when squared, remains the same at any given point, the simple extension follows as the same assumption is the focus of ARCH/GARCH models.

Thus the result of our research shows that the coefficient estimates of GARCH models are seriously affected by additive outliers thereby extending a corresponding effect on forecast performance.

The research also reveals that the severity of bias decreases with the outlier distance from the forecast origin and so the least squares estimate is unaffected if the outlier occurs at the last observation. Thus, the impact of an unrecognized outlier that occurs 5 or more periods prior to forecast origin (or end of series) is automatically discounted.

Since the GARCH (1,1) model is directly set up to forecast for just one period, it turns out that based on the one-period forecast, a two-period forecast can be made. Ultimately, by repeating this step, long-horizon forecasts can be constructed. Thus, the GARCH models are mean reverting and conditionally heteroskedastic, but have a constant unconditional variance.

Finally, although the effect of an additive level outlier is bounded and independent of GARCH model for the series, the difference in estimation between the outlier corrected and uncorrected series is always significant. Hence, outlier detection plays an important role in applied time series modeling, testing and inference as they can lead to model misspecification, biased parameter estimation, poor forecasts and inappropriate decomposition of the series.

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