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Fuzzy Multi-Echelon Inventory System

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Abstract: This study applies fuzzy set theory to multi-echelon inventory system. Supply chain management means co-coordinating, scheduling and controlling procurement, production, inventories and delivers of products and service of customers. Within manufacturing research, the supply chain concept grew largely out of two stage multi-echelon inventory models. The objective of this study is to modulate and simulate the behaviour of an supply chain in an uncertain environment. Numerical examples are provided and sensitivity analysis is also given to emphasize the results in crisp and fuzzy environment.

Key words: Fuzzy sets, multi-echelon, supply chain management, inventory system

INTRODUCTION

In this study we considered a series of inventories in a supply chain in fuzzy environment. This system is a multi-echelon inventory control system with individual decision makers. It is very much essential for companies to manage the resources that are available to them. These resources include labor, equipment, money, land and information. The company which manages these resources better can compete and confidently with other companies. Mathematical models can help in providing the solution to manage these resources. SCM is the technical name for him above described activities. A logical progression of the inventory models is to investigate the SC that consists of suppliers, manufacturers, distributors and retailers. Each one of them holds inventory in some form to support the requirement at the end of the chain. SCM is also called as multi-echelon inventory theory as each stage is considered as an Echelon. The intermediate inventory in the SC is essential to support the customers. The question therefore to be answered is What is the optimal inventory theory that each company should hold along a chain such that the resource of each company are used the best and the inventory cost of the chain are reduced.

Recent years have witnessed increasing interest in SC management problems (Cadenas and Verdegay, 2000). An SC is generally viewed as a network of facilities that performs the procurement of raw material, its transformation to intermediate and end-products, distribution and selling of the end-products to end customers. The subsystems in an SC, explicitly recognizable, including a raw material inventory, production facilities, in-process and end-products stocks and selling point inventories, are coupled and interrelated in such a way that the control of one subsystem affects the performance of others.

Scanning the not very long history of SC literature one may see that it is a scene of rapid changes. Various SC configurations and different aspects of SC management and control problems have been treated in the literature. Here are only a few interesting historical facts. Industrial dynamics theory has been used for examining SC dynamic behavior. Forrester proved that small variations of customer demand cause demand variations amplification along an SC and create system instability. The customers, distribution, manufacturing and procurement orders along an SC show great swings as answers to the small variation of retail sales. A good deal of work has been done in the field of optimal SC control. Most of the models developed treated only isolated parts of an SC. Among the earliest was Mung-Fung Yang model for a serial two stage production and inventory control. An inspirative and representative analytical model for the whole SC, based on stochastic optimization, was developed by Cinler (1958). A number of SC analyses reported applied simulation techniques. For example, Giannoccaro *et al.* (2003) used a discrete

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event simulation to assess the impact of uncertain data, such as customer orders and a factory lead time, on SC performances.

A real SC operators in an uncertain environment. Different sources and types of uncertainty exist along the SC. They are random events, uncertainty in judgment, lack of evidence, lack of certainty of evidence that appear in customer demand, production and supply. Each facility in the SC must deal with uncertain demand imposed by succeeding facilities and uncertain delivery of the preceding facilities and uncertain delivery of the preceding facilities in the SC. The SC models developed so far either ignored uncertainty or considered it approximately through the use of probability concepts.

The objective of this study is to model and simulate behavior of an SC in an uncertain environment. Uncertain demand and uncertain supply are described by vague linguistic phrases, such as demand is about d products per week or the supplier is very reliable in delivering. Our position is that fuzzy set theory is an adequate methodology of accounting for vagueness of different types. In this paper uncertainties are formally represented by fuzzy sets and calculus on them is performed according to the fuzzy arithmetic rules. Supply chain management means co-coordinating, scheduling and controlling procurement, production, inventories and delivers of products and service of customers. Within manufacturing research, the supply chain concept grew largely out of two stage multi-echelon inventory models.

MODEL DESCRIPTION

Problem statement: We can write the multi-criteria problem as follows:

Vector Max $F(x) = \{f_1(x), f_2(x),...,f_P(x)\}$
subject to
$g_i(x) \leq 0 \; \forall_j = 1,..., M$
Where x is a n-vector of decision variable and $f_i(x)$, i = 1, ..., p are the criterion functions that are to be maximized simultaneously.
Let
$S = \{x/g_i(x) \leq 0, i = 1, 2, ..., M\}$ be the decision space of the problem.
$Y = \{y/F(x) = y, x \in S\}$ be the objective space

Efficiency: A solution $x^0 \in S$ is said to be efficient if there does not exist any point $x \in S$ such that $f_k(x) \geq f_k(x^0)$ $\forall k = 1, ..., p$. Efficient points have the property that no criterion can be improved from its present value without sacrificing on at least one criterion. Let N be the set of all efficient solutions.

Ideal solution: Let f_i^*, $\forall i = 1,...., p$ represent the solution found by solving the problem.
Maximum $f_i(x)$ subject to $x \in S$.

Then the vector solution $F^* = \{f_1^*, f_2^*, ..., f_p^*\}$ is called ideal solution to the problem. The ideal solution to a multi-criteria problem is not achievable since the criteria conflict with one another.

In this study we consider linear multi objective mathematical programming problem concerning SCM, in which the coefficients defining the multi objective function are given as fuzzy numbers. This is reasonable, because each objective can be defined by a different decision maker (manufacturer, distributor and retailer) with which the respective ways of comparing with fuzzy numbers involved are to be taken into account in order to give operation methodology the main objective of this study is total number of orders per year and total inventory capital per year.

CRISP MODEL

A serial supply chain is one in which the companies are arranged in a series (Fig. 1). It is assumed that there is one product that is sold at the retailer level. Product flows from one company to the next, starting furthest from the retailer and moving towards the retailer. The model assumes that there does not exist any reverse flow of the product, nor is there any backlog at any stage.

The problem that analyzed here is unconstrained. Constraints such as capacity and cost are not considered here. The Decision Makers (DM's) at each company play an important role in the final solution to this problem. It is assumed that the cost, capacity and any other constraint information which is present in each individual company is implicitly known to the DM's. Since the solution is an interactive one, the DM's can steer the problem to the best compromise point of their choice. The unconstrained serial supply chain problem is a multi criteria problem in two levels. Level one of the problem is a multi criteria problem pertaining to the individual companies of the supply chain. We assume that each individual company consists of a single decision maker, deciding the inventory policy for the company and operating independently. Level two of the problem is a multi criteria problem where the entire supply chain is taken into consideration. The supply chain is made up a number of decision makers. We assume that the number of decision makers in a supply chain is equal to the number of companies in the supply chain (since each company consists of a decision maker). Both the levels are modeled using two criteria. The first criterion is based on the order quantity of each company. The second criterion is based on the number of orders made by each company to satisfy

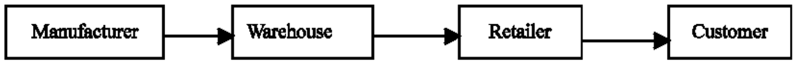

Fig. 1: Serial supply chain system

demand. Since a supply chain multi criteria problem exists at both levels, it is important for companies to collaborate with one another to reach a compromise solution that is not only efficient at the individual company level, but also efficient at the supply chain.

INDIVIDUAL COMPANY MULTI CRITERIA PROBLEM

This study describes the multi criteria individual company problem and explains how it can be solved. At the end of this solution process, each individual company would have a most preferred non-dominated solution.

Assumptions:

- Single product model.
- Demand is assumed to be deterministic and continuous.
- Backlog or shortage is not considered.
- Lead time is assumed to be constant or zero.
- Based on the demand, each company would choose an ordering policy that consists of a fixed order at regular intervals.

Objective functions: Let the maximum inventory per cycle of length T of a company be Q, the cost of an item be C and the demand, D, per year. The inventory level follows a saw tooth function and is shown in Fig. 2.

The two criteria that are considered for solving the individual company problem are inventory.

Capital and number of orders

Inventory capital: The inventory capital of a single company is defined as the product of its average inventory per unit time to the cost of the product. The inventory capital per cycle is given as (1/2 DT)C. Therefore, inventory capital per unit time is QC/2

Number of orders: Number of orders for a single company is defined as the number of orders placed annually by the company. The annual number of orders is D/Q.

Multi criteria inventory model for individual company: The individual company bi-criteria problem can be written as

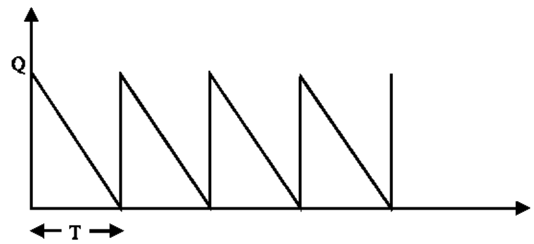

Fig. 2: Inventory cycle of a single company

$$\text{Min } Z_1 = \frac{QC}{2} \quad (1)$$

$$\text{Min } Z_2 = \frac{D}{Q} \quad (2)$$

such that $Q \geq 0$.

We assume that the Decision Maker (DM) has a quasi concave utility function is given by Maximum $U(Z_1, Z_2)$.

The assumption of quasi concave 'U' is consistent with a diminishing marginal return on the tradeoffs. The problem is to determine Q that maximizes U. Note that U increases monotonically as Z_1, Z_2 decreases. Since the DM utility function is not known, we cannot find the point that maximizes $U(Z_1, Z_2)$ directly. We use an interactive method that assesses U through continuous interaction with DM.

Methods for generating efficient points: One method is to generate the efficient points is given by Pλ problem.

Pλ-problem: Maximum $\sum_{i=1}^{p} \lambda_i f_i(x)$

Subject to $x \in S$

Where λ_i are non negative parameters. The λ_i can be normalized using $\sum_{i=1}^{p} \lambda_i = 1$

Sufficient condition: If $\lambda_i > 0$ and x^0 is an optimal solution to the P_λ problem then x^0 is an efficient solution.

Necessary condition: If the constraint region S is a closed convex set and f_i are concave on S and one of the f_i is strictly concave then x^0 is efficient if and only if x^0 is optimal for the P_λ problem for $0<\lambda<1$. When the decision space is not convex then the efficient points can be classified as supported and unsupported points.

Generating efficient points for individual company multi-criteria problem: Using Eq. 1 and 2 the P_λ problem (Cinler, 1958; Giannoccaro *et al.*, 2003) for the single company can be written as

$$\text{Min } Z(\lambda) = \lambda\left(\frac{QC}{2}\right) + (1-\lambda)\frac{D}{Q}$$
$$\text{subject to } AQ \le B \quad Q > 0 \quad 0<\lambda<1 \tag{3}$$

The above problem satisfies the necessary and sufficient condition of the P_λ -problem. The objective functions are convex and the decision space is a closed convex set. Therefore, it is possible to find every efficient solution to the problem by solving the P_λ-problem. Since the both objective function are convex, z (λ) is convex, for a fixed λ between (0, 1) and we can find the minimum by equating the first differential to zero.

The first differential of Z with respect to Q is

$$\frac{dZ}{dQ} = \frac{C\lambda}{2} - (1-\lambda)\frac{D}{Q^2}$$

The second differential with respect to Q is

$$\frac{d^2Z}{dQ^2} = 2(1-\lambda)\frac{D}{Q^3}$$

Given that D and Q are always positive, the second differential is always positive for $0<\lambda<1$. Therefore equating the first differential to 0 gives in the minimum value of the function z (λ).

$$Q = \sqrt{\frac{2D\ (1-\lambda)}{C\lambda}} \tag{4}$$

For every λ, $0<\lambda<1$, the Eq. 4 gives an efficient point to the single company multi-criteria problem. For different values of λ we get different values of Q. Since the demand is known we can calculate the number of orders once we know the order quantity. Therefore, for every Q, that the DM chooses we get a corresponding value of number of orders per year. This combination is always efficient. This shows that each company has a infinite number of points which make up its efficient frontier.

The efficient frontier for a single company is shown in the (Fig. 3).

We can also see that for any inventory level Q that a company chooses, we can get a corresponding value for λ.

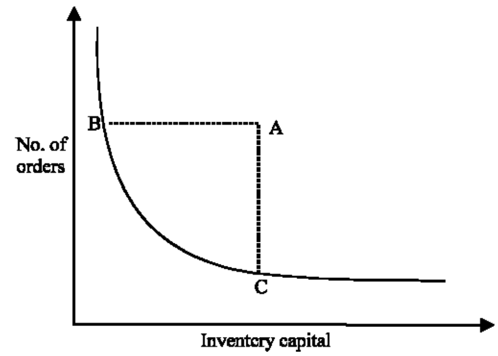

Fig. 3: Efficient frontier

$$\text{That is } \lambda = \frac{2D}{Q^2C + 2D}$$

The Fig. 3 shows that any point above the curve is dominated. For example policies B and C are better than A, since the company can do better by operating at a lower inventory capital with the same number of order B or a lower number of orders for the same inventory capital C. Any point below the curve is infeasible.

Numerical example: The computation of optimal ordering quantity for the values A = 2 sq. m,
Z = 500, P = 800, C = 200, D = 100, B = 750, λ = 0.3.
From the Eq. 4, we get Q = 1.5275.

FUZZY MODEL

In the above crisp model, we considered, the optimization problem as a non-linear programming problem of the type

$$\text{Min } Z(\lambda) = \lambda\left[\frac{QC}{2}\right] + (1-\lambda)\left[\frac{D}{Q}\right]$$
$$\text{subject to } AQ \le B \quad 0<\lambda<1 \quad Q>0 \tag{5}$$

where A : Space required by each unit (in sq. m)
B : Maximum available warehouse space (in sq. m)
Q : Maximum inventory
D : Demand

After fuzzification of the crisp parameters (cost, demand and storage area) we get,

$$\text{Minimize } \tilde{Z}(\lambda) = \lambda\left[\frac{Q\tilde{C}}{2}\right] + (1-\lambda)\left[\frac{\tilde{D}}{Q}\right]$$

subject to $AQ \leq \tilde{B}$

$Q > 0$

The corresponding fuzzy non linear programming problem is

Maximize α

Subject to the constraints

$$\mu_C^{-1}(a)\left[\frac{\lambda Q}{2}\right] + \mu_D^{-1}(a)\left[\frac{1-\lambda}{Q}\right] \leq \mu_Z^{-1}(a)$$

$$AQ \leq \mu_B^{-1}(a) \qquad a \in (0,1)$$

Where μ_c, (x), μ_D (x) and μ_Z(x) are the membership functions for cost, demand and maximum inventory, respectively.

Here the Lagrange function is

$$L(a,Q,\Delta) = a - \Delta_1\left[\mu_C^{-1}(a)\left[\frac{\lambda Q}{2}\right] + \mu_D^{-1}(a)\left[\frac{1-\lambda}{2}\right] - \mu_Z^{-1}(a)\right] - \Delta_2\left[AQ - \mu_B^{-1}(a)\right] \quad (6)$$

Here $\Delta_1$and Δ_2 are Lagrange multipliers.

Fuzzy goal, costs and storage area are represented by following linear membership function.

The membership function of goal, cost and storage area are defined as,

$$\mu_C(x) = \begin{cases} 1 & \text{if } x \geq C \\ 1 - \dfrac{C-x}{P_C} & \text{if } C - P_C \leq x < C \\ 0 & \text{if } x < C - P_C \end{cases} \quad (7)$$

$$\mu_Z(x) = \begin{cases} 1 & \text{if } x \leq Z \\ 1 - \dfrac{x-Z}{P} & \text{if } Z < x \leq Z + P \\ 0 & \text{if } x > Z + P \end{cases} \quad (8)$$

$$\mu_B(x) = \begin{cases} 1 & \text{if } x \leq C \\ 1 - \dfrac{x-B}{P_B} & \text{if } B < x \leq B + P_B \\ 0 & \text{if } x > B + P_B \end{cases} \quad (9)$$

$$\mu_D(x) = \begin{cases} 1 & \text{if } x \leq C \\ 1 - \dfrac{x-D}{P_D} & \text{if } D < x \leq D + P_D \\ 0 & \text{if } x > B + P_D \end{cases} \quad (10)$$

Here P_C, P, P_D and P_B are maximally acceptance violation of the aspiration levels for cost, goal and warehouse space, respectively. The above Eq. 7-10 are

$$\left.\begin{aligned} \mu_C^{-1}(a) &= C - (1-a)P_C \\ \mu_Z^{-1}(a) &= Z - (1-a)P \\ \mu_B^{-1}(a) &= B - (1-a)P_B \\ \mu_D^{-1}(a) &= D - (1-a)P_D \end{aligned}\right\} \quad (11)$$

substitute the Eq. 11 in 6, we get,

$$L(a,Q,\Delta) = a - \Delta_1\left[\begin{matrix}\left(C-(1-a)P_C\right)\left[\frac{\lambda Q}{2}\right] + \left(D-(1-a)P_D\right) \\ \left[\left[\frac{1-\lambda}{2}\right] - \left(Z-(1-a)P\right)\right.\end{matrix}\right] - \Delta_2\left[AQ - B - (1-a)P_B\right] \quad (12)$$

Using Kuhn-Tucker conditions in (12), we get

$$\frac{\partial L}{\partial \Delta_1} = 0 \text{ and } \frac{\partial L}{\partial \Delta_2} = 0$$

$$\left(C-(1-\alpha)P_C\right)\left[\frac{\lambda Q}{2}\right] + \left(D-(1-\alpha)P_D\right)\left[\frac{1-\lambda}{2}\right] - \left(Z-(1-\alpha)P\right) = 0 \quad (13)$$

$$AQ - B - (1-\alpha)P_B = 0 \quad (14)$$

From Eq. 13, we get

$$a = 1 - \left[\frac{Z - C\dfrac{\lambda Q}{2} - D\left(\dfrac{1-\lambda}{2}\right)}{P - P_C\dfrac{\lambda Q}{2} - P_D\left(\dfrac{1-\lambda}{2}\right)}\right] \quad (15)$$

Using (15) in (14), we get

$$AQ - B - \left[\frac{Z - C\frac{\lambda Q}{2} - D\left(\frac{1-\lambda}{2}\right)}{P - P_C\frac{\lambda Q}{2} - P_D\left(\frac{1-\lambda}{2}\right)} \right] P_B = 0$$

Then the optimal ordering quantity is,

$$Q^* = \frac{U \pm \sqrt{U^2 - 4\left(\frac{AP_C}{2}\right) V\lambda}}{A P_C \lambda} \quad (16)$$

where

$$U = AP - AP_D\left(\frac{1-\lambda}{2}\right) + \frac{BP_C\lambda}{2} + \frac{C\lambda}{2} \quad (17)$$

$$V = BP - BP_D\left(\frac{1-\lambda}{2}\right) + P_B\left(Z - D\left(\frac{1-\lambda}{2}\right)\right) \quad (18)$$

Numerical example: Let us illustrate the model with fuzzy environment by an example. In addition to the above parametric values, we consider the maximum acceptance violation of the aspiration level for inventory cost and the floor space. Let $P_D = 600$, $P_C = 700$, $P_B = 900$.
From the Eq. 17 and 18, we get
U = 79960 and V = 861000
Q* = 11.0909

SENSITIVITY ANALYSIS

Sensitive analysis elucidates the determination of ranges of variation. The post optimal analysis done in this chapter attempts to analyze the optimal Q* for different tolerant levels P_C and P_D in the fuzzy environment. The values of Q* for different maximum tolerant levels of cost and inventory capital is shown in the following Table 1 and 2.

Table1: Effect of variation in P_C for $P_B = 900$ and $P_D = 600$

P_B	P_D	P_C	U	V	Q*
900	600	400	46210	861000	19.6333
		450	51835	861000	17.3988
		500	57460	861000	15.6214
		550	63085	861000	14.1737
		600	68710	861000	12.9717
		650	74335	861000	11.9578
		700	79960	861000	11.0909
		750	85585	861000	10.3413
		800	91210	861000	9.6867

Table 2: Effect of variation in P_D for $P_B = 900$ and $P_C = 700$

P_B	P_C	P_D	U	V	Q*
900	700	400	80100	913500	11.0704
		450	80065	900375	11.0755
		500	80030	887250	11.0806
		550	79995	874125	11.0858
		600	79960	861000	11.0909
		650	79925	847875	11.0961
		700	79890	834750	11.1013
		750	79855	821625	11.1064
		800	79820	808500	11.1116

- From the Table 1, we observe that for fixed values of P_B and P_D the value of Q* decreases with increasing value of P_C.
- From the Table 2, we observe that for fixed values of P_B and P_C the value of Q* increases with increasing value of P_D. But the changes are not significant.

SUMMARY

In this study, we considered an individual company (manufacturer or distributor or retailer) multi-criteria inventory control system, which is the building block to construct serial supply chain inventory model. We modeled it as a fuzzy inventory control with multi criteria optimization principle. The results are obtained in both crisp and fuzzy environment. The sensitivity analysis in fuzzy model have been done. In future this result may be extended to serial connected multiple companies taking independent decisions.

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