Temperature Distribution of an Arrheniusly Reacting Unsteady Flow Through a Porous Medium with Variable Permeability

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Abstract: We consider the energy equation of an Arrheniusly reacting flow through a porous medium with variable permeability. A self similar solution was obtained for the unsteady case of the flow using shooting numerical method. We report the effects of the coefficient of the variable permeability and the Frank-Kamenetskii parameter on the temperature field of the system.

Key words: Reacting flow, porous medium, variable permeability, unsteady flow, temperature field, Frank-Kamenetskii parameter

INTRODUCTION

Water and oil which are indispensable to human daily activities exist in the porous domains– underground media. In order to get water and oil in abundance we dig wells and sink bore-holes. There are light oils and heavy oils. Light oils can flow easily and can easily be recovered. Some areas of the world only have heavy oils exist in the porous underground media or reservoirs. In such cases, it may become necessary to introduce in-situ combustion in the processes of oil recovery. The in-situ combustion involves burning the heavy hydrocarbons (heavy oils) by introducing heat into the system in the presence of oxygen so that the system yields light oils, water and gas. That is:

$$\underset{\text{When heated}}{\underline{\text{Heavy+Oxygen}}} \longrightarrow \text{Light oils+Water+Gas}$$
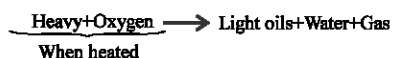
Of special interest to the scientists is the research done on physico-chemical and biological processes especially those of the enhanced oil recovery as well as processes in connection with organic pollutants and their removal from the subsurface. For instance, water flooding is at present the basic technological process of oil recovery. However, this method involves great loses of oil in the reservoir, i.e. much oil is not recovered. Consequently, more attention is being drawn to various processes of Enhanced Oil Recovery (EOR), such as the use of hot water, steam or water with added chemicals (called additives) to displace oil from the reservoir. The essence of these processes is that the properties of the water, oil and/or the porous media are so altered that the displacement produced under the influence of the physico-chemical processes becomes more efficient and more oil is recovered (Barenblatt *et al.*, 1990).

In 1998, Barenblatt and Vazquez revisited the theory of filtration through a horizontal porous stratum under the conditions of gently sloping fluid height profile. A model for flooding followed by natural outflow through the end-wall of the stratum was considered. The proposed model leads to a kind of free boundary problem for the porous medium equation which was analysed using phase plane techniques. Ayeni (2003) presented a phase plane analysis of a liquid front moving through a hot porous rock. The mechanism by which a relatively cold liquid vaporizes as it invades a hot permeable rock and the high rise in the pressure of the system were broadly discussed. Woods (1994). in his lecture series on porous media at a workshop on fluid mechanics organized by Abdus Salam International Centre for Theoretical Physics (ICTP), Italy, expounded much on dispersion and diffusion in porous media flows. He also explained some types of reactions driven by fluid flows in the porous rock and discussed the equations of energy transfer in such media.

Details of the systematic treatment of the mathematical theory of fluid flows in natural reservoirs as well as the physico-chemical hydrodynamics of porous media were contained in the study of Barenblatt *et al.* (1990) . Okoya (2002) considered the thermal stability for a reactive non-Newtonian viscous flow in a slab and obtained an approximate analytic solution of the velocity of the steady case of the system. Also, Mendes *et al.* (2002) carried out an experimental and theoretical flow of a Bingham material through ideal porous media. The

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pressure distribution in a layered reservoir with lateral wells was studied by Adewole *et al.* (2003). Recently, Ogunsola (2005) studies the reacting flows in porous media with variable permeability. In this study, we consider the non-isothermal case of the study of Barenblatt and Vazquez (1998) with a non-constant permeability distribution of the medium in the presence of an Arrhenius type reaction.

GOVERNING EQUATIONS

The unsteady non-isothermal flow of a fluid through a porous medium in the presence of a chemical reaction is governed as derived in (Barenblatt and Vazques, 1998; Barenblatt *et al.*, 1990 Ogunsola, 2005) by the equations:

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}(\frac{k\rho g}{2m\mu}\frac{\partial h^2}{\partial x}), x_0 < x < x_f \qquad (1)$$

$$\rho c_p \left(\frac{\partial T}{\partial t} - \frac{k\rho g}{\mu}\frac{\partial h}{\partial x}\frac{\partial T}{\partial x} \right) = \Gamma\frac{\partial^2 T}{\partial x^2} + \frac{Q}{t}e^{-\frac{E}{RT}} \qquad (2)$$

accompanied with the boundary conditions:

$$h(x,0) = h_0, h(0,t) = 0, \ t > 0, \frac{\partial h^2}{\partial x}\bigg|_{x=x_f} = 0 \qquad (3)$$

$$T(0,t) = T_1, \quad t > 0, \ T(x,0) = T_0, \ T_1 > T_0, \frac{\partial T}{\partial x}\bigg|_{x_f} = T_0 \quad (4)$$

where h is the hydraulic head of the fluid in the medium, ρ is the density, g is acceleration due to gravity, μ is fluid viscosity, k is permeability of the medium, m is porosity, t is time, T is temperature, with Γ the thermal conductivity of the medium, c_p the specific heat capacity, E the activation energy, Q the heat released per unit mass and R the universal gas constant. The subscripts o and f had been chosen to denote the initial and the final points, respectively. The second term of the right hand side of Eq (2) is the reaction term for the system. We assume that the medium is layered and thereby has a variable permeability distribution (Adewole *et al.*, 2003) which yields to pressure and depends on time. The permeability of a medium, (in a reacting system) may be varied by the results of chemical reactions occurring due to and in the fluid flowing through the medium. For instance, when a petroleum-well is drilled (during oil recovery), the pressure in the porous rock near the well decreases, altering the thermodynamic state of the petroleum. Light

hydrocarbons volatilize, which causes the temperature and solubility of the heavy hydrocarbons to decrease. In the case of waxy crudes, paraffins tend to precipitate off the oil (Mendes *et al.*, 2002). These flow reactions lead to either precipitations or dissolutions which in turn result in variations in the permeability of the medium. We therefore consider a permeability distribution that varies with space and time in the form:

$$k(x,t) = \frac{\lambda x}{t^\delta} \qquad (5)$$

as in (Ogunsola, 2005) where λ is the coefficient of the variable permeability and δ a real constant parameter. Equations (1) and (2) then become:

$$\frac{\partial h}{\partial t} = \frac{\lambda\rho g}{2m\mu}\left(t^{-\delta}\frac{\partial h^2}{\partial x} + \frac{x}{t^{-\delta}}\frac{\partial^2 h^2}{\partial x^2} \right) \quad x_0 < x < x_f \qquad (6)$$

$$\rho c_p \left(\frac{\partial T}{\partial t} - \frac{\rho g}{\mu}\frac{\lambda x}{t^\delta}\frac{\partial h}{\partial x}\frac{\partial T}{\partial x} \right) = \Gamma\frac{\partial^2 T}{\partial x^2} + \frac{Q}{t}e^{-\frac{E}{RT}} \qquad (7)$$

subject to the conditions (3) and (4).

METHOD OF SOLUTION

Let

$$\theta = \frac{E}{RT_0^2}(T - T_0) \text{ and } \in = \frac{RT_0}{E} \qquad (8)$$

Equation (7) becomes

$$\frac{\partial\theta}{\partial t} - (\frac{\lambda x}{t^\delta})\frac{\rho g}{\mu}\frac{\partial h}{\partial x}\frac{\partial\theta}{\partial x} = \frac{\Gamma}{\rho c}\frac{\partial^2\theta}{\partial x^2} + \frac{Q}{\rho c \in T_0}e^{-\frac{E}{RT_0}}e^{\frac{\theta}{1+\in\theta}} \quad (9)$$

Following Barenblatt and Vazquez (1998), we take the free boundaries to be:

$$x_f(t) = At^\beta, \ x_0(t) = \lambda_0 At^\beta \qquad (10)$$

and seek self similar solutions:

$$h = t^{-\alpha}f(\eta) \ , \ \eta = \frac{x}{At^\beta} \qquad (11)$$

$$\theta = t^{-\alpha}\phi(\eta), \ \eta = \frac{x}{At^\beta} \qquad (12)$$

The free boundaries correspond to $\eta_f = 1, \eta_0 = \lambda_0$ in the new space variable and the similarity solutions exist if $\beta = \frac{1}{2}, \alpha = 0$ and $\delta = \frac{1}{2}$. Then we obtain:

$$a\left((f^2)' + \eta(f^2)''\right) + \beta\eta f' + \alpha f = 0 \qquad (13)$$

$$D\phi'' + B\eta f'\phi' + \frac{\eta}{2}\phi + \psi e^{\frac{\phi}{1+\epsilon\phi}} = 0 \qquad (14)$$

where $a = \frac{\lambda\rho g}{2m\mu A}$, $\psi = \frac{Q_0 \ell^{-\frac{E}{RT_0}}}{\rho c \in T_0}$ and $D = \frac{\Gamma}{\rho c A}$

Following (Barenblatt and Vazques, 1998) the boundary conditions at now become:

$$f(1) = 0, \, f'(1) = \frac{-1}{4a} \qquad (15)$$

and

$$\phi(0) = T_1, \, \phi'(1) = T_0 \qquad (16)$$

The parameter ψ is the Frank-Kamenetskii parameter and ϵ is the activation energy parameter for the reacting system.

As in Ogunsola (2005) and Pakdemirli and Suhubi (1992) where series solutions of non-linear equations were obtained, we solved Eq (13) subject to (15) using series techniques and obtained an approximate solution for the function $f(\eta)$ such that:

$$f'(\eta) = \frac{1}{\eta}(1 - \frac{0.25}{a}\eta + \frac{0.046875}{a^2}\eta^2)$$
$$+ \ln(\eta)(\frac{0.09375}{a^2}\eta - \frac{0.25}{a}) \qquad (17)$$

Using (17) in (14) we solve Eq. 14 and 16 numerically using a Runge-Kutta method with shooting numerical techniques.

RESULTS AND DISCUSSION

The results obtained in this research are as presented on the graphs displayed in Fig. 1 and 2. We plot the graph of the temperature $\phi(n)$ for various values of the parameters φ and a, the results show that the temperature of a reacting system increases as the Frank-Kamenetskii parameter φ increases and that changes in the values of the parameter a which contain the coefficient of the
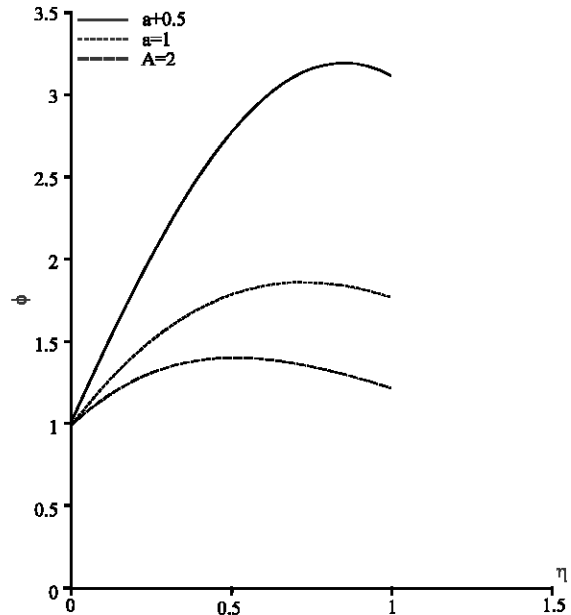

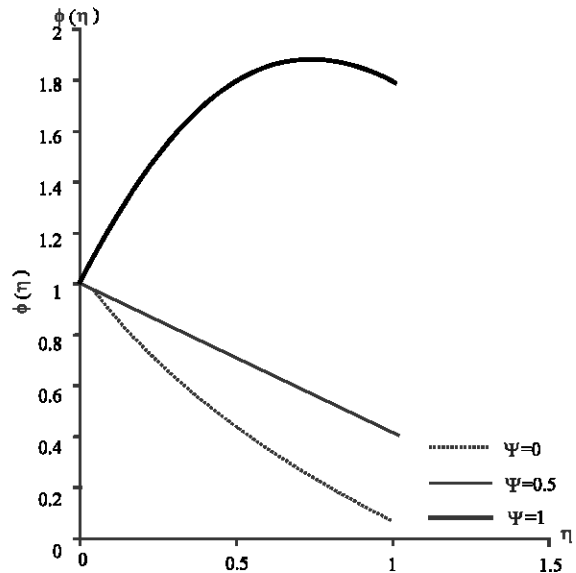Fig. 1: Graph of temperatyre ϕ againt η for various values of constant a

Fig. 2: The graph of temperature ϕ (η) for various values of parameter W Ψ

variable permeability affect the temperature of the system. The significance of the effects of the reaction parameters φ is most envisaged in the processes of recovery of heavy oils from porous rocks in which in-situ combustion is necessary. When the parameter φ which is the measure of exothermicity of the reaction in the Arrheniusly reacting system is high, the rate of conversion of heavy

oils into light oils, water and gas is high and consequently, the recovery rate is boosted. This is of great economic importance.

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