Research Journal of Applied Sciences 2 (3): 215-220, 2007

A Comparative Study of the Performances of the OLS and Some GLS Estimators When Stochastic Regressors are Correlated with the Error Terms

[1]Kayode Ayinde and [2]B.A. Oyejola
[1]Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, P.M.B. 4000, Ogbomoso, Oyo State, Nigeria
[2]Department of Statistics, University of Ilorin, P.M.B 1515, Ilorin, Kwara State, Nigeria

Abstract: The estimates of the OLS estimator of the Classical Linear Regression Model are known to be inconsistent when regressors are correlated with the error terms. However, this does not imply that inference is impossible. In this study, we compare the performances of the OLS and some Feasible GLS estimators when stochastic regressors are correlated with the error terms through Monte Carlo studies at both low and high replications. The performances of the estimators are compared using the following small sampling properties of estimators at various levels of correlation: bias, absolute bias, variance and more importantly the mean squared error of the model parameters. Results show that the OLS and GLS estimators considered in the study are equally good in estimating the model parameters when replication is low. However with increased replication, the OLS estimator is most efficient even though the performances of all the estimators exhibit no significant difference when the correlation between regressor and error terms tends to ±1.

Key words: Stochastic regressors, correlation between stochastic regressor and error terms, OLS estimator, feasible GLS estimators

INTRODUCTION

In the Classical Linear Regression Model (CLRM), regressors are not only assumed to be non-stochastic but also uncorrelated with the error terms. The assumption of non-stochastic regressors are not always valid especially in economics and social sciences where regresssors are not always assumed fixed in repeated sampling but are generated by a stochastic mean beyond control (Fomby *et al*., 1984). Many authors including Neter and Wasserman (1974), Chartterjee *et al*. (2000) and Maddala (2002) emphasized that in practice the uncorrelated assumption of regressor with error terms may not always be satisfied especially in econometric where variables are often measured with errors. Errors in the dependent variable pose no problem in that the errors are added to the model random error component. However when they are noted to be present in the regressors, they cause the regressors to be correlated with the error terms (Neter and Wasserman, 1974; Maddala, 2002).

The Ordinary Least Square (OLS) estimator of the CLRM is still unbiased with stochastic regressors provided the regressors and the error terms are independent. Also, the variance of the OLS estimator and the test of significance are valid if only they are viewed as conditional in the values of regressors. Furthermore, if the dependent variable and the regressors are assumed to be jointly distributed, all the test statistics are not only valid but the results are also identical to the OLS estimator (Formby *et al*., 1984; Neter and Wasserman, 1974; Maddala, 2002).

The Ordinary Least Square (OLS) estimator $\hat{\beta}$ of β given as

$$\hat{\beta} = \left(X^{1}X\right)^{-1} X^{1}Y \qquad (1)$$

and its variance-covariance matrix given as

$$V\left(\hat{\beta}\right) = \sigma^{2}\left(X^{1}X\right)^{-1} \qquad (2)$$

When regessors and error terms are correlated, the estimates of the OLS estimator are known to be biased and lack property of consistency. Maddala (2002)

Corresponding Author: Kayode Ayinde, Department of Pure and Applied Mathematics, Ladoke Akintola Univeristy of Technology, P.M.B. 4000, Ogbomoso, Oyo State, Nigeria

emphasized that the fact that consistent estimates of the parameters can not be obtained does not imply no inference is possible. Also, he did not only point out that one can obtain expressions for the direction of biases and consistent bound for the parameter, but also that sometimes we can use some extra information available in the dynamics of the equations or the structure of other equations in the model to get consistent estimates of the parameters.

When the homoscedastic assumption (i.e., $E(UU^1) = \sigma^2 I_n$) of the CLRM is violated, the resulting model the Generalized Least Squares (GLS) Model with heteroscedastic error variance (i.e., $E(UU^1) = \sigma^2\Omega$). Aitken (1935) has shown that the GLS estimator $\hat{\beta}$ of β given as

$$\hat{\beta} = \left(X^1\Omega^{-1}X\right)^{-1} X^1\Omega^{-1}Y \quad (3)$$

is efficient among the class of linear unbiased estimators of β with variance-covariance matrix of $\hat{\beta}$ given as

$$V\left(\hat{\beta}\right) = \sigma^2\left(X^1\Omega^{-1}X\right)^{-1} \quad (4)$$

where Ω in Eq. 3 and 4 is usually unknown. It is often estimated by Ω to have what is known as Feasible GLS estimator. Fomby *et al.* (1984) pointed out that many consistent estimate of Ω can be obtained.

When the error terms of the GLS Model are autocorrelated and follow Autoregressive of order one (AR (1)), some of the Feasible GLS estimators available in literatures are the Cochrane and Orcutt (1949), Hildreth and Lu (1960), Paris and Winsten (1954), Thornton (1982), Durbin (1960), Theil's (1971), the maximum likelihood estimator and the maximum likelihood grid estimator (Beach and Mackinnon, 1978). Some of these estimators have now been incorporated into White's SHAZAM program (White, 1978) and the new version of the time series processor (TSP, 2005). However, all of these estimators are known to be asymptotically equivalent but the question on which is to be preferred in small samples is the worry of researchers (Fomby *et al.*, 1984).

Assuming no autocorrelation of the error terms, we therefore examine and compare the performances of some of these Feasible GLS estimators with that of the OLS estimator when stochastic regressor is correlated with the error terms.

MATERIALS AND METHODS

Consider the CLRM with stochastic regressors of the form

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + e_t \quad (5)$$

where x_1 is said to have ρ correlation with $|\rho|<1$

i.e., $x_{1t} = g(e_t, \rho) \quad t = 1, 2, \ldots, n \quad \varepsilon_t \sim N(0, \sigma^2 I_n)$

As emphasized before, the OLS method can be used to estimate the parameters of the model.

Equation 5 can equivalently be written as a GLS model with AR (1) of the form

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t \quad (6)$$

where $u_t = \rho_1 u_{t-1} + \varepsilon_t, \quad |\rho_1| < 1, \quad t = 1, 2, \ldots, n$

and x_1 is said to have ρ correlation with $e|\rho|<1$ by setting $\rho_1 = 0$.

Now, suppose $W_i \sim N(\mu_i, \sigma_i^2)$ $i = 1, 2$. If these variables are correlated, then W_1 and W_2 can be generated with the equations

$$W_1 = \mu_1 + \sigma_1 z_1$$
$$W_2 = \mu_2 + \rho\sigma_2 z_1 + \sigma_2 z_2\sqrt{1-\rho^2} \quad (7)$$

where $Z_i \sim N(0,1)$ $i = 1, 2$ and $|\rho| < 1$ is the value of the correlation between the two variables. This is proved in Appendix 1.

Monte Carlo experiments were therefore performed for n = 20, a small sample size representative of many time series study (Park and Mitchell, 1980) with four Replication (R) levels (R = 10, 40, 80, 120) and nine various levels of correlation between regressor and error terms (ρ = -0.99, -0.75, -0.5,... 0.99). At a particular choice of ρ and R (a scenario), each replication was first by obtained generating $e_t \sim N(0,1)$. Next, $x_{1t} \sim N(0,1)$ was generated such that it had ρ correlation with $e_t \sim N(0,1)$ already generated using (7). $x_{2t} \sim N(0,1)$ was then generated. The values of y_1 in Eq. (5) were also calculated by setting the true regression coefficients as $\beta_0 = \beta_1 = \beta_2 = 1$. This process continued until all replications in this scenario were obtained. Another scenario then started until all the scenarios were completed.

The performances of the OLS estimator with the following feasible GLS estimators: Cochrane Orcutt

(CORC), Hildreth-Lu (HILU), Maximum Likelihood (ML) and the Maximum Likelihood Grid (MLGD) estimators were examined and compared under Eq. 5. Evaluation and comparison of estimators were examined using the following finite sampling properties of estimators: Bias (B), Absolute Bias (AB), Variance (Var) and the more importantly the Mean Squared Error (MSE) of the estimated parameter of the model. Mathematically, for any estimator $\hat{\beta}_i$ of β_i of (5)

$$\bar{\hat{\beta}}_i = \frac{1}{R}\sum_{j=1}^{R}\hat{\beta}_{ij} \quad (8)$$

$$B\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right) = \bar{\hat{\beta}}_i - \beta_i \quad (9)$$

$$AB\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left|\hat{\beta}_{ij} - \beta_i\right| \quad (10)$$

$$Var\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \bar{\hat{\beta}}_i\right)^2 \quad (11)$$

$$MSE\left(\hat{\beta}_i\right) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right)^2 = Var\left(\hat{\beta}_i\right) + \left[B\left(\hat{\beta}_i\right)\right]^2 \quad (12)$$

for i =0, 1, 2 and j=1,2,…,R.

A computer program was written using TSP software for each of the five methods of estimations to estimate the model parameters and evaluate the performances of the estimators based on the criteria. The four replication levels were further grouped into low (R = 10, 40) and high (R = 40, 80) and the effect of correlation between stochastic regressor and the error terms on the methods (estimators) were examined via the analysis of variance of the criteria of each of the model parameters in the two replication groups. This was accomplished by performing the LSD test of the estimated marginal mean of the criteria whose interaction effect is significant. At a particular level of correlation, the estimated marginal mean that is most preferred was compared with others. An estimator is most preferred at particular level of correlation if its estimated marginal mean is the smallest

APPENDIX 1:

Theorem: Suppose $W_i \sim N\left(\mu_i, \sigma_i^2\right) \quad i = 1,2$. If these variables are correlated, then W_1 and W_2 can be generated with the equations

$$W_1 = \mu_1 + \sigma_1 z_1$$
$$W_2 = \mu_2 + \rho\sigma_2 z_1 + \sigma_2 z_2\sqrt{1-\rho^2} \quad (I)$$

where $Z_i \sim N(0,1) \quad i = 1,2$ and $|\rho| < 1$ is the value of the correlation between the two variables.
In order to prove this theorem, we use this lemma.

Lemma: A matrix A is positive definite if and only if there exist a non singular matrix F such that A = FF[1]. A (real) symmetric matrix A of rank r is semi definite if and only if their exist a matrix F of rank r such that A = FF[1] (Neil, 1975).

This lemma indicates the existence of a matrix F such that A = FF[1]. To determine a matrix F, we use the Cholesky factorization procedure available in Forsythe (1951).
Suppose a matrix product may be defined as

$$A = TT^1 = \sum_{i=1}^{n} t_i t_i^1 \quad (ii)$$

so that

$$A - \sum_{i=1}^{n} t_i t_i^1 = 0 \quad (iii)$$

then, the Cholesky or square root factorization procedure constructs t_i^1 in a systematic manner until (iii) is obtained.

Proof:

Suppose $\quad \underset{\sim}{Z_t} = \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} \sim N_2(0, I)$

Then $\quad \underset{\sim}{W_i} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \Sigma^{\frac{1}{2}} Z_t \quad (iv)$

Where $\quad \underset{\sim}{W_i} \sim N_2\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \Sigma\right), \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$

Hence from (iv), we need to seek for $\Sigma^{\frac{1}{2}}$. This, we do by applying the lemma and the Cholesky factorization procedure since it is obvious that

$$\Sigma = \Sigma^{\frac{1}{2}}\Sigma^{\frac{1}{2}^1} = TT^1 \quad (v)$$

Using the Cholesky factorization and noting that $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} a_1^1 \\ a_2^1 \end{bmatrix}$$

$$t_1^1 = \frac{a_1^1}{\sqrt{a_{11}}} = \frac{1}{\sigma_1}\begin{bmatrix} \sigma_1^2 & \sigma_{12} \end{bmatrix} = \begin{bmatrix} \sigma_1 & \rho_{12}\sigma_2 \end{bmatrix}$$

$$t_1 t_1^1 = \begin{bmatrix} \sigma_1 \\ \rho_{12}\sigma_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & \rho_{12}\sigma_2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \rho_{12}^2\sigma_2^2 \end{bmatrix}$$

Thus, $A = \Sigma - t_1 t_1^1 = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_2^2\left(1-\rho_{12}^2\right) \end{bmatrix} = \begin{bmatrix} b_1^1 \\ b_2^1 \end{bmatrix}$

Similarly,

$$t_2^1 = \frac{b_2^1}{\sqrt{b_{22}}} = \frac{1}{\sigma_2\sqrt{1-\rho_{12}^2}}\begin{bmatrix} 0 & \sigma_2^2\left(1-\rho_{12}^2\right) \end{bmatrix} = \begin{bmatrix} 0 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix}$$

$$t_2 t_2^1 = \begin{bmatrix} 0 \\ \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix} \begin{bmatrix} 0 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_2^2\left(1-\rho_{12}^2\right) \end{bmatrix}$$

Thus,

$$B = A - t_2 t_2^1 = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_2^2\left(1-\rho_{12}^2\right) \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \sigma_2^2\left(1-\rho_{12}^2\right) \end{bmatrix} =$$

This has satisfied the Cholesky factorization procedure in (iii

Therefore,

$$T^1 = \begin{bmatrix} t_1^1 \\ t_2^1 \end{bmatrix} = \begin{bmatrix} \sigma_1 & \rho_{12}\sigma_2 \\ 0 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix}, \qquad T = \begin{bmatrix} \sigma_1 & 0 \\ \rho_{12}\sigma_2 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix}$$

CHECK: It is expected that $TT^1 = \Sigma^{\frac{1}{2}}\Sigma^{\frac{1}{2}^1} = \Sigma$.

$$TT^1 = \begin{bmatrix} \sigma_1 & 0 \\ \rho_{12}\sigma_2 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix}\begin{bmatrix} \sigma_1 & \rho_{12}\sigma_2 \\ 0 & \sigma_2\sqrt{1-\rho_{12}^2} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \rho_{12}^2\sigma_2^2 + \sigma_2^2\left(1-\rho_{12}^2\right) \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$

Consequently, $T = \Sigma^{\frac{1}{2}}$ and hence (iv) becomes

$$\underset{\sim}{W_i} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \Sigma^{\frac{1}{2}} Z$$

$$\underset{\sim}{W_i} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \sigma_1 & 0 \\ \rho_{12}\sigma_2 & \sigma_2\sqrt{1-\rho_{12}} \end{bmatrix}\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} \qquad \text{(vi)}$$

Thus, it follows that if $W_i \sim N\left(\mu_i, \sigma_i^2\right) \quad i = 1,2$. If these variables are correlated, then W_1 and W_2 can be generated with the equations

$$W_1 = \mu_1 + \sigma_1 z_1$$

$$W_2 = \mu_2 + \rho\sigma_2 z_1 + \sigma_2 z_2\sqrt{1-\rho^2} \qquad \text{(vii)}$$

where $Z_i \sim N(0,1) \quad i = 1,2$ and $|\rho| < 1$ is the value of the correlation between the two variables.

RESULTS AND DISCUSSION

The summary of our findings (via ANOVA) on the performances of the estimators as they are affected by correlation between regressor and error terms in all the criteria of each of the model parameters under the two replication groups is given in Table 1.

From Table 1, it is observed that the error sum of square and hence the mean square error (if estimated) reduces with increased replications. Thus, the performances of the estimators improve with increased replication.

At the low replication group, the performances of the estimators in estimating the model parameters in all the criteria are not affected by correlation between regressor and error terms since their interaction effects are not significant (p-value >0.05) even though some of the main effects are significant (p-value <0.05).

Based on variance and mean squared error criteria under the high replication group, the performances of the estimators in estimating the parameters of the model are affected by the correlation except in estimating $\beta_1$under the mean squared error criterion where the interaction effect is not significant (p-value >0.05). In estimating β_2 under the absolute bias criterion, the interaction effect is significant (p-value <0.05).

The estimated marginal means of parameters based on criteria whose interaction effects are significant are showed in Appendix 2. From Appendix 2, it is observed that the estimated marginal means of the significant interaction effect decrease as $|\rho|$ increases. The OLS estimator is the most efficient estimator in estimating all the parameters of the model in all the levels of the correlation. However, on the basis of variance and the mean squared error, it is observed that when $|\rho|\to 1$ all the estimators are equally in estimating all model parameters.

Table 1: Summary of the ANOVA TABLE showing the sum of squares of the model parameters based on the criteria in the two replication groups

				Type III Sum of squares			
Parameter	Replication group	Source	d.f	Bias	Absolute bias	Variance	Mean squared error
0	Low	R	8	1.10E-03	0.311**	4.278E-02**	4.334E-02**
		M	4	5.61E-04	7.175E-03**	1.570E-03*	1.599E-03**
		R*M	32	3.54E-03	3.98E-03	1.22E-03	1.22E-03
		Error	45	2.84E-02	9.78E-03	2.99E-03	2.80E-03
		Total	89	3.36E-02	0.332	4.86E-02	4.90E-02
	High	R	8	5.286E-03**	0.280**	3.244E-02**	3.303E-02**
		M	4	1.70E-04	1.119E-03**	2.519E-04**	2.606E-04**
		R*M	32	1.87E-04	4.71E-04	1.197E-04*	1.194E-04**
		Error	45	2.19E-03	7.86E-04	8.11E-05	5.91E-05
		Total	89	7.84E-03	0.283	3.29E-02	3.35E-02
	Low	R	8	39.353**	7.127**	6.312E-02**	9.897
		M	4	8.32E-03	2.24E-03	5.340E-03**	9.12E-03
		R*M	32	1.46E-02	1.12E-02	3.74E-03	1.06E-02
		Error	45	4.38E-02	3.01E-02	3.81E-03	4.28E-02
		Total	89	39.42	7.170	7.60E-02	9.959
	High	R	8	36.665**	6.986**	3.668E-02**	9.688**
		M	4	1.52E-04	9.879E-04**	1.697E-03**	2.069E-03**
		R*M	32	1.27E-03	1.80E-03	7.757E-04**	2.01E-03
		Error	45	3.18E-03	1.65E-03	3.18E-04	2.75E-03
		Total	89	36.67	6.990	3.95E-02	9.694
	Low	R	8	4.361E-02**	0.182**	2.347E-02**	2.684E-02**
		M	4	4.157E-03*	1.42E-02	5.424E-03**	5.287E-03**
		R*M	32	1.24E-02	5.33E-03	3.08E-03	3.27E-03
		Error	45	1.45E-02	6.82E-02	7.81E-03	8.71E-03
		Total	89	7.47E-02	0.270	3.98E-02	4.41E-02
	High	R	8	3.383E-02**	0.364**	4.693E-02**	5.103E-02**
		M	4	3.68E-04	4.200E-03**	1.767E-03**	1.624E-03**
		R*M	32	1.69E-03	1.094E-03**	6.285E-04**	5.851E-04**
		Error	45	2.62E-03	2.08E-04	1.05E-04	8.51E-05
		Total	89	3.85E-02	0.370	4.94E-02	5.33E-02

* ⇒Computed F value is significant at $\alpha = 0.05$. ** ⇒Computed F value is significant at $\alpha = 0.01$. ρ = R ⇒ Correlation between regressor and error terms, M⇒ Methods (Estimators)

APPENDIX 2

Summary of the results on the LSD test of the estimated marginal means based on criteria whose interaction effects are significant

		Replication = High		
		β_0		β_1
ρ	Methods	Estimated Marginal Means: VBO	Estimated Marginal Means: MBO	Estimated Marginal Means: VB1
-0.99	OLS	1.235E-03	1.241E-03	8.223E-03
	CORC	1.262E-03	1.267E-03	1.037E-02
	HILU	1.274E-03	1.280E-03	1.046E-02
	ML	1.256E-03	1.261E-03	1.001E-02
	MLGD	1.256E-03	1.261E-03	9.534E-03
-0.75	OLS	2.362E-02	2.386E-02	2.747E-02
	CORC	2.633E-02+	2.674E-02+	3.388E-02+
	HILU	2.645E-02+	2.686E-02+	3.382E-02+
	ML	2.442E-02	2.460E-02	3.087E-02
	MLGD	2.450E-02	2.470E-02	3.092E-02
-0.5	OLS	3.901E-02	3.946E-02	4.156E-02
	CORC	4.262E-02+	4.320E-02+	5.106E-02+
	HILU	4.249E-02+	4.309E-02+	5.129E-02+
	ML	3.962E-02	3.993E-02	4.752E-02+
	MLGD	3.939E-02	3.970E-02	4.754E-02+
-0.25	OLS	4.924E-02	4.974E-02	4.909E-02
	CORC	5.312E-02+	5.355E-02+	6.789E-02+
	HILU	5.347E-02+	5.390E-02+	6.857E-02+
	ML	4.847E-02	4.875E-02	6.055E-02+
	MLGD	4.860E-02	4.887E-02	6.063E-02+

Appendix 2: Countinued

		Replication = High		
		β_0		β_1
ρ	Methods	Estimated Marginal Means: VBO	Estimated Marginal Means: MBO	Estimated Marginal Means: VB1
0	OLS	5.140E-02	5.191E-02	5.095E-02
	CORC	5.815E-02+	5.858E-02+	7.530E-02+
	HILU	5.834E-02+	5.877E-02+	7.546E-02+
	ML	5.265E-02	5.299E-02	6.715E-02+
	MLGD	5.272E-02	5.306E-02	6.680E-02+
0.25	OLS	4.524E-02	4.579E-02	4.710E-02
	CORC	5.230E-02+	5.285E-02+	6.260E-02+
	HILU	5.262E-02+	5.318E-02+	6.362E-02+
	ML	4.673E-02	4.724E-02	5.749E-02+
	MLGD	4.668E-02	4.720E-02	5.637E-02+
0.5	OLS	3.613E-02	3.663E-02	4.812E-02
	CORC	4.221E-02+	4.263E-02+	6.714E-02+
	HILU	4.210E-02+	4.249E-02+	6.680E-02+
	ML	3.628E-02	3.670E-02	5.389E-02+
	MLGD	3.640E-02	3.679E-02	5.419E-02+
0.75	OLS	2.319E-02	2.345E-02	3.471E-02
	CORC	2.628E-02+	2.652E-02+	4.105E-02+
	HILU	2.636E-02+	2.660E-02+	4.103E-02+
	ML	2.418E-02	2.435E-02	3.713E-02
	MLGD	2.428E-02	2.444E-02	3.761E-02
0.99	OLS	1.215E-03	1.218E-03	8.695E-03
	CORC	1.265E-03	1.268E-03	1.128E-02
	HILU	1.267E-03	1.271E-03	1.163E-02
	ML	1.279E-03	1.282E-03	1.090E-02
	MLGD	1.284E-03	1.287E-03	1.146E-02

Appendix 2: Countinued

	Replication = High β_2			
ρ	Methods	Estimated Marginal Means:ABB2	Estimated Marginal Means:VB2	Estimated Marginal Means:MB2
-0.99	OLS	2.975E-02	1.962E-03	1.963E-03
	CORC	3.337E-02	2.077E-03	2.078E-03
	HILU	3.323E-02	2.005E-03	2.006E-03
	ML	3.170E-02	1.907E-03	1.907E-03
	MLGD	3.184E-02	1.964E-03	1.964E-03
-0.75	OLS	0.127	2.526E-02	2.635E-02
	CORC	0.151+	3.626E-02+	3.656E-02+
	HILU	0.151+	3.649E-02+	3.678E-02+
	ML	0.141+	3.109E-02+	3.169E-02+
	MLGD	0.141+	3.101E-02+	3.163E-02+
-0.5	OLS	0.159	3.930E-02	4.202E-02
	CORC	0.185+	5.543E-02+	5.635E-02+
	HILU	0.185+	5.519E-02+	5.611E-02+
	ML	0.174+	4.733E-02+	4.874E-02+
	MLGD	0.174+	4.725E-02+	4.862E-02+
-0.25	OLS	0.178	4.832E-02	5.188E-02
	CORC	0.200+	6.486E-02+	6.642E-02+
	HILU	0.200+	6.429E-02+	6.597E-02+
	ML	0.185+	5.460E-02+	5.658E-02+
	MLGD	0.185+	5.466E-02+	5.668E-02+
0	OLS	0.189	5.313E-02	5.654E-02
	CORC	0.209+	7.032E-02+	7.242E-02+
	HILU	0.209+	7.039E-02+	7.241E-02+
	ML	0.202+	6.165E-02+	6.447E-02+
	MLGD	0.201+	6.122E-02+	6.405E-02+
0.25	OLS	0.189	5.306E-02	5.545E-02
	CORC	0.210+	6.930E-02+	7.203E-02+
	HILU	0.212+	7.062E-02+	7.338E-02+
	ML	0.197+	6.111E-02+	6.430E-02+
	MLGD	0.195+	6.093E-02+	6.408E-02+
0.5	OLS	0.172	4.545E-02	4.661E-02
	CORC	0.194+	6.211E-02+	6.397E-02+
	HILU	0.195+	6.273E-02+	6.457E-02+
	ML	0.176+	5.023E-02+	5.198E-02+
	MLGD	0.176+	4.972E-02+	5.145E-02+
0.75	OLS	0.131	2.678E-02	2.703E-02
	CORC	0.147+	3.527E-02+	3.590E-02+
	HILU	0.148+	3.540E-02+	3.606E-02+
	ML	0.134	2.837E-02	2.887E-02
	MLGD	0.134	2.825E-02	2.876E-02
0.99	OLS	2.886E-02	1.855E-03	1.870E-03
	CORC	3.254E-02	2.005E-03	2.008E-03
	HILU	3.227E-02	1.968E-03	1.972E-03
	ML	3.086E-02	1.814E-03	1.819E-03
	MLGD	3.064E-02	1.859E-03	1.883E-03

+⇒Estimate that is significantly different from the most preferred one at $\alpha = 0.05$

CONCLUSION

The performances of both the OLS and some Feasible GLS estimators considered in this study exhibit no significant difference when stochastic regressors are correlated with the error terms at low replication. With increased replication, the OLS estimator is most efficient in estimating all the parameters of the model even though when $|\rho|\rightarrow 1$ all the estimators are equally good.

REFERENCES

Aitken, A.C., 1935. On Least Squares and Linear combinations of observations. Proceedings of the royal statistical society. Edinburgh, 55: 42-48.

Beach, C.M. and J.S. Mackinnon, 1978. A Maximum Likelihood Procedure for Regression with autocorrelated errors. Econometrica, 46: 51-57.

Chatterjee, S., A.S. Hadi and B. Price, 2000. Regression Analysis by Example. (3rd Edn.), A Wiley-Interscience Pubication. John Wiley and Sons.

Cochrane, D. and G.H. Orcutt, 1949. Application of Least Square to Relationship Containing Autocorrelated Error Terms. J. Am. Stat. Assoc., 44: 32-61.

Durbin, J., 1960. Estimation of Parameters in Time-Series Regression Models. J. Royal Stat. Soc. B., 22: 139-153.

Fomby, T.B., R.C. Hill and S.R. Johnson, 1984. Advanced Econometric Methods. Springer-Verlag, New York Berlin Heidelberg London Paris Tokyo.

Forsythe, G.E., 1951. Theory of selected methods of finite matrix inversion and decomposition. Washington, D.C: National Bureau of Standard, INA Report 52-5.

Hildreth, C. and J.Y. Lu, 1960. Demand Relationships with Autocorrelated Disturbances. Michigan State University. Agric. Exp. Stat. Bull. East Lansing, Michigan, pp: 276.

Maddala G.S. (2002). Introduction to Econometrics. John Wiley and sons Ltd, England (3rd Edn.).

Neter, J. and W. Wasserman, 1974. Applied Linear Model. Richard D. Irwin, Inc.

Neil, H.T., 1975. Multivariate Analysis with application in Education and Psychology. A Division of Wadsworth Publishing Company, Inc., pp: 73-75.

Park, R.E. and B.M. Mitchell, 1980. Estimating the Autocorrelated Error Model with Trended Data. J. Econometrics, 13: 185-201.

Paris, S.J. and C.B. Winstein, 1954. Trend Estimators and Serial Correlation. Unpublished Cowles Commission, Discussion Paper, Chicago.

Theil, H., 1971. Principle of Econometrics. New York, John Wiley and Sons.

Thornton, D.L., 1982. The appropriate autocorrelation transformation when the autocorrelation process has a finite past. Federal Reserve Bank of St. Louis Working, pp: 82-002.

TSP, 2005. Users Guide and Reference Manual. Time Series Processor. New York.

White, K.J., 1978. A General Computer Program for Econometric Methods-SHAZAM. Econometrica, 46: 239-240.