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# Understanding Techniques in Constructing Life Tables

[1]Aminul Islam, [1]Md. Rafiqul Islam, [1]Md. Nazrul Islam Mondal, [2]Md. Rahidul Islam, [1]Md. Shariful Islam, [3]Md. Nuruzzaman Khan, [4]Md. Nazrul Hoque and [5]Md. Mahedi Hasan

[1]*Department of Population Science and Human Resource Development, University of Rajshahi, 6205 Rajshahi, Bangladesh*

[2]*Institute of Bangladesh Studies, University of Rajshahi, 6205 Rajshahi, Bangladesh*

[3]*Research Center for Generational Health and Ageing, School of Public Health and Medicine, Faculty of Health and Medicine, The University of Newcastle, Australia*

[4]*Hobby Center for Public Policy, University of Houston, 306 McElhinney Hall, Houston, TX 77204-5021, USA*

[5]*Department of Accounting and Information Systems, Jessore University of Science and Technology, 7408 Jessore, Bangladesh*

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**Corresponding Author:**
Md. Nazrul Islam Mondal
*Department of Population Science and Human Resource Development, University of Rajshahi, 6205 Rajshahi, Bangladesh*

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**Abstract:** A life table is one of the most influential discoveries in demography which is primarily used to express the pattern of human mortality. Among the various uses, it plays an imperative role for analysis of mortality, population projection, life expectancy design and to determine key factors responsible for the highest mortality within the population. Public health employees, demographers, actuaries and many others use life tables for innumerable purposes. Life table can be constructed for a country or an area on the basis of various demographic characteristics including sex, occupation, race and so on. It can be classified according to reference year of the table, age detail and a number of factors comprehended by the table. The purpose of this study is to introduce life table including its various types and construction procedures.

## INTRODUCTION

In demography, a life table provides an estimate of the probability that a member of the cohort will die at a precise age. It affords to understand the changes in population due to different ages throughout their lives. A life table associates the mortality rates at different ages. It is an analytical technique in studying the calculation of mortality of an organism and individuals. As a life table is concerned with survival and life span, sometimes it is viewed as 'mortality table'[1]. The assumptions to which life table is based on may not be attainable in actual life but still, it has proved itself to be the most beneficial and authentic model and no superior model has been recommended to study various demographic components.

**Perspective history of life table:** A life table concept was first originated from John Graunt. He tabulated the number of deaths based on the analysis of 'Bills of Mortality'. Now, his life table was defective as it was constructed based on mortality experience only[2].

Halley[3] who was a British mathematician and astronomer first informed how to construct and calculate a life table properly. Halley[3] published his first recognized life table based on birth and death registration data for Breslau. His life table contained most of the columns of modern life table and the life table, we use today methodologically is similar to Halley's life table but his life table was not correct as it was constructed based on the assumption that population remained stationary. In 17 and 18th centuries, several attempts were made to construct a life table based on limited data. In 1815, Milne first published a life table using population and death data by age. In 1912, King was the first person to show us a method of getting the rates of the life table at pivotal values using interpolation formula. In 1939, a short method for calculation of abridged life table was given by Reed and Merrell[4]; Chiang suggested a method to calculate person-years lived. During the past 100 years, there were many contributions to the development of life table by demographers, actuaries and mathematician and so on.

**Types of life table:** According to reference year of the table, life table can be distinguished into two types: the generation or cohort life table and the current or period life table. Again, it can be classified into two categories: complete or unabridged and abridged life table according to the length of the age interval in which the data are presented. Moreover, single or multiple decrement life tables are also considered according to the number of decrement characteristics considered in a life table.

**Cohort life table:** Cohort life table reflects the summary presentation of the death history of the actual cohort. According to this type of life table, the death history of the actual cohort would be observed from their moment of birth through each successive age and continue until all of them die. It requires along period of time as the span of the cohort can be anywhere near 100 years or more. However, cohort life table is suitable to study mortality of those who possess short span of life like insects and plants. It should be kept in mind that cohort life expectancies tend to be higher than period life expectancies because they include any assumptions about future improvements in mortality rates.

**Period life table:** Period life table represents the mortality experience of the synthetic cohort over a short period of time such as 1 year, 3 years or an intercensal period if it experienced the mortality conditions of a particular period. This is also referred to as cross-section life table or current life table[5]. A period life table is constructed using current population data. This type of life table signifies mortality experience by age of the members of the hypothetical cohort rather than actual cohort in a particular period of time. Therefore, period life table may be viewed as an excellent summary description of mortality for a short period of time.

**Complete life table:** Complete life table presents decrementation due to deaths and other functions of life table for a single year of age from birth to the last applicable age. This type of life table can be constructed from the observed data or by suitable interpolation of an abridged life table.

**Abridged life table:** A life table in which value of the life table functions is presented for a certain age group usually 5 or 10 years interval are termed as abridged life table. Abridged life tables are found to be satisfactory and convenient for most of the situations in demographic analysis.

**Single decrement life table:** Single decrement life table concerns with single attrition factor and considers only one characteristic at a time. This type of table is a process in which the members of the cohort have only one recognized mode of departure from a well-defined state. Mortality is one such process.

**Multiple decrements life table:** Multiple decrement life tables summarize the mortality experience of cohort where the members of the cohort lost their membership cause of two or more attrition factors[6]. In such table, separate and combined effects of more than one factor are treated.

## MATERIALS AND METHODS

A life table is constructed based on age-specific death rates in a specific population. As it is constructed to trace the life history of the cohort, it is assumed that all those born should die and no one can be included at an age in between. To construct both single and multiple decrement life table, there is required some assumptions; the cohort originate from some standard number of birth say 10,000 or 100,000 which is called the radix of the life table. As mortality experience is found to be different for males and females, a life table is constructed separately for them. Death is uniformly distributed within the interval and the members of the cohort will die according to pre-determined mortality schedule at each age.

A single decrement life table has the following functions:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| (x, x+n) | ${}_nq_x$ | ${}_nP_x$ | $l_x$ | ${}_nd_x$ | ${}_nL_x$ | $T_x$ | $e^0_x$ |

The functions shown in the table are briefly described below:

**Age interval:** Age interval from exact age x to age x+n.

**Mortality rate $_nq_x$:** The probability that the persons in the cohort alive at the beginning of age interval x and would die during the next age. The value of $_nq_x$ is obtained using the following formula:

$$_nq_x = \frac{n._nm_x}{1+(n-{}_na_x)_nm_x}$$

where n refers to the length of the age interval, $_na_x$ refers to the person-years lived in the interval x to x+n by those dying in the interval[7] and $_nm_x$ is the observed mortality rate.

**Survival rate, $_np_x$:** The probability of surviving from exact age x to x+n and mathematically written as:

$$_np_x = 1-{}_nq_x = \frac{l_{x+n}}{l_x}$$

**Survivors, $l_x$:** The number of persons living at the beginning of age interval out of assumed number of birth $l_0$ which is the radix of the life table which is assumed number of birth to age 0 and is usually taken as 100,000. The $l_x$ is also known as decreasing function:

$$l_{x+n} = l_x \times {}_np_x$$

$$\text{Or, } l_{x+n} = l_x(1-{}_nq_x)$$
$$\text{Or, } l_{x+n} = l_x - {}_nd_x$$

**Number of deaths, $_nd_x$:** The number of deaths between exact ages x and x+n out of total number of births assumed in the table:

$$_nd_x = l_x - l_{x+n}$$

**Person-years lived, $_nl_x$:** Number of person-years lived by the cohort assumed number of birth $l_0$ between ages x and x+n. It is also called the life table population[8]. Mathematically it can be written as:

$$_nL_x = nl_{x+1} + {}_na_{xn}d_x$$
$$_\infty L_x = \frac{l_x}{{}_\infty m_x}$$

**Person-years lived after age, x, $T_x$:** This column represents the total number of person-years that would be lived by the cohort $l_x$ after attaining the age x:

$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

**The complete expectation of life, $e_x^0$:** The last column of life table implies the average remaining lifetime in years for a person aged x can expect to live including part years. It is defined mathematically as following formula:

$$e_x^0 = \frac{T_x}{l_x}$$

It is related to the curtate expectation of life by the following formula:

$$e_x^0 = e_x + \frac{1}{2}$$

Here, $e_x$ is the curtate expectation of life which represents the average number of complete years of life lived after attaining age x. Thus:

$$e_x = \frac{\sum_{t=1}^{\infty} l_{x+t}}{l_x}$$

Some sophisticated short-cut methods used for the construction of abridged life table are briefly narrated in the following:

- Reed-Merrell method
- Greville's method
- Keyfitz-Frauenthal method
- Methods of reference to a standard table

**Reed-Merrell method:** Reed and Merrell[4] introduced a shortcut method to us for calculating abridged life tables. This method has been found to give more accurate results. In this method, at first it is required to convert the value of observed mortality rate ($_nm_x$) to the probability of dying ($_nq_x$) and the rest of the functions of life table are calculated on the basis of values $_nq_x$. The value $_nq_x$ of is obtained by the following exponential equation:

$$_nq_x = 1 - e^{-n._nm_x - an^3{}_nm_x^2}$$

where, n is the size of the age interval, $_nm_x$ is the observed death rate and is a constant where a value of a = 0.008 gives acceptable results. After calculation of $_nq_x$, the value of $l_x$ and $_nd_x$ is calculated by following formulas:

$$l_{x+n} = l_x \times {}_nP_x$$

$$_nd_x = l_x - l_{x+n}$$

Further, $T_x$ values are directly obtained from $l_x$ values for 5 or 10 years age interval by using the following equation:

$$T_x = -0.20833l_{x-5} + 2.5l_x + 0.20833l_{x+5} + 5\sum_{i=1}^{\alpha} l_{x+5i}$$

for 5 years age interval:

$$T_x = 4.166667l_x + 0.833l_{x+10} + 10\sum_{i=1}^{\alpha} l_{x+10i}$$

for 10-year age interval. To obtain the value of ${}_nL_x$ under age 10, Reed and Merrell suggested the following equations:

$$L_0 = 0.276l_0 + 0.724l_1$$
$$L_1 = 0.410l_1 + 0.590l_2$$
$${}_4L_1 = 0.034l_0 + 1.184l_1 + 2.782l_5$$
$${}_5L_5 = -0.003l_0 + 2.242l_5 + 2.761l_{10}$$

However, for ages 10 and over ${}_nL_x$ values are obtained from differences between the $T_x$ values and $e^x_0$ is calculated as the ratio of $T_x$ and $l_x$.

**Greville's method:** Greville[9] suggested a method to construct an abridged life table in an easy way. In this method, the following equation has been suggested to convert the value of observed central death rate (${}_nm_x$) to the probability of dying (${}_nq_x$):

$${}_nq_x = \frac{{}_nm_x}{\frac{1}{n} + {}_nm_x\left[\frac{1}{2} + \frac{n}{12}\left({}_nm_x - \ln C\right)\right]}$$

where, InC is assumed to be about 0.095. In Greville's method it is recommended that the value of central death rates both in life table and observed population are same[9]. Further, ${}_nL_x$ column is obtained using below equation:

$${}_nL_x = \frac{{}_nd_x}{{}_nm_x}$$

$${}_\infty L_x = \frac{l_x}{{}_\infty m_x}$$

for the last age interval. The rest of the functions of life table are calculated as usual notation.

**Keyfitz-Frauenthal method:** Keyfitz-Frauenthal[10] method is viewed as the simplification of the Greville[9] method[11]. Keyfitz and Frauenthal[10] suggested the following procedure to convert the value of observed death rate to the probability of dying on which the accuracy of life table depends:

$${}_nq_m = 1 - \exp\left[-n\left({}_nm_x + \frac{\left({}_nP_{x-n} - {}_nP_{x+n}\right)\left({}_nm_{x+n} - {}_nm_{x-n}\right)}{48\,{}_nP_x}\right)\right]$$

where p is observed population. The desired value of ${}_nL_x$ is calculated using the following formula:

$${}_nL_x = \frac{n\left(l_x - l_{x+n}\right)}{\ln l_x - \ln l_{x+n}}\left[1 + \frac{n}{24}\left({}_nm_{x+n} - {}_nm_{x-n}\right)\right]$$

Further, $T_x$ and $e^0_x$ are calculated using as usual way.

**Method of reference to a standard table:** Abridged life tables can be constructed by reference to a standard table. According to this method, mortality is assumed to be atacomparable level in both tables. Now, the value of ${}_ng_x$ is obtained using ${}_nq_x$ and ${}_nm_x$ of the standard life table by following formula:

$${}_ng_x = \frac{n}{{}_nq_x} - \frac{1}{{}_nm_x}$$

where, ${}_ng_x$ signifies the average number of years lived in the interval of age group by those dying in the interval[12]. Then the ${}_nq_x$ values for new life table are obtained using the formula:

$${}_nq_x = \frac{n_{\times n}m_x}{1 + {}_n g_{x\times n}m_x}$$

For the calculation of ${}_nL_x$, first it is required to compute a factor may designate by ${}_nG_x$ using following formula:

$${}_nG_x = \frac{nl_x - {}_nL_x}{{}_nd_x}$$

where, ${}_nG_x$ signifies the distribution of deaths in the interval x to x+n[12]. Now, ${}_nL_x$ values for new life table is obtained as:

$${}_nL_x = nl_x - {}_nG_{x\times n}d_x$$

For open-ended age group, ${}_\infty L_x$ is computed from a special formula. In that case, a factor $r_x$ is required to calculate for standard life table by using the formula:

$$r_x = \frac{{}_\infty m_{x\times\infty}L_x}{l_x}$$

Now ${}_\infty L_x$ for new life table is calculated as follows:

$${}_\infty L_x = \frac{l_x \times r_x}{{}_\infty m_x}$$

Other functions of life table can be calculated as usual way. Furthermore, some life tables recognized as model life tables have been indicated in this study given below:

- United Nations model lie tables
- Coale and Demeny regional model life tables
- Ledermann's system of model life tables
- Brass logit-life table system
- United Nations model life tables for developing countries

## CONCLUSION

A life table is a design that displays various pieces of information about mortality experience of cohort and can be functional in an extensive variety of population studies. Demographers, actuaries and many others use life tables in the analysis of fertility, migration, reproductively; estimation and projection of population and in the analysis of various social and economic characteristics. Life table can also be used for comparison purposes. Various measures of mortality can be compared in a different group of the population or among different countries. That is why life table is considered one of the best known and influential discoveries in demography and public health study.

## REFERENCES


01. Etikan, I., S. Abubakar and R. Alkassim, 2017. A review of life table construction. Biom. Biostatistics Intl. J., 5: 1-3.
02. Ramakumar, R. and Y.S. Gopal, 1986. Technical Demography. Wiley Eastern Limited, New Delhi, India, ISBN:9780852267431, Pages: 219.
03. Halley, E., 1963. An estimate of the degrees of mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslaw with an attempt to ascertain the price of annuities on lives. Philos. Trans., 17: 596-610.
04. Reed, L.J. and M. Merrell, 1939. A short method for constructing an abridged life table. Am. J. Hyg., 30: 33-62.
05. Rowland, D.T., 2003. Demographic Methods and Concepts. Oxford University Press, Oxford, UK., ISBN:9780198752639, Pages: 546.
06. Hinde, A., 1998. Demographic Methods. Arnold Publishers, London, UK., ISBN:9780340718926, Pages: 305.
07. Preston, S.H., P. Heuveline and M. Guillot, 2001. Demography: Measuring and Modeling Population Processes. Blackwell Publishers, Malden, Massachusetts, ISBN:9781557862143, Pages: 291.
08. Pollard, A.H., F. Yusuf and G.N. Pollard, 1974. Demographic techniques. 2nd Edn., Pergamon Press, Australia, ISBN-10: 0080173780, pp: 161.
09. Greville, T.N.E., 1943. Short methods of constructing abridged life tables. Rec. Am. Inst. Actuaries, 32: 29-42.
10. Keyfitz, N. and J. Frauenthal, 1975. An improved life table method. Biom., 31: 889-899.
11. Poston, D.L. and M. Micklin, 2006. Handbook of Population. Springer, Boston, Massachusetts, ISBN-13:978-0387-25702-0, Pages: 924.
12. Sirken, M.G., 1966. Comparison of two methods of constructing abridged life tables by reference to a standard table. Vital Health Stat., 2: 1-11.