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# A Class of Variable Payment Life Insurance Actuarial Model with the Stochastic Interest Rate under the Weibull Distribution of Death

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**Abstract:** Owing to the steady fluctuation of the Wiener process and the sudden disturbance of the negative binomial distribution, we proposed an interest force accumulation function model with a Wiener process and a negative binomial distribution. We built a n-years variable payment life insurance Actuarial Model with this interest force accumulation function. Meanwhile, we obtained the level net premium based on the semi-continuous variable payment life insurance policy with the hypothesis of Weibull mortality. Finally, we simulated model in a numerical example with Matlab and concluded the impact of different parameters upon the level net premium. That further verified the feasibility of the model.

**Key words:** Stochastic interest rate, Wiener process, level net premium, Actuarial Model, Weibull mortality, China

## INTRODUCTION

As the risk caused by the randomness of the interest rate for the insurance company is quite great, the majority of scholars pay more and more attention to the study on the randomness of the interest rate. According to traditional Actuarial Methods, insurance companies spread the risk caused by the randomness of mortality through selling a great many policies. It is impossible to spread the stochastic interest risk with the same method. Consequently, the mortality risk is greater than the stochastic interest risk (Ming, 2005).

In 1976, Boyle considered life insurance and annuity with the stochastic mortality and the stochastic interest rate. In the 1990's, some scholars used Perturbation Method in modeling. Beekman, respectively used the O-U process and Wiener process in modeling in 1990 and 1991. He Wenjiong proposed an interest force accumulation function model with a Gauss process and obtained each order moment of the present value of a class of increasing life insurance with immediate payment (Chunzeng and Xiuyu, 2008). Liu Lingyun modeled the interest force accumulation function with a guass process combined with a poisson process and also obtained each order moment of the present value of a class of increasing life insurance with immediate payment (Wenguang and Yujuan, 2007). Wu Yaohua proposed the formula for the calculation of the level net premium with the fixed interest rate (Niannian and Changqing, 2009). Wang Liyan proposed an actuarial model for family combined insurance, survival conditions with three variables and the formula for the calculation of the level net premium with the interest force accumulation function modeled with Wiener process (Zhiming, 2010). According to the stable volatility of the Wiener process and the sudden disturbance of the negative binomial distribution, we proposed an interest force accumulation function modeled with a Wiener process combined with a negative binomial distribution (Fangxian and Qingwu, 2001). We calculate the level net premium paid immediately when the assured died and the reserve of the semi-continuous variable payment life insurance policy (Zhigang and Tianxiong, 2000). Finally, there make simulation analysis of the level net premium in the model through Matlab Simulation Technology.

## MATERIALS AND METHODS

Assumed the insured aged x, insurance period is n-years variable payment endowment insurance (Zhizhong and Fusheng, 2003). If the insured is still alive at the end of the nth year, the insurance company pays L (L = 0, 1, 2, ...) units of amount insured, else the insurance company pays the corresponding variable payment C(t) ($t \geq 0$) right after the insured dead (Xiaolin, 1999). Assume the interest force accumulated function is R(t) and the discount to present value function is $v_t = e^{-R(t)}$, T is the future lifetime of the n-years insured and it is a random variable then we denote the present value of the random variable by $Z_T$:

$$Z_T = \begin{cases} C(T)e^{-R(T)}, & 0 \leq T < n \\ Le^{-R(n)}, & T \geq n \end{cases} \quad (1)$$

Let interest force accumulated function be:

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$$R(t) = \delta t + \beta|W(t)| + \gamma N(t) \quad 0 \le t < \infty \tag{2}$$

Where:

| | |
|---|---|
| W (t) | = A normal Wiener distribution |
| N (t) | = A negative binominal distribution |
| W (t) and N (t) | = Independent of each other |
| $\delta, \beta, \gamma \ge 0$ | = All real constants |

When, $\beta = 0$, $\gamma = 0$, the interest rate is a constant, the single net premium is:

$$\bar{A}_{x:\overline{n}|} = E(Z(t)) = E_W E_N E_T(Z(t)) \tag{3}$$

The life annuity is:

$$\ddot{a}_{x:\overline{n}|} = E(\ddot{a}_{\overline{[T]+1}|}) = E_W E_N E_T(\sum_{k=0}^{n-1} I_{(T>k)} e^{-R(k)}) \tag{4}$$

The level net premium is:

$$\bar{P}(\bar{A}_{x:\overline{n}|}) = \frac{\bar{A}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \tag{5}$$

The reserve at t is:

$${}_t\bar{V}(\bar{A}_{x:\overline{n}|}) = \bar{A}_{x+t:\overline{n-t}|} - \bar{P}(\bar{A}_{x:\overline{n}|})\ddot{a}_{x+t:\overline{n-t}|} \tag{6}$$

With the hypothesis of Weibull mortality, the mortality force is denoted by $\mu_x$:

$$\mu_x = k_1 x^m \quad (x \ge 0, k_1 > 0, m > 0) \tag{7}$$

The survival function is:

$$s(x) = \exp(-bx^{m+1}) \quad (x \ge 0, b = \frac{k_1}{m+1}) \tag{8}$$

Then:

$${}_t p_x = \frac{s(x+t)}{s(x)} = e^{-b[(x+t)^{m+1} - x^{m+1}]} \tag{9}$$

$$\mu_{x+t} = k_1(x+t)^m \tag{10}$$

**Theorem 1:** Let N (t) be a negative binominal distribution with parameters ($\alpha$, p), $\gamma$ is a constant, q = 1-p then:

$$E\left(e^{-\gamma N(t)}\right) = \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha} \tag{11}$$

**Theorem 2:** Let W (t) be a Wiener process, X~W (t) has the following relation, $\beta$ is a constant:

$$E_W(e^{-\beta|W(t)|}) = e^{\frac{1}{2}\beta^2 t} \tag{12}$$

**Actuarial Model with the hypothesis of Weibull mortality:** The single net premium with the hypothesis of Weibull mortality is:

$$\bar{A}_{x:\overline{n}|} = \int_0^n C(t) \cdot e^{(\frac{1}{2}\beta^2 - \delta)t - b[(x+t)^{m+1} - x^{m+1}]} \cdot \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha} \cdot k_1(x+t)^m dt + Le^{(\frac{1}{2}\beta^2 - \delta)n - b[(x+n)^{m+1} - x^{m+1}]} \cdot \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha} \tag{13}$$

The life annuity with the hypothesis of Weibull mortality is:

$$\begin{aligned} \ddot{a}_{x:\overline{n}|} &= E(\ddot{a}_{\overline{[T]+1}|}) \\ &= E_W E_N E_T(\sum_{k=0}^{n-1} I_{(T>k)} e^{-R(k)}) \\ &= \sum_{k=0}^{n-1} {}_k p_x E_W E_N(e^{-\delta k - \beta|W(k)| - \gamma N(k)}) \\ &= \sum_{k=0}^{n-1} {}_k p_x e^{-\delta k} E_W(e^{-\beta|W(k)|}) E_N(e^{-\gamma N(k)}) \\ &= \sum_{k=0}^{n-1} e^{(\frac{1}{2}\beta^2 - \delta)k - b[(x+k)^{m+1} - x^{m+1}]} \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha} \end{aligned} \tag{14}$$

The level net premium with the hypothesis of Weibull mortality is:

$$\bar{P}(\bar{A}_{x:\overline{n}|}) = \frac{\int_0^n C(t) e^{(\frac{1}{2}\beta^2 - \delta)t - b[(x+t)^{m+1} - x^{m+1}]} \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha} k_1(x+t)^m dt}{\sum_{k=0}^{n-1} e^{(\frac{1}{2}\beta^2 - \delta)k - b[(x+k)^{m+1} - x^{m+1}]} \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha}} + \frac{Le^{(\frac{1}{2}\beta^2 - \delta)n - b[(x+n)^{m+1} - x^{m+1}]} \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha}}{\sum_{k=0}^{n-1} e^{(\frac{1}{2}\beta^2 - \delta)k - b[(x+k)^{m+1} - x^{m+1}]} \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha}}$$

The reserve at t with the hypothesis of Weibull mortality is:

$$_{t}\bar{V}(\bar{A}_{x:\overline{n}|}) = \bar{A}_{x+t:\overline{n-t}|} - P(\bar{A}_{x:\overline{n}|})\ddot{a}_{x+t:\overline{n-t}|}$$

$$= \int_0^{n-t} C(t)\cdot e^{(\frac{1}{2}\beta^2-\delta)t-b[(x+t)^{m+1}-x^{m+1}]} \cdot \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha} \cdot k_1(x+t)^m dt +$$

$$Le^{(\frac{1}{2}\beta^2-\delta)n-b[(x+n)^{m+1}-x^{m+1}]} \cdot \left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha} -$$

$$P(\bar{A}_{x:\overline{n}|})\cdot \sum_{k=0}^{n-t} e^{(\frac{1}{2}\beta^2-\delta)k-b[(x+k)^{m+1}-x^{m+1}]}\left(\frac{p}{1-qe^{-\gamma}}\right)^{\alpha}$$

## RESULTS AND DISCUSSION

Suppose the insured, aged 30 buy's n-years variable payment endowment insurance. If the insured is still alive at the end of the nth year, the insurance company pays L (L = 0, 1, 2, ...) units of amount insured, else the insurance company pays the corresponding variable payment C(t) ($t \geq 0$) right after the insured dead. Furthermore, suppose the ultimate age of the insured is denoted by w and w = 110 (n = 10, 20, 30, 40, 50, 60, 70, 80).

The interest force accumulated function was modeled with normal Wiener process and negative binominal distribution. Assume the variable premium function C(t) = 1+bt that is considers a class of variable life insurance actuarial model. Simulating the model under the hypothesis of Weibull mortality. First giving the initial values of parameters due to the requirements of Matlab, we denoted the parameters as follows:

$$b1 = \alpha = 0.01,\ b2 = \beta = 0.1,\ b3 = \delta = 0.1$$
$$b4 = \gamma = 0.1,\ b5 = p = q = 0.5$$
$$b6 = k_1 = 0.01,\ b7 = m = 0.05$$

Figure 1 compared the values of the level net premium with different values of b in function C(t) = 1+bt, the level net premium increases correspondingly when the b increases. That is to say the level net premium increases correspondingly when the variable payment increases, this is more consistent with the actual life insurance business.

Figure 2-4 compared the values of the level net premium with different values of β and γ, the influence of the parameter β values on the level net premium is not obvious. Influence of the parameter γ values has a significant impact on the level net premium with the increasing of γ values, the level net premium is also increasing, this shows that the emergency has greater impact on the bank interest rates which brings a greater risk for the life insurance companies. Figure 5, the same values of other parameters when δ takes different values

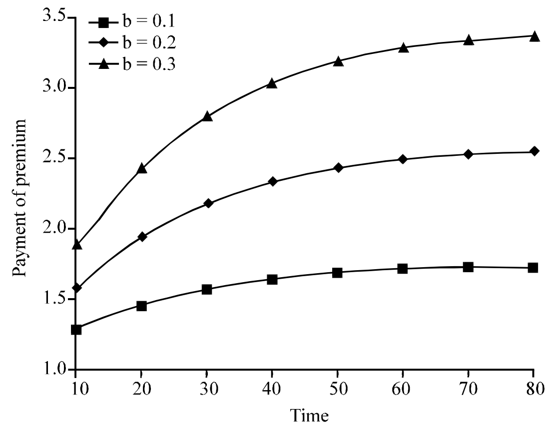


Fig. 1: Curves of life insurance level net premium when value of parameter b is different

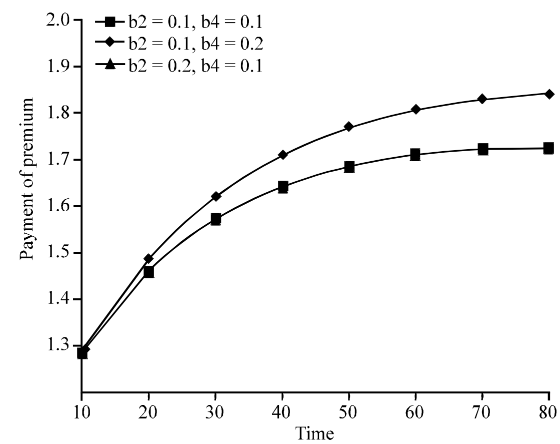


Fig. 2: Curves of life insurance level net premium when values of parameter δ and γ are different

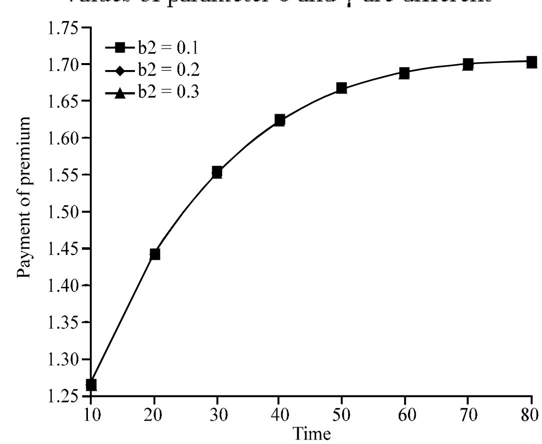


Fig. 3: Curves of life insurance level net premium when value of parameter β is different

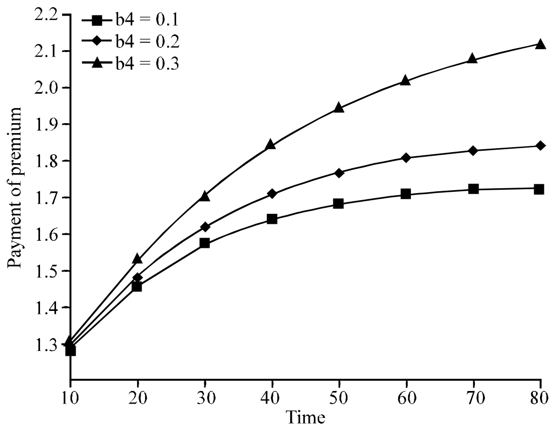


Fig. 4: Curves of life insurance level net premium when value of parameter γ is different

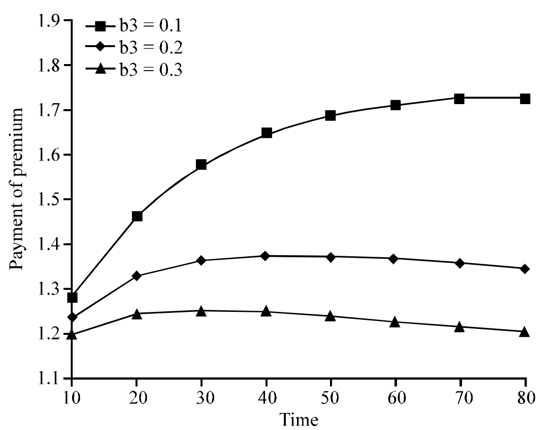


Fig. 5: Curves of life insurance level net premium when value of parameter δ is different

with the increasing of fix interest rate δ, the level net premium is decreasing. That is the impact of interest rate on insurance company can not be ignored.

## CONCLUSION

Let the interest force accumulation function:

$$R(t) = \delta t + \beta|W(t)| + \gamma N(t) \quad 0 \leq t < \infty$$

where, $W(t)$ and $|W(t)|$ represent a Brownian motion and a reflected Brownian motion. $N(t)$ subjects to the negative binomial distribution. $W(t)$ and $N(t)$ are mutually independent. $\delta$, $\beta$, $\gamma \geq 0$ and they are all constants. Researchers considered the impact of different parameters upon the level net premium. Simulating the numerical example with Matlab and conclude the following results:

- The interest rates are affected by a number of factors. The randomness of the interest rate has directly more effect on the level net premium and as a result insurance companies must monitor changes of the interest rate (Niannian and Changqing, 2009)
- In view of non-normal fluctuation of interest caused by external factors, insurance companies should actively take reasonable measures to spread the risk caused by the sudden change of the interest rate
- Researchers apply the model under the Weibull distribution of death to specific practical examples. The results conform to real life insurance practices which we draw through simulation analysis of the effect on the model with parameter changing

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