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Statistical Theory of Certain Distribution Functions in MHD Turbulent Flow Undergoing a First Order Reaction in Presence of Dust Particles

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Abstract: In this study, an attempt is made to study the distribution functions for simultaneous velocity, magnetic, temperature, concentration fields and reaction in MHD turbulent flow undergoing a first order reaction in presence of dust particles. The transport equations for evolution of distribution functions have been derived. The various properties of the distribution function have been discussed. Finally, a comparison of the obtained equation for one-point distribution functions with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

Key words: Distribution functions, MHD turbulence, concentration, first order reaction, dust particles, Bangladesh

INTRODUCTION

The kinetic theory of gases and the statistical theory of fluid mechanics are the two major and distinct areas of investigations in statistical mechanics. In the past, several researchers discussed the distribution functions in the statistical theory of turbulence. Lundgren (1967) derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions which resemble with BBGKY hierarchy of equations of Wu (1966) in the kinetic theory of gasses. Kishore (1978) studied the distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope (1981) derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kishore and Singh (1984a) derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh (1984b) have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. Dixit and Upadhyay (1989) considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the Coriolis force. Kollman and Janica (1982) derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient flux model. But at this stage, one is met with the difficulty that the N-point distribution function depends upon the N+1-point distribution function and thus result is an unclosed system. This so-called closer problem is encountered in turbulence, Kinetic theory and other non-linear system. Sarker and Kishore (1991a) discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Also Sarker and Kishore (1999) studied the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid. Sarker and Islam (2002) studied the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid in a rotating system. Islam and Sarker (2007) also studied distribution functions in the statistical theory of MHD turbulence for velocity and concentration undergoing a first order reaction.

In this study, the researchers have studied the distribution function for simultaneous velocity, magnetic, temperature, concentration fields and reaction in MHD turbulence in presence of dust particles. Finally, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed. The resulting one-point equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

MATERIALS AND METHODS

Basic equations: The equations of motion and continuity for viscous incompressible dusty fluid MHD turbulent flow, the diffusion equations for the temperature and concentration undergoing a first order chemical reaction are shown by:

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\frac{\partial w}{\partial x_\alpha}+\nu\nabla^2 u_\alpha+f\left(u_\alpha-v_\alpha\right) \quad (1)$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta-u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha \quad (2)$$

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$$\frac{\partial\theta}{\partial t}+u_\beta\frac{\partial\theta}{\partial x_\beta}=\gamma\nabla^2\theta \tag{3}$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta}=D\nabla^2C-RC \tag{4}$$

with

$$\frac{\partial u_\alpha}{\partial x_\alpha}=\frac{\partial v_\alpha}{\partial x_\alpha}=\frac{\partial h_\alpha}{\partial x_\alpha}=0 \tag{5}$$

Where:

$u_\alpha(x, t)$	=	α-component of turbulent velocity
$h_\alpha(x, t)$	=	α-component of magnetic field
$\theta(x, t)$	=	Temperature fluctuation
C	=	Concentration of contaminants
v_α	=	Dust particle velocity
R	=	Constant reaction rate
$f = KN/\rho$	=	Dimension of frequency
N	=	Constant number of density of the dust particle
$w(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\lvert\bar{h}\rvert^2$	=	Total pressure
$p(x, t)$	=	Hydrodynamic pressure
ρ	=	Fluid density
ν	=	Kinetic viscosity
$\lambda=(4\pi\mu\sigma)^{-1}$	=	Magnetic diffusivity
$\gamma=k_T/\rho c_p$	=	Thermal diffusivity
c_p	=	Specific heat at constant pressure
k_T	=	Thermal conductivity
σ	=	Electrical conductivity
μ	=	Magnetic permeability
D	=	Diffusive co-efficient for contaminants

The repeated suffices are assumed over the values 1, 2 and 3 and unrepeated suffices may take any of these values. Here u, h and x are vector quantities in the whole process. The total pressure w which occurs in Eq. 1 may be eliminated with the help of the equation obtained by taking the divergence of Eq. 1:

$$\nabla^2w=-\frac{\partial^2}{\partial x_\alpha\partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\left[\frac{\partial u_\alpha}{\partial x_\beta}\frac{\partial u_\beta}{\partial x_\alpha}-\frac{\partial h_\alpha}{\partial x_\beta}\frac{\partial h_\beta}{\partial x_\alpha}\right] \tag{6}$$

In a conducting infinite fluid only the particular solution of the Eq. 6 is related, so that:

$$w=\frac{1}{4\pi}\int\left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha}-\frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{\partial\bar{x}'}{\lvert\bar{x}'-\bar{x}\rvert} \tag{7}$$

Hence, Eq. 1-4 becomes:

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha}-\frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{d\bar{x}'}{\lvert\bar{x}'-\bar{x}\rvert}+\nu\nabla^2u_\alpha+f\left(u_\alpha-v_\alpha\right) \tag{8}$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta-u_\alpha h_\beta\right)=\lambda\nabla^2h_\alpha \tag{9}$$

$$\frac{\partial\theta}{\partial t}+u_\beta\frac{\partial\theta}{\partial x_\beta}=\gamma\nabla^2\theta \tag{10}$$

$$\frac{\partial C}{\partial t}+u_\beta\frac{\partial C}{\partial x_\beta}=D\nabla^2C-RC \tag{11}$$

Formulation of the problem: The researchers consider the turbulence and the concentration fields are homogeneous, the chemical reaction and the local mass transfer have no effect on the velocity field and the reaction rate and the diffusivity are constant. They also consider a large ensemble of identical fluids in which each member is an infinite incompressible reacting and heat conducting fluid in turbulent state. The fluid velocity u, Alfven velocity h, temperature θ and concentration C are randomly distributed functions of position and time and satisfy their field. Different members of ensemble are subjected to different initial conditions and the aim is to find out a way by which we can determine the ensemble averages at the initial time. Certain microscopic properties of conducting fluids such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). The present aim is to construct the distribution functions, study its properties and derive an equation for its evolution of this distribution functions.

Distribution function in MHD turbulence and their properties: It may be considered that the fluid velocity u, Alfven velocity h, temperature θ, concentration C and constant reaction rate R at each point of the flow field in MHD turbulence. Lundgren (1967) and Sarker and Kishore (1991a, b) has studied the flow field on the basis of one variable character only (namely the fluid u) but we can study it for two or more variable characters as well. The corresponding to each point of the flow field, we have four measurable characteristics. We represent the four variables by v, g, ϕ and ψ and denote the pairs of these variables at the points:

$$\bar{x}^{(1)}, \bar{x}^{(2)}, -----, \bar{x}^{(n)}$$

as,

$$\left(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}\right)$$

$$\left(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}\right), ------\left(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}\right)$$

at a fixed instant of time. It is possible that the same pair may be occur more than once; therefore, we simplify the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the distribution as:

$$\left\{\begin{matrix}\left(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}\right);\left(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}\right); \\ ------\left(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}\right)\end{matrix}\right\}$$

Instead of considering discrete points in the flow field if we consider the continuous distribution of the variables $\bar{v},\bar{g},\phi$ and ψ over the entire flow field, statistically behavior of the fluid may be described by the distribution function $F(\bar{v},\bar{g},\phi,\psi)$ which is normalized so that:

$$\int F\left(\bar{v}, \bar{g}, \phi, \psi\right) d\bar{v}, d\bar{g} d\phi d\psi = 1$$

Where the integration ranges over all the possible values of v, g, ϕ and ψ. We shall make use of the same normalization condition for the discrete distributions also. The distribution functions of the above quantities can be defined in terms of Dirac Delta-functions.

The one-point distribution function $F_1^{(1)}(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)})$ defined so that $F_1^{(1)}(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)})\, dv^{(1)}, dg^{(1)}, d\phi^{(1)}, d\psi^{(1)}$ is the probability that the fluid velocity, Alfven velocity, temperature and concentration field at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $dg^{(1)}$ about $g^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$ and $d\phi^{(1)}$ about $\psi^{(1)}$, respectively and is given by:

$$F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right) = \left\langle \begin{matrix}\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right) \\ \delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\end{matrix} \right\rangle \quad (12)$$

where δ is the Dirac delta-function defined as:

$$\int \delta\left(\bar{u} - \bar{v}\right) d\bar{v} = \begin{cases}1 & \text{at the point } \bar{u}=\bar{v} \\ 0 & \text{elsewhere}\end{cases}$$

Two-point distribution function is given by:

$$F_2^{(1,2)} = \left\langle \begin{matrix}\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right) \\ \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)\end{matrix} \right\rangle \quad (13)$$

and three point distribution function is shown by:

$$F_3^{(1,2,3)} = \left\langle \begin{matrix}\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right) \\ \delta\left(C^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right) \\ \times\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right) \\ \delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(C^{(3)} - \psi^{(3)}\right)\end{matrix} \right\rangle \quad (14)$$

Similarly, we can define an infinite numbers of multi-point distribution functions $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on. The distribution functions so constructed have the following properties:

Reduction properties: Integration with respect to pair of variables at one-point, lowers the order of distribution function by one. For example:

$$\iiint\int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} = 1$$

$$\iiint\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

$$\iiint\int F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} = F_2^{(1,2)} \text{ etc.}$$

Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as:

$$\int F_1^{(1)} dv^{(1)} = \left\langle \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\right\rangle$$

$$\int F_1^{(1)} dg^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\right\rangle$$

$$\int F_1^{(1)} d\phi^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\right\rangle$$

And

$$\int F_2^{(1,2)} dv^{(2)} = \left\langle \begin{matrix}\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right) \\ \delta\left(C^{(1)} - \psi^{(1)}\right)\delta\left(h^{(2)} - g^{(2)}\right) \\ \delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)\end{matrix} \right\rangle$$

Separation properties: The pairs of variables at the two points are statistically independent of each other if these points are far apart from each other in the flow field i.e.,

$$\text{Lim}_{|\bar{x}^{(2)}\to\bar{x}^{(1)}|\to\infty} F_2^{(1,2)} = F_1^{(1)}F_1^{(2)}$$

and similarly,

$$\text{Lim}_{|\bar{x}^{(3)}\to\bar{x}^{(2)}|\to\infty} F_3^{(1,2,3)} = F_2^{(1,2)}F_1^{(3)} \text{ etc.}$$

Co-incidence property: When two points coincide in the flow field, the components at these points should be obviously the same that is $F_2^{(1,2)}$ must be zero. Thus:

$$\bar{v}^{(2)} = \bar{v}^{(1)},\ g^{(2)} = g^{(1)},\ \phi^{(2)} = \phi^{(1)}$$

And

$$\psi^{(2)} = \psi^{(1)}$$

but $F_1^{(1,2)}$ must also have the property.

$$\iiint F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

And hence it follows that:

$$\text{Lim}_{|\bar{x}^{(2)}\to\bar{x}^{(1)}|\to\infty} \int F_2^{(1,2)} = F_1^{(1)}\delta\left(v^{(2)} - v^{(1)}\right)\delta\left(g^{(2)} - g^{(1)}\right)$$
$$\delta\left(\phi^{(2)} - \phi^{(1)}\right)\delta\left(\psi^{(2)} - \psi^{(1)}\right)$$

Similarly:

$$\text{Lim}_{|\bar{x}^{(3)}\to\bar{x}^{(2)}|\to\infty} \int F_3^{(1,2,3)} = F_2^{(1,2)}\delta\left(v^{(3)} - v^{(1)}\right)\delta\left(g^{(3)} - g^{(1)}\right)$$
$$\delta\left(\phi^{(3)} - \phi^{(1)}\right)\delta\left(\psi^{(3)} - \psi^{(1)}\right) \text{ etc.}$$

Symmetric conditions:

$$F_n^{(1,2,r,------s,-----n)} = F_n^{(1,2,-----s,---r,---n)}$$

Incompressibility conditions:

$$\iint \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} v_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$$

$$\iint \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} h_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$$

Continuity equation in terms of distribution functions: An infinite number of continuity equations can be derived for the convective MHD turbulent flow and the continuity equations can be easily expressed in terms of distribution functions and are obtained directly by div u = 0. Taking ensemble average of Eq. 5:

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \iiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$
$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \iiint F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$
$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiint \left\langle u_\alpha^{(1)} \right\rangle \left\langle F_1^{(1)} \right\rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (15)$$
$$= \frac{\partial}{\partial x_\alpha^{(1)}} \iiint v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$
$$= \iiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

And similarly;

$$0 = \iiint \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (16)$$

Which are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by:

$$\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right)$$

And if we take the ensemble average, we obtain:

$$o = \left\langle \begin{matrix} \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right) \\ \delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right) \end{matrix} \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle$$
$$= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle \begin{matrix} \delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right) \\ \delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right) \end{matrix} u_\alpha^{(1)} \right\rangle$$
$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \left\langle \begin{matrix} u_\alpha^{(1)}\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right) \\ \delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right) \\ \times\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right) \\ \delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(C^{(2)} - \psi^{(2)}\right) \end{matrix} \right\rangle$$

$$o = \frac{\partial}{\partial x_\alpha^{(1)}} \iiint v_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (17)$$

and similarly;

$$o = \frac{\partial}{\partial x_\alpha^{(1)}} \iiint g_\alpha^{(1)} F_2^{(1,2)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (18)$$

The Nth-order continuity equations are:

$$o = \frac{\partial}{\partial x_\alpha^{(1)}} \iiint v_\alpha^{(1)} F_N^{(1,2,---,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \quad (19)$$

and $$o = \frac{\partial}{\partial x_\alpha^{(1)}} \iiint\int g_\alpha^{(1)} F_N^{(1,2,\ldots\ldots,N)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{20}$$

The continuity equations are symmetric in their arguments i.e.,

$$\begin{aligned} &\frac{\partial}{\partial x_\alpha^{(r)}} \iiint\int \left(v_\alpha^{(r)} F_N^{(1,2,\ldots\ldots,r,N)} dv^{(r)} dg^{(r)} d\phi^{(r)} d\psi^{(r)} \right) \\ &= \frac{\partial}{\partial x_\alpha^{(s)}} \iiint\int v_\alpha^{(s)} F_N^{(1,2,\ldots\ldots r,s,\ldots\ldots N)} dv^{(s)} dg^{(s)} d\phi^{(s)} d\psi^{(s)} \end{aligned} \tag{21}$$

Since, the divergence property is an important property and it is easily verified by the use of the property of distribution function as:

$$\begin{aligned} &\frac{\partial}{\partial x_\alpha^{(1)}} \iiint\int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \\ &= \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \right\rangle = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha (1)} \right\rangle = 0 \end{aligned} \tag{22}$$

Equations for evolution of distribution functions: The Eq. 8-11 will be used to convert these into a set of equations for the variation of the distribution function with time.

This, in fact is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect to time making some suitable operations on the right hand side of the equation so obtained and lastly replacing the time derivative of v, h, θ and C from the Eq. 8-11. Differentiating Eq. 12 and then using Eq. 8-11 we get:

$$\frac{\partial F_1^{(1)}}{\partial t} = \frac{\partial}{\partial t} \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \right\rangle$$

$$= \left\langle \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial}{\partial t} \delta\left(u^{(1)} - v^{(1)}\right) \right\rangle + \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \times \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial}{\partial t} \delta\left(h^{(1)} - g^{(1)}\right) \right\rangle +$$

$$\left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \frac{\partial}{\partial t} \delta\left(C^{(1)} - \psi^{(1)}\right) \right\rangle$$

$$= \left\langle -\delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial u^{(1)}}{\partial t} \frac{\partial}{\partial v^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \right\rangle +$$

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial h^{(1)}}{\partial t} \frac{\partial}{\partial g^{(1)}} \delta\left(h^{(1)} - g^{(1)}\right) \right\rangle +$$

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(C^{(1)} - \psi^{(1)}\right) \frac{\partial \theta^{(1)}}{\partial t} \frac{\partial}{\partial \phi^{(1)}} \delta\left(\theta^{(1)} - \phi^{(1)}\right) \right\rangle +$$

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \frac{\partial C}{\partial t} \frac{\partial}{\partial \psi^{(1)}} \delta\left(C^{(1)} - \psi^{(1)}\right) \right\rangle \tag{23}$$

Using Eq. 8-11 in the Eq. 23, we get:

$$\frac{\partial F_1^{(1)}}{\partial t} = \left\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\left\{-\frac{\partial}{\partial x_\beta}\left(u_\alpha^{(1)}u_\beta^{(1)} - h_\alpha^{(1)}h_\beta^{(1)}\right) - \frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\bar{x}'}{|\bar{x}' - \bar{x}|} + \nu\nabla^2 u_\alpha^{(1)} + f\left(u_\alpha^{(1)} - v_\alpha^{(1)}\right)\right\}\times$$

$$\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\Big\rangle + \left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\left\{-\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)} - u_\alpha^{(1)}h_\beta^{(1)}\right) + \lambda\nabla^2 h_\alpha^{(1)}\right\}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right)\Big\rangle + \left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\times\right.$$

$$\left.\left\{-u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta} + \gamma\nabla^2\theta^{(1)}\right\}\times\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)} - \phi^{(1)}\right)\Big\rangle + \left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\times\delta\left(\theta^{(1)} - \phi^{(1)}\right)\left\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}} + D\nabla^2 C\right\}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)} - \psi^{(1)}\right)\Big\rangle\right.$$

$$= \Big\langle\ \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\Big\rangle + \left\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha}\delta\left(u^{(1)} - v^{(1)}\right)\Big\rangle +\right.$$

$$\left\langle\ \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\times\frac{d\bar{x}'}{|\bar{x}' - \bar{x}|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\Big\rangle + \left\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\nu\nabla^2 u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\Big\rangle +\right.$$

$$\left\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)f\left(u_\alpha^{(1)} - v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\Big\rangle + \Big\langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right)\Big\rangle +\right.$$

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha}\delta\left(h^{(1)} - g^{(1)}\right)\Big\rangle + \left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right)\Big\rangle +\right.$$

$$\left\langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)} - \phi^{(1)}\right)\Big\rangle + \left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)} - \phi^{(1)}\right)\Big\rangle +\right.$$

$$\left\langle\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)u_\beta^{(1)}\frac{\partial C^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)} - \psi^{(1)}\right)\Big\rangle + \left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)D\nabla^2 C^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)} - \psi^{(1)}\right)\Big\rangle +\right.$$

$$\left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)RC^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)} - \psi^{(1)}\right)\Big\rangle\right. \tag{24}$$

Various terms in the Eq. 24 can be simplified as that they may be expressed in terms of one point and two point distribution functions. The first term on the right hand side of the earlier equation is simplified as follows:

$$\left\langle\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\right\rangle = \left\langle u^{(1)}{}_\beta\,\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\right\rangle$$

$$= \left\langle -u^{(1)}{}_\beta\,\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\right\rangle = \left\langle -u^{(1)}{}_\beta\,\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\right\rangle, \left(\text{since}\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}} = 1\right)$$

$$= \left\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)u_\beta^{(1)}\frac{\partial}{\partial x_\alpha^{(1)}}\delta\left(u^{(1)} - v^{(1)}\right)\right\rangle \tag{25}$$

Similarly, 7th, 10th and 12th terms of right hand side of Eq. 24 can be simplified as follows:

$$\left\langle\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right)\right\rangle = \left\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(C^{(1)} - \psi^{(1)}\right)u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(h^{(1)} - g^{(1)}\right)\right\rangle \tag{26}$$

10th term,

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial\theta^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle = \left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle \tag{27}$$

and 12th term,

$$\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle = \left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle \tag{28}$$

Adding Eq. 25-28, we get:

$$\left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle + \left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle +$$

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle + \left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)u_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$= -\frac{\partial}{\partial x_{\beta}^{(1)}}\left\langle u^{(1)}{}_{\beta}\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle\right\rangle = -\frac{\partial}{\partial x_{\beta}^{(1)}}v_{\beta}^{(1)}F_{1}^{(1)}$$

Using the properties of distribution functions:

$$= -v_{\beta}^{(1)}\frac{\partial F_{1}^{(1)}}{\partial x_{\beta}^{(1)}} \tag{29}$$

Similarly 2nd and 8th terms on the right hand side of the Eq. 24 can be simplified as:

$$\left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial h_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle = -g_{\beta}^{(1)}\frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}F_{1}^{(1)} \tag{30}$$

and

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial u_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}}\delta(h^{(1)}-g^{(1)})\right\rangle = -g_{\beta}^{(1)}\frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}\frac{\partial}{\partial x_{\beta}^{(1)}}F_{1}^{(1)} \tag{31}$$

Fourth term can be reduced as:

$$\left\langle -v\nabla^{2}u_{\alpha}^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle = -v\frac{\partial}{\partial v_{\alpha}^{(1)}}\left\langle \nabla^{2}u_{\alpha}^{(1)}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right]\right\rangle$$

$$= -v\frac{\partial}{\partial v_{\alpha}^{(1)}}\frac{\partial^{2}}{\partial x_{\beta}^{(1)}\partial x_{\beta}^{(1)}}\left\langle u^{(1)}{}_{\alpha}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right]\right\rangle = -v\frac{\partial}{\partial v_{\alpha}^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^{2}}{\partial x_{\beta}^{(2)}\partial x_{\beta}^{(2)}}$$

$$\left\langle u_{\alpha}^{(2)}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right]\right\rangle = -v\frac{\partial}{\partial v_{\alpha}^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^{2}}{\partial x_{\beta}^{(2)}\partial x_{\beta}^{(2)}}\left\langle \iiint\int u_{\alpha}^{(2)}\delta\left(u^{(2)}-v^{(2)}\right)\right.$$

$$\left.\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(C^{(2)}-\psi^{(2)}\right)\times\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right.$$

$$= -v\frac{\partial}{\partial v_{\alpha}^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^{2}}{\partial x_{\beta}^{(2)}\partial x_{\beta}^{(2)}}\iiint\int v_{\alpha}^{(2)}F_{2}^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)} \tag{32}$$

9th, 11th and 13th terms of the right hand side of Eq. 24:

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle = \left\langle -\lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle$$

$$= -\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)} \tag{33}$$

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle = \left\langle -\gamma\nabla^2\theta^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$

$$= -\gamma\frac{\partial}{\partial\phi^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}-\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)} \tag{34}$$

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)D\nabla^2 C^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle = \left\langle -D\nabla^2 C^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$

$$= -D\frac{\partial}{\partial\psi^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}\overline{\to}\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)} \tag{35}$$

Now, we reduce the 3rd term of right hand side of Eq. 24:

$$\left\langle \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$= \frac{\partial}{\partial v_\alpha^{(1)}}\frac{1}{4\pi}\iiint\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)} \tag{36}$$

6th term of right hand side of Eq. 24:

$$\left\langle \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle = -\left\langle f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\left[\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right]\right\rangle$$

$$= -f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}\left\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle = -f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}F_1^{(1)} \tag{37}$$

And, the last term of the Eq. 24 reduces to:

$$\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)RC^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(C^{(1)}-\psi^{(1)}\right)\right\rangle = -R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)} \tag{38}$$

Substituting the results 25-38 in Eq. 24, we get the transport equation for one point distribution function $F_1^{(1)}$ (v, g, ϕ, ψ) in MHD turbulence for concentration undergoing a first order reaction in a rotating system in presence of dust particles as:

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\iiint\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]+$$

$$\nu\frac{\partial}{\partial v_\alpha^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{\mathrm{Lim}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{\mathrm{Lim}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}+$$

$$\gamma\frac{\partial}{\partial\phi^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{\mathrm{Lim}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}+D\frac{\partial}{\partial\psi^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{\mathrm{Lim}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}+$$

$$f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(1)}}F_1^{(1)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \tag{39}$$

Similarly, an equation for two-point distribution function $F_2^{(1,2)}$ in MHD dusty fluid turbulence for concentration undergoing a first order reaction can be derived by differentiating Eq. 13 and using Eq. 2, 3, 4, 8 and simplifying in the same manner which is:

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\right)F_2^{(1,2)}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}+g_\beta^{(2)}\left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(2)}}F_2^{(1,3)}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\iiint\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(1)}\right|}\right)\times\right.$$

$$\left.\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]-\frac{\partial}{\partial v_\alpha^{(2)}}\left[\frac{1}{4\pi}\iiint\int\frac{\partial}{\partial x_\alpha^{(2)}}\left(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(2)}\right|}\right)\left(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right)\times\right.$$

$$\left.F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\right]+\nu\left(\frac{\partial}{\partial v_\alpha^{(1)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(1)}}{\mathrm{Lim}}+\frac{\partial}{\partial v_\alpha^{(2)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(2)}}{\mathrm{Lim}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiint\int v_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}+$$

$$\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(1)}}{\mathrm{Lim}}+\frac{\partial}{\partial g_\alpha^{(2)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(2)}}{\mathrm{Lim}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiint\int g_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{3)}d\psi^{(3)}+\gamma\left(\frac{\partial}{\partial\phi^{(1)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(1)}}{\mathrm{Lim}}+\frac{\partial}{\partial\phi^{(2)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(2)}}{\mathrm{Lim}}\right)+$$

$$\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiint\int\phi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}+D\left(\frac{\partial}{\partial\psi^{(1)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(1)}}{\mathrm{Lim}}+\frac{\partial}{\partial\psi^{(2)}}\underset{\bar{x}^{(3)}\to\bar{x}^{(2)}}{\mathrm{Lim}}\right)\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\iiint\int\psi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}+$$

$$f\left(u_\alpha^{(1)}-v_\alpha^{(1)}\right)\frac{\partial}{\partial v_\alpha^{(2)}}F_2^{(1,2)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \tag{40}$$

Continuing this way, we can derive the equations for evolution of $F_3^{(1,2,3)}$, $F_4^{(1,2,3,4)}$ and so on. Logically, it is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed. It seems that certain approximations will be required thus obtained.

RESULTS AND DISCUSSION

If the fluid is clean then $f = 0$, the transport equation for one point distribution function in MHD turbulent flow Eq. 39 becomes:

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiint\int\left(\frac{\partial}{\partial x_\alpha^{(1)}}\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]+$$

$$\nu\frac{\partial}{\partial v_\alpha^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{\mathrm{Lim}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\underset{\bar{x}^{(2)}\to\bar{x}^{(1)}}{\mathrm{Lim}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}+$$

$$\gamma\frac{\partial}{\partial\phi^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}+D\frac{\partial}{\partial\psi^{(1)}}\mathrm{Lim}_{\bar{x}^{(2)}\to\bar{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\iiint\int\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \tag{41}$$

Which was obtained earlier by Sarker and Islam (2002). We can exhibit an analogy of this equation with the first equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is shown as:

$$\frac{\partial F_1^{(1)}}{\partial t}+\frac{1}{m}v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}F_1^{(1)}=n\iint\frac{\partial\psi_{1,2}}{\partial x_\alpha^{(1)}}\frac{\partial F_2^{(1,2)}}{\partial v_\alpha^{(1)}}d\bar{x}^{(2)}d\bar{v}^{(2)} \tag{42}$$

Where $\psi_{1,2}=\psi\left|v_\alpha^{(2)}-v_\alpha^{(1)}\right|$ is the inter molecular potential. If we drop the viscous, magnetic and thermal diffusive, concentration terms and constant reaction terms from the one point evolution Eq. 41, we have:

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\iiint\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(2)}-\bar{x}^{(1)}\right|}\right)\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}\right]=0 \tag{43}$$

The existence of the term:

$$\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}$$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at a single point $x^{(1)}$.

In order to close the system of equations for the distribution functions, some approximations are required. If we consider the collection of ionized particles i.e., in plasma turbulence case, it can be provided closure form easily by decomposing $F_2^{(1,2)}$ as $F_1^{(1)}F_1^{(2)}$. But such type of approximations can be possible if there is no interaction or correlation between two particles. If we decompose $F_2^{(1,2)}$ as:

$$F_2^{(1,2)}=(1+\varepsilon)F_1^{(1)}F_1^{(2)}$$

$$F_3^{(1,2,3)}=(1+\varepsilon)^2F_1^{(1)}F_1^{(2)}F_1^{(3)}$$

Where ϵ is the correlation coefficient between the particles. If there is no correlation between the particles, ϵ will be zero and distribution function can be decomposed in usual way. Here, we are considering such type of approximation only to provide closed from of the equation i.e., to approximate two-point equation as one point equation.

The transport equation for distribution function of velocity, magnetic, temperature, concentration and reaction have been shown here to provide an advantageous basis for modeling the turbulent flows in presence of dust particles. Here, we have made an attempt for the modeling of various terms such as fluctuating pressure, viscosity and diffusivity in order to close the equation for distribution function of velocity, magnetic, temperature, concentration and reaction. It is also possible to construct such type of distribution functions in variable density follows. The advantage of constructing such type hierarchy is to provide simultaneous information about velocity, magnetic temperature, concentration and reaction without knowledge of scale of turbulence.

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