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On Efficiency of Some Combined Ratio Estimators in Stratified Random Sampling

[1]M.O. Gali, [2]A.A. Adewara, [3]M.O. Oladosu, [4]O.M. Olayiwola and [5]L.A. Nafiu
[1]FHA Homes Ltd., P.M.B. 414, Garki, Abuja, FCT, Nigeria
[2]Department of Statistics, University of Ilorin, Ilorin, Kwara State, Nigeria
[3]Code of Conduct Bureau, Osogbo, Osun State, Nigeria
[4]Department of Physical Sciences, Ajayi Crowther University, Oyo, Oyo State, Nigeria
[5]Department of Maths and Computer Science, Federal University of Technology, Minna, Niger State, Nigeria

Abstract: In this study, we compared two newly proposed combined ratio estimators in stratified random sampling to the conventional combined stratified random estimator. These two newly proposed combined ratio estimators are in line with the conventional combined stratified random estimator. Four data sets called populations 1-4 were used to justify this research work. While, population 1 is based on the total monthly income and total monthly expenditure of 86 occupants of Kubwa Federal Housing Authority, Phase IV Estate, Abuja, Nigeria, Population 2 is based on the total monthly income and total monthly expenditure of 95 occupants of Gwarinpa II Estate, Abuja, Nigeria. Also, while population 3 is based on the total monthly income and total monthly savings of 86 occupants of Kubwa Federal Housing Authority, Phase IV Estate, Abuja, Nigeria, Population 4 is based on the total monthly income and total monthly savings of 95 occupants of Gwarinpa II Estate, Abuja, Nigeria and from the variance estimates obtained, it was observed that each of this newly proposed estimator was better and preferred based on the conditions attached to their preference.

Key words: Efficiency, stratified, ratio, random, estimator

INTRODUCTION

The intention of this study is to compare the performances of these proposed combined ratio estimators in stratified random sampling with the traditional combined stratified random estimator described by Okafor (2002), Cochran (1977), Raj and Chandhok (1999) and Hansen *et al.* (1946) etc.

There are lots of improvement on estimators modification (Kalu, 2007; Singh *et al.*, 2008), etc. in sample surveys, this study will also contribute a little to this innovation.

ON THE CONVENTIONAL COMBINED STRATIFIED RANDOM ESTIMATOR

In planning surveys, stratified random sampling has often proved useful in improving the precision of other stratified and unstratified strategies to estimate the finite population mean,

$$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}$$

Singh and Vishwakarma (2006).

When the population mean $\bar{x}$ of the auxiliary variate x is known, Hansen *et al.* (1946) suggested a combined ratio estimator,

$$\bar{y}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)$$

Singh and Vishwakarma (2006). Where,

$$\bar{y}_{st} = \sum_{i=1}^{n_h} W_h \bar{y}_h \text{ and } \bar{x}_{st} = \sum_{i=1}^{n_h} W_h \bar{x}_h$$

are the sample means of the variates in stratified sampling.

$\bar{y}_h$ = The sample mean of variate of interest in stratum h.
$\bar{x}_h$ = The sample mean of auxiliary variate in stratum h.
N_h = The population size for stratum h.
n_h = The sample size for stratum h.

Hence,

$$v(\bar{y}_{RC}) = \sum_{h=1}^{L} W^2 \left(\frac{N_h - n_h}{N_h n_h}\right)(s^2_{hy} - 2\hat{R}s_{hxy} + \hat{R}^2 s^2_{hx}) \quad (1)$$

Corresponding Author: M.O. Gali, FHA Homes Ltd., P.M.B. 414, Garki, Abuja, FCT, Nigeria

where, $W_h = N_h/N$ and

$$\hat{R} = \frac{\bar{y}_{st}}{\bar{x}_{st}} = \frac{\sum_h w_h \bar{y}_h}{\sum_h w_h \bar{x}_h}$$

ON OUR PROPOSED COMBINED RATIO ESTIMATOR IN STRATIFIED RANDOM SAMPLING

In line with Hansen *et al*. (1946) combined ratio estimator, we now proposed our combined ratio estimators in stratified random sampling as:

$$\bar{y}_{RC1} = \bar{y}_{st}\left(\frac{\bar{X}}{\theta_1 \bar{x}_{st}}\right)$$

and

$$\bar{y}_{RC2} = \bar{y}_{st}\left(\frac{\bar{X}}{\theta_2 \bar{x}_{st}}\right)$$

where,

$$\theta_1 = \frac{n_h}{n_h - 1}$$

and

$$\theta_2 = \frac{n_h}{n_h + 1}$$

Hence,

$$v(\bar{y}_{RC1}) = \sum_{h=1}^{L} W_h^2 \left(\frac{N_h - n_h}{N_h n_h}\right)(s^2_{hy} - 2\theta_1 \hat{R} s_{hxy} + \theta_1^2 \hat{R}^2 s^2_{hx}) \quad (2)$$

and

$$v(\bar{y}_{RC2}) = \sum_{h=1}^{L} W_h^2 \left(\frac{N_h - n_h}{N_h n_h}\right)(s^2_{hy} - 2\theta_2 \hat{R} s_{hxy} + \theta_2{}^2 \hat{R}^2 s^2_{hx}) \quad (3)$$

Data used

Population 1: The data used for population 1 is based on the total monthly income and total monthly expenditure distributions of 86 occupants of Kubwa Federal Housing Authority, Phase IV Estate, Abuja, Nigeria, which were stratified into two on the basis of their marital status (single and married) of the head of household. Then samples of size 50 are drawn altogether from these two stratum using proportional allocation method,

$$n_h = nW_h = n\left(\frac{N_h}{N}\right)$$

Summary of estimates

Stratum I (single occupants)	Stratum II (married occupants)
$N_1 = 24$	$N_2 = 62$
$n_1 = 14$	$n_2 = 36$
$\bar{x} = 38542.21$	$\bar{x} = 66406.94417$
$\bar{y} = 45649.36$	$\bar{y} = 81712.49972$
$s^2_y = 4.2\times10^8$	$s^2_y = 3.1\times10^9$
$s^2_x = 2.9\times10^8$	$s^2_x = 1.6\times10^9$
$s_{xy} = 3.4\times10^8$	$s_{xy} = 2.2\times10^9$

Population 2: The data used for population 2 is based on the total monthly income and total monthly expenditure distributions of 95 occupants of Gwarinpa II Estate, Abuja, Nigeria, which were stratified into two on the basis of their marital status (single and married) of the head of household. Then samples of size 50 are drawn altogether from these 2 stratum using proportional allocation method,

$$\left(n_h = nW_h = n\left(\frac{N_h}{N}\right)\right)$$

Summary of estimates

Stratum I (single occupants)	Stratum II (married occupants)
$N_1 = 19$	$N_2 = 76$
$n_1 = 10$	$n_2 = 40$
$\bar{x} = 30000$	$\bar{x} = 48258.75$
$\bar{y} = 58165.82$	$\bar{y} = 77494.61275$
$s^2_y = 4.8\times10^9$	$s^2_y = 4.9\times10^9$
$s^2_x = 7.8\times10^8$	$s^2_x = 9.9\times10^9$
$s_{xy} = 1.7\times10^9$	$s_{xy} = 1.7\times10^9$

Population 3: The data used for population 3 is based on the total monthly income and total monthly savings distributions of 86 occupants of Kubwa Federal Housing Authority, Phase IV Estate, Abuja, Nigeria, which were stratified into two on the basis of their marital status (single and married) of the head of household. Then samples of size 50 are drawn altogether from these two stratum using proportional allocation method,

$$n_h = nW_h = n\left(\frac{N_h}{N}\right)$$

Summary of estimates

Stratum I (single occupants)	Stratum II (married occupants)
$N_1 = 24$	$N_2 = 62$
$n_1 = 14$	$n_2 = 36$
$\bar{x} = 7107.143$	$\bar{x} = 15305.5556$
$\bar{y} = 45649.36$	$\bar{y} = 81712.49972$
$s^2_y = 4.2\times10^8$	$s^2_y = 3.1\times10^9$
$s^2_x = 2.4\times10^7$	$s^2_x = 3.4\times10^8$
$s_{xy} = 7.9\times10^7$	$s_{xy} = 1.9\times10^8$

Population 4: The data used for population 4 is based on the total monthly income and total monthly savings distributions of 95 occupants of Gwarinpa II Estate, Abuja, Nigeria, which were stratified into two on the basis of their marital status (single and married) of the head of household. Then samples of size 50 are drawn altogether from these two stratum using proportional allocation method,

$$\left(n_h = nW_h = n\left(\frac{N_h}{N}\right)\right)$$

Summary of estimatess

Stratum I (single occupants)	Stratum II (married occupants)
$N_1 = 19$	$N_2 = 76$
$n_1 = 10$	$n_2 = 40$
$\bar{x} = 28639.51$	$\bar{x} = 33015.84775$
$\bar{y} = 58165.82$	$\bar{y} = 77494.61275$
$s^2_y = 4.8\times10^9$	$s^2_y = 4.9\times10^9$
$s^2_x = 2.04\times10^9$	$s^2_x = 2.6\times10^8$
$s_{xy} = 3.00\times10^9$	$s_{xy} = 3.2\times10^8$

RESULTS AND DISCUSSION

From the estimates obtained on the data sets above, we have this summary:

Case A: when $(\bar{y}_{RC2})$ is preferred

Estimator	Population 1	Population 2
$v(\bar{y}_{RC})$	731963.1	1.75×10^7
$v(\bar{y}_{RC1})$	839440.6	1.79×10^7
$v(\bar{y}_{RC2})$	652504.6	1.73×10^7
$\hat{\beta}_1$	1.1724	2.1795
$\hat{\beta}_2$	1.3750	1.7172
$\hat{\rho}_1$	0.9742	0.8786
$\hat{\rho}_2$	0.9878	0.7719
$\hat{R}$	1.2220	1.6506

Here,

- For population 1,

$$\bar{y}_{RC2}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_2\bar{x}_{st}})]$$

is preferred to both

$$\bar{y}_{RC}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\bar{x}_{st}})]$$

and

$$\bar{y}_{RC1}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_1\bar{x}_{st}})]$$

since,

$$\hat{R} > \hat{\rho}_1 > \hat{\rho}_2 > \hat{\beta}_1 < \hat{\beta}_2$$

- For population 2,

$$\bar{y}_{RC2}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_2\bar{x}_{st}})]$$

is preferred to both

$$\bar{y}_{RC}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\bar{x}_{st}})]$$

and

$$\bar{y}_{RC1}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_1\bar{x}_{st}})]$$

since,

$$\hat{R} > \hat{\rho}_2 > \hat{\rho}_1 < \hat{\beta}_2 < \hat{\beta}_1$$

We observed here that $\hat{R}$ must be less than one or both of $\hat{\beta}_h$ but greater than both $\hat{\rho}_h$ for $\bar{y}_{RC2}$ to be preferred.

Case B: when $(\bar{y}_{RC1})$ is preferred

Estimator	Population 3	Population 4
$v(\bar{y}_{RC})$	2.11×10^7	3.28×10^7
$v(\bar{y}_{RC1})$	1.9×10^7	2.93×10^7
$v(\bar{y}_{RC2})$	2.3×10^7	3.72×10^7
$\hat{\beta}_1$	3.2917	1.4706
$\hat{\beta}_2$	2.6765	1.2308
$\hat{\rho}_1$	0.7869	0.9587
$\hat{\rho}_2$	0.8864	0.8965
$\hat{R}$	5.5040	2.2908

Also here,

- For population 3,

$$\bar{y}_{RC1}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_1\bar{x}_{st}})]$$

is preferred to both

$$\bar{y}_{RC}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\bar{x}_{st}})]$$

and

$$\bar{y}_{RC2}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_2\bar{x}_{st}})]$$

since,

$$\hat{R} > \hat{\rho}_1 > \hat{\rho}_2 > \hat{\beta}_1 > \hat{\beta}_2$$

- For population 4,

$$\bar{y}_{RC1}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_1\bar{x}_{st}})]$$

is preferred to both

$$\bar{y}_{RC}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\bar{x}_{st}})]$$

and

$$\bar{y}_{RC2}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_2\bar{x}_{st}})]$$

since,

$$\hat{R} > \hat{\rho}_2 > \hat{\rho}_1 > \hat{\beta}_2 > \hat{\beta}_1$$

We observed here that $\hat{R}$ must be greater than both $\hat{\beta}_h$ and $\hat{\rho}_h$ for $\bar{y}_{RC1}$ to be preferred.

CONCLUSION

In conclusion, from the estimates obtained above using populations 1-4, one can say that:

- For

$$\bar{y}_{RC2}\,[\text{i.e.}\,\bar{y}_{st}(\frac{\bar{X}}{\theta_2\bar{x}_{st}})]$$

to be preferred, $\hat{R}$ must be greater than both $\hat{\rho}_h$ but less than one or both of $\hat{\beta}_h$

- For

$$\bar{y}_{RC1}[\text{i.e. } \bar{y}_{st}(\frac{\bar{X}}{\theta_1\bar{x}_{st}})]$$

to be preferred, $\hat{R}$ must be greater than both $\hat{\beta}_h$ and $\hat{\rho}_h$.

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