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Finite-Time Linear Filtering and Control of A Communication Satellite Driven by White Noise Sequence

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Abstract: This study models the dynamics of a communication satellite driven by a white noise sequence and proposes a finite-time filtering solution in which state estimation was addressed. The dynamics of the satellite assumed linear and reducible to an equivalent interconnection of subsystems enhanced computation of the filter gain matrix from the recursive relation. The significant practical benefits of using the stationary form of the Kalman filter in the computation were established.

Key words: Communication satellite, finite-time filtering solution, error covariance matrix, Kalman filter, unbiased estimates

INTRODUCTION

For many space applications involving data communication, a large number of satellites have been launched into the earth orbit. For effectiveness in their operation, the attitude must be observable and the dynamics stabilizable (Flanagan, 1969; Sohn, 1959). By the former is meant that it should be possible to orient the satellites in a preferred or specified attitude and by the latter is meant that if the satellite is uncontrolled, it should be possible to obtain a time history of its attitude by suitable instrumentation and telemetry.

Since measurement of the physical parameters made by satellite-borne instruments is strongly dependent on the orientation of the instrument, it is necessary to either control the attitude precisely or provide information regarding the attitude to enhance realistic interpretation of the results.

In general, the satellites and the environment in which they operate are far from isotropic (Flanagan, 1969) consequently, in the case of scientific satellites. In the case of communications satellites of active relay type, the effective power transmitted from the satellites depends on the satellite transmitter power, transmission efficiency and antenna gain. In such systems, many of the design techniques based on state variable approach assume that values are available for all the states for a given control vector.

However, in most practical situations, it is not possible to measure all the states and furthermore, the measurements that are available often contain significant amounts of random noise and/or systematic errors. A near-earth satellite, for example orbiting in the attitude range of 150-450 km (Flanagan, 1969) encounters small but non-negligible aerodynamic forces due essentially to gravity waves set up in a stably stratified atmosphere (Obinabo, 1978).

The influence of major environmental forces on the attitude response of gravity gradient satellites using essentially both analytical and numerical techniques presents a problem of major interest to the process engineer.

Several aspects concerning formulation of the problem which rely on current measurements of the process variables and the ease with which detailed characterization of drag resistance effects of gravity waves in stably stratified atmosphere especially as it affects free and forced rotation of astronomical satellites through it have been addressed in the existing literature.

In absence of correct measurement, any change in the control vector can hardly influence the dynamics of interconnected subsystems thereby making it difficult to establish directly the true value of the process data since all measurements are unavoidably subject to noise (Oyediran *et al*., 2010). In these situations, on-line estimation techniques produce estimates of the true process values from computation of a suitable process model.

To this end, the Kalman filter has received considerable attention in the existing literature (Kalman, 1960; Hsiao and Wang, 2000; Oyediran *et al*., 2010) and has been applied successfully in the aerospace industry

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(Athans, 1971). Furthermore, there has been a number of theoretical investigations and simulation studies of its use in process control applications. Given a signal model that consists of a linear dynamic system driven by stochastic white noise processes, the Kalman filter (Kalman, 1960; Foster and Saffman, 1970) exploits a state space model for optimal filtration of noisy measurements. An interesting description of the history of Kalman filtering theory (Grimble, 1981; Ljung, 1979) and the applications include navigation and guidance, global positioning systems, target tracking, communications and signal processing and electrical machines. A precursor to the Kalman filter was the Weiner filter which was derived independently by Weiner and Kolmogorov (Oyediran *et al*., 2010) and which gives a method of optimally attenuating noise in process measurements. However, the Weiner filter is limited to time-invariant problems involving stationary noise sequence. The filter algorithm is not computationally straightforward as the Kalman filter.

This study presents solution to the filtering problem in a communication satellite process due essentially to drag resistance effects (Obinabo, 1978) associated with gravity waves in the satellite's atmosphere. The overall study shows that a filter can be postulated to define a covariance matrix to yield unbiased measurements of the process variables.

MATERIALS AND METHODS

Generally, estimation of process variables contaminated with noise are formulated on the basis of maximum likelihood (Athans, 1971; Kalman, 1960) using statistical information in terms of joint probability distribution functions.When the additive, zero-mean white gaussian measurement noise is defined in terms of mean values and variances which will be appropriate for many practical applications, the least-squares solution is formulated as a deterministic problem (Bacher *et al*., 1981; Ho, 1962; Sage, 1967) with appropriate weighting that leads to the maximum likelihood estimate. The modeling is enhanced using the discrete-time stochastic sequence as follows:

$$\begin{aligned} \underline{x}(k+1) &= \underline{\underline{\phi}}\underline{x}(k)+\underline{\underline{\Lambda}}\underline{u}(k)+\underline{\underline{\theta}}\underline{d}(k)+\underline{\underline{\Gamma}}\underline{w}(k) \\ y(k) &= \underline{\underline{C}}\underline{x}(k)+\underline{v}(k) \end{aligned} \quad (1)$$

Which is the linear stochastic time-invariant state space model where x is the n×1 state vector, $\underline{u}$ is the m×1 control vector, $\underline{d}$ is the p×1 disturbance vector, $\underline{y}$ is the l×1 output vector and $\underline{\underline{\phi}}, \underline{\underline{\Lambda}}, \underline{\underline{\theta}}, \underline{\underline{\Gamma}}$ and $\underline{\underline{C}}$ are constant matrices. Vectors are here denoted by single underlines and matrices by double underlines. It is assumed that the measurement noise vector v and the s-dimensional process noise vector $\underline{w}$ are zero mean, uncorrelated and have covariance matrices of $\underline{\underline{Q}}$ and R, respectively.

The Kalman filter generates a state estimate $\hat{\underline{x}}$ which is the solution to the optimal estimation problem in Eq. 1. Now, with the Kalman filter, the known or assumed noise covariance matrices, $\underline{\underline{Q}}$ and $\underline{\underline{R}}$ and the initial state covariance matrix $\underline{\underline{P}}(0)$, generate an estimate which is linear with respect to the measured outputs $\underline{y}$ and which provides a minimal variance estimate.

Suppose the control law of interest is a proportional state feedback controller with a constant gain matrix K as follows:

$$\underline{u}(k) = \underline{\underline{K}}\underline{x}(k) \quad (2)$$

It can be shown that the multivariable controller in Eq. 2 is easily extended to include integral, feed forward and set point control modes. In order to implement the control law in Eq. 2, measurements of all n state variables are required but in many practical control problems, only measurements of the l input variables are available where l<n .

One possible strategy is to use a state estimation technique to generate an estimate of the state vector, $\hat{\underline{x}}$ and then substitute the estimate in the control law of Eq. 2. This approach results in the controller:

$$\underline{u}(k) = \underline{\underline{K}}\hat{\underline{x}}(k)$$

The filtration procedure generates an optimal estimate defined as follows:

$$\hat{\underline{x}}(k) = \overline{\underline{x}}(k)+\underline{\underline{K}}(k)\left[\underline{y}(k)-\underline{\underline{C}}\overline{\underline{x}}(k)\right] \quad (3)$$

Where $\overline{\underline{x}}(k)$ is calculated from the deterministic model using the state estimation plus the measured process inputs from the previous time interval, i.e.,

$$\overline{\underline{x}}(k) = \underline{\underline{\phi}}\hat{\underline{x}}(k-1)+\underline{\underline{\Lambda}}\underline{u}(k-1)+\underline{\underline{\theta}}\underline{d}(k-1) \quad (4)$$

RESULTS AND DISCUSSION

The filter gain matrix, $\underline{\underline{K}}(k)$ is generated from a recursive relation. If it is assumed that the observation period is long compared to the dominant time constants of the process then the gain matrix in Eq. 3 becomes a time-invariant matrix, $\underline{\underline{K}}$. There is a significant practical advantage in using the stationary form of the Kalman filter since only a single matrix, $\underline{\underline{K}}$, rather than a sequence of time-varying matrices $\underline{\underline{K}}(k)$, need be stored. To this end, we consider the scalar system (Fig. 1) which reduces Eq. 1 to the following:

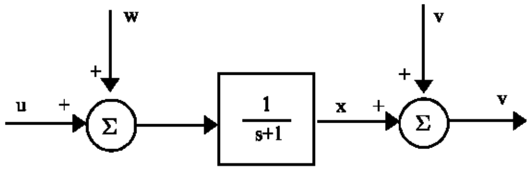

Fig. 1: Linear and reducible to an eduivalent interconnection of sub-systems

$$x(k+1) = ax(k) + cw(k)$$
$$z(k+1) = hx(k+1) + v(k+1) \quad (5)$$

Where z (k+1) are the measured outputs and w (k) and v (k) are zero-mean stationary white noise sequences:

$$E\left[w^2(k)\right] = q$$
$$E\left[v^2(k)\right] = r$$
$$E\left[v(k)w(j)\right] = 0 \text{ for all k, j}$$

The notation $\hat{x}(k/j)$ will be used to mean an estimate of the state x (k) based on all information up to the time j,

- k>j-prediction
- k = j-filtering
- k<j-smoothing

In this study, we consider filtering problem by determining an estimate $\hat{x}(k+1)/(k+1)$ of the state $x(k+1)$, of the given model that is a linear combination of the previous state and the measurement. We assume the estimate $\hat{x}(k/k)$ is available. From estimation theory, we know that the best estimate of x (k+1) gives measurement up to time k is the minimum variance or the conditional mean E [x (k+1)/k]. From Eq. 1:

$$\hat{x}((k\_1)/k) = E\left[x(k+1)k\right]$$
$$= aE\left[x(k)/k\right] + cE\left[w(k)/k\right] \quad (6)$$

By definition:

$$E\left[x(k)/k\right] = \hat{x}(k/k)$$

because w (k) is a zero mean white noise process:

$$E\left[w(k)/k\right] = 0$$

Hence Eq. 6 becomes:

$$\hat{x}((k+1)/k) = a\hat{x}(k/k) \quad (7)$$

$$\hat{z}(k+1) = h\hat{x}((k+1)/k) \quad (8)$$

With the measurement $\hat{z}(k+1)$ now carried out, the error in the prediction of Eq. 8 was determined as follows:

$$\tilde{z}(k+1) = z(k+1) - \hat{z}(k+1) \quad (9)$$

The prediction of Eq. 7 can be improved by using the information that we have now got available at time (k+1) and adding a proportion $\tilde{z}$ of to $\hat{x}((k+1)/k)$;

$$\hat{x}((k+1)/(k+1)) = \hat{x}((k+1)/k) + k\tilde{z}(k+1) \quad (10)$$

This is a predictor-corrector type of equations which tries to drive $\tilde{z}$ to zero. Substituting Eq. 8 and 9 into 10 yields:

$$\hat{x}((k+1)/(k+1)) = (1+kh)\hat{x}((k+1)/k) + kz(k+1) \quad (11)$$

$$\hat{x}((k+1)/(k+1)) = (1+kh)a\hat{x}(k/k) + kz(k+1) \quad (12)$$

Equation 12 is the defining equation of the state estimator which estimates x (k+1) from only the current measurements z (k+1), z (k) etc., and the previous estimate $\hat{x}(k/k)$. . The only problem remaining is to select k. this was done by defining the estimation error as follows:

$$\tilde{x}(k+1) = x(k+1) - \hat{x}((k+1)/(k+1)) \quad (13)$$

The filter gain k was chosen such that:

$$E\left[\left(x(k+1) - \hat{x}(k+1/k+1)\right)^2\right] = \text{Minimum} \quad (14)$$

We now define the variance as:

$$p(k+1) = E\left[\tilde{x}^2(k+1)\right] \quad (15)$$

Squaring Eq. 13 and taking expectation, noting that the cross product terms E [x (k) w (k)], E [w (k) v (k+1)] etc., are all zero:

$$p(k+1) = (1-kh)^2 a^2 p(k) + (1-kh)^2 c^2 q + k^2 r$$
$$= (1-kh)^2 p^*(k+1) + k^2 r \quad (16)$$

Where:

$$p^*(k+1) = a^2 p(k) + c^2 q \quad (17)$$

Differentiating Eq. 16 with respect to k and setting it to zero:

$$\frac{dp(k+1)}{dk} = z(-h)(1-kh)p^*(k+1) + 2kr = 0$$

giving:

$$k = \frac{p^*(k+1)h}{h^2 p^*(k+1) + r} \quad (18)$$

Substituting Eq. 18 into 16:

$$p(k+1) = \frac{rp^*(k+1)}{h^2 p^*(k+1) + r} = (1 - kh)p^*(k+1) \quad (19)$$

Which gives the Kalman-Bucy filter as follows:

$$p^*(k) = a^2 p(k-1) + c^2 q \quad (20)$$

$$k(k) = \frac{hp^*(k)}{h^2 p^*(k) + r} \quad (21)$$

$$\hat{x}(k/k) = a\hat{x}((k-1)/(k-1)) + k(k)\left[z(k) - ah\hat{x}((k-1)/(k-1))\right] \quad (22)$$

$$p(k) = (1 - k(k)h)p^*(k) \quad (23)$$

Example: Here, we consider the dynamics of the satellite (Fig. 1) assuming they are linear and reducible to an equivalent interconnection of subsystems. The state and output equations were determined as follows:

$$\dot{x} = -x + u + w$$
$$y = x + v$$

From the Riccati equation with A = A^T = −1, c = G = 1 and Q = 2R , the co-state variables:

$$\dot{p} = -2p - \frac{p^2}{R} + Q$$

were obtained. For steady state filter $\dot{p} = 0$ giving:

$$-2p - \frac{p^2}{R} + Q = 0$$

$$p = \frac{2 \pm \sqrt{4 + \frac{4Q}{R}}}{-\frac{2}{R}} = -\frac{R}{2}\left(2 \pm \sqrt{12}\right) = \frac{R}{2}\left(\sqrt{12} - 2\right)$$

Therefore,

$$k = pR^{-1} = \frac{\sqrt{12} - 2}{2} = \sqrt{3} - 1$$

Which is the steady state gain of the filter associated with the problem.

CONCLUSION

Researcher concluded that prediction of discrete-time stochastic processes can be uniquely sought by recourse to the assumption that some or all of the parameters may be unknown even though the structure of the differential equation characterizing the system as well as the initial and boundary conditions may be available. The output of the data communication measuring instrument is an approximation of the true value. The computation of the optimal estimates relied on convergence of the iteration employed which was accomplished through sequential filtration of the estimate. There are no useful convergence results available in the existing literature for the plant situation described in this study which do not operate on the basis of data filtration. The plant itself is represented by a stable, linear, parameter-dependent state space model the solution of which is a Kalman filter. The most interesting results of this study have been concerned with the solution of the linear estimation problem and this was considered in detail as it applies to communication satellites driven by a stochastic white noise sequence. Computation of the estimates relied on convergence of the iteration employed which was accomplished through sequential filtration.

REFERENCES

Athans, M., 1971. The role and use of the stochastic linear quadratic Gaussian problem in control system design. IEEE Trans. Automatic Control, 16: 529-552.

Bacher, R.A., W. Gray and D.J. Murray-Smith, 1981. Time domian syaytem identification applied to non-invasive estimation cardiopulmonary quantities. IEE Proc. Control Theory Appl., 128: 56-64.

Flanagan, R.C., 1969. Effects of environmental forces on the attitude dynamics of gravity oriented satellites. Ph.D. Thesis, University of British Columbia.

Foster, M.R. and P.G. Saffman, 1970. The drag of a body moving transversely in a confined stratified fluid. J. Fluid Mech., 43: 407-418.

Grimble, M.J., 1981. Adaptive kalman filter for control systems with unknown disturbance. IEE Proc., 128: 263-267.

Ho, Y.C., 1962. On the stochastic approximation method and optimal filtration theory. J. Math. Anal. Appl., 6: 152-154.

Hsiao, C.H. and W.J. Wang, 2000. State analysis and parameter estimation of bilinear systems via haar wavelets. IEEE Trans. Circuits Syst. Fundam. Theory Appl., 47: 246-250.

Kalman, R.E., 1960. A new approach to linear filtering and prediction problems. Trans. ASME. J. Basic Eng., 82: 35-45.

Ljung, I., 1979. Asymptotic behavior of the extended kalman filter as a parameter estimation for linear systems. IEEE Trans. Automatic Control, 24: 36-50.

Obinabo, E.C., 1978. Drag resistance caused by gravity waves in a stably stratified fluid. B.Sc. Thesis, University of Portsmouth, England, UK.

Oyediran, E.O., J.J. Biebuma and E.C. Obinabo, 2010. A deterministic approach to process noise attenuation in a communication satellite driven by white noise sequence. J. Eng. Applied Sci., 5: 72-77.

Sage, A.P., 1967. Least squares curve fitting and discrete optimum fitting. IEEE Trans. Educ., 10: 29-38.

Sohn, R.L., 1959. Attitude stabilization by means of solar radiation pressure. ARS J., 29: 371-373.