Journal of Mobile Communication 1 (2): 54-58, 2007

LMMSE Based Kalman Filter Algorithm Used on Channel Estimation in OFDM Systems

C. Venkatesh and S. Sathish
Department of ECE, Adhiyamaan College of Engineering Hosur,
Tamil Nadu, 635 109, India

Abstract: OFDM Scheme widely used in high data rate communication systems over widest channels. It requires the tracking of the fading radio channel. In this study addresses channel estimation algorithm based on time domain channel statistics. Using a general model for a slowly fading channel, we present the LMMSE based channel estimators. This is mainly used for complexity and improves the performance of the channel.

Key words: OFDM, LMMSE, channel estimation, BPSK

INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) (Biglieri and Mclane, 1991) the first and foremost multicarrier communication system has been adopted as the major technique for the current and future broadband communication system like wireless LAN, wireless MAN, DAB (Brajal and Chouly, 1994) and DVB. The principle of OFDM is to split a high date rate stream into lower date rate stream that are transmitted. OFDM signaling in fading channel environments has gained a broad interest. For instance, its digital TV broadcasting in OFDM system is currently being investigated (Cimini, 1985). The use of Binary Phase Shift Keying (BPSK) in OFDM systems avoids the tracking of a time varying channel. However, this will limit the number of bits per symbol and results in a 3 dB loss in Signal-to-Noise Ratio (SNR) (Cost 207 Management Committe, 1989). If the receiver contains a channel estimator, multiamplitude signaling schemes can be used. A decision directed channel tracking method, which allows the use of multi amplitude schemes in a sloe Rayleigh-Fading environment is analyzed in (Du *et al.*, 1993).

In the design of wireless OFDM systems, the channel is usually assumed to have a finite-length impulse response. A cyclic extension, longer than this impulse response, is put between consecutive blocks in order to avoid interblock interference and preserve Orthogonality of the tones (Helard and Floch, 1991). Generally, the OFDM system is designed so that the cyclic extension is a small percentage of the total symbol length. In this study mainly focused on LMMSE based Kalman filter algorithm proposed for channel estimation in OFDM system, that use of this property of the channel impulse response.

SYSTEM MODEL

The OFDM system model shown in Fig 1. The random data generator generates the data system. This input serial data stream is formatted into the word size required for transmission and then shifted into a parallel format. The data to be transmitted on each carrier is then mapped into a Phase Shift Keying (PSK) format and the data on each symbol is mapped to a phase angle based on the modulation method. The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading.

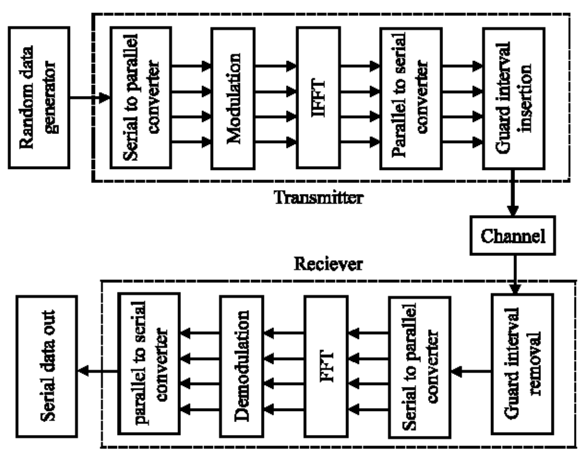

Fig. 1: OFDM system model

Corresponding Author: C. Venkatesh, Department of ECE, Adhiyamaan College of Engineering Hosur, Tamil Nadu, 635 109, India

After the required spectrum is worked out, an Inverse Fourier Transform is used to find the corresponding time domain waveform. The IFFT mathematical analysis represented as

$$X(n = \sum_{n=o}^{N-1} x(k)\sin(2\pi kn/N) - j\sum_{n=o}^{N-1} x(k)\cos(2\pi kn/N) \quad (1)$$

Here, x(n) are the coefficient of the sine and cosine of the frequency 2πkn/N, where k is the index of the frequencies over the N frequencies and n is the time index. x(k) is the value of this spectrum for the k^{th} frequency and x(n) is the signal at the time n. The inverse FFT takes this spectrum and converts the whole thing back to time domain signal by again successively multiplying it by a range of sinusoids. The guard period is then added to the start of each symbol. The mathematical analysis of transmitted OFDM symbol at t = T is defined as

$$S(t) = \sum_{n=0}^{N-1}\{Dn\exp(j2n\,II\,fs\,T/N)\}; t_S \le t \le t_S + T \quad (2)$$

Where Dn represents the complex data symbols, N is number of carriers and T is symbol duration.

The level of multipath robustness can be further increased by the addition of a guard period between transmitted symbols. The guard period allows time for multipath signals form the previous symbol to the die away before the information from the copy of previous symbol to die away before the information from the current symbol is gathered. The guard period used, can be made up of two sections,. Half of the guard period time is a zero amplitude transmission called Zero padding and the other half of the guard period is a cyclic extension of the symbol to be transmitted. After the guard has been added, the symbols are then converted back to a serial time waveform. This is then the base band signal for the OFDM transmission. A channel model is then applied to the transmitted signal. The model allows for the signal to noise ratio. It is set by adding a known amount of white noise to the transmitted signal.

Path from the transmitter to the receiver either has reflections, we can get fading effects. In this case, the signal reaches the receiver from many different routes each of these rays has a slightly different delay and slightly different gain. The time delays result in phase shifts added to be degraded. Therefore the Orthogonality of channel impulse response is defined as

$$hc(t) = \sum_{n=0}^{N-1} \alpha_K \, x\, \delta(t - Tn) \quad (3)$$

where α_k is the complex path gain, To is normalized path delay relative to LOS, Δk = Tn-To difference in path time.

The FFT of each symbol is then taken to find the original transmitted spectrum i.e., defined as

$$x(k) = \sum_{n=0}^{N-1} x(n)\sin(2\pi kn/N) + j\sum_{n=0}^{N-1} x(n)\cos(2\pi kn/N) \quad (4)$$

The phase angle of each transmission carrier is then evaluated and converted back to the data word by demodulating the received phase. The data words are then combined, which gives the same word size as that of original data.

CHANNEL ESTIMATION

In Fig. 2, X(n) is the transmitted sequence, Y(n) is actual received signal, $\hat{Y}(n)$ is estimated signal and e(n) is defined as error signal. The channel estimation procedure utilizes the underlying channel model in conjunction with the available data based estimate. We adopt the Jakes model for the channel, assigns an autocorrelation and power spectral density to the time varying tap gain processes of the channel. The Jakes model is represented as an auto-regressive process where the current value of the process is a weighted sum of previous values plus plant noise. The weights for the AR Jakes model can be calculated, by solving the Yule-Walker set of equations or from a shaping filter. This AR Jakes model enables us to predict the tap gain process independently of the data based estimator.

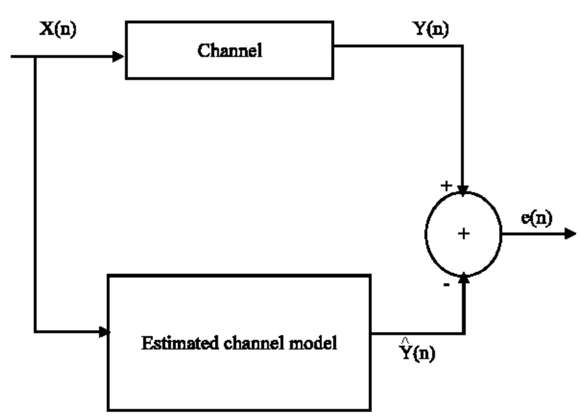

Fig. 2: A general channel estimation procedure

The system equation and the observation equation jointly form a State-Space representation of the dynamics of the tap-gain process. This state-space representation of the overall process is used to formulate the parameters of a Kalman filter. The channel estimation algorithm in the form of a Kalman filter produces LMMSE (Linear Minimized Mean Squared Error) estimates of the tap-gain process. Since it uses both the data estimator and the Jakes channel model. We will show through simulations that it performs better than the data only estimator.

CHANNEL ESTIMATION USING KALMAN FILTER ALGOTITHM

Modeling the channel tap gain as auto-regressive process: Any stationary random process can be represented as an infinite tap AR process (Proakia, 1983). The an infinite tap AR process model is impractical. It is truncated to an N-tap form so We will use the truncated AR process model to represent the underlying model driving the tap-gain process.

An AR process has the form for the for the general Auto-regressive process model is represented by a difference equation of the form

$$S(n)=\sum_{n=0}^{N-1} \varnothing i S(n-i)+W(n) \quad (5)$$

Where S(n) is a complex Gaussian process, Øi are the parameters of the model, N is the number of delays in the autoregressive model, W(n) A sequence of identically distributed zero mean Complex Gaussian random variables. The sequence W(n) is called white Gaussian noise because its spectrum is broad and uniform frequency range. Thus the AR process is simply another name for a linear difference equation model driven by white Gaussian noise.

The N^{th} order difference Equation can easily be reduced to a state model in the vector form.

$$\hat{S}(n) = F\hat{S}(n\text{-}1) + \hat{W}(n) \quad (6)$$

Where $\hat{S}(n)$ and $\hat{W}(n)$ are column vector of size (N×1) and F is an (N×N) matrix. We now find the mean, variance autocorrelation of the autoregressive process:

Mean:

$$\mu s = E[S(n)] = E\left[\sum_{i=1}^{N} \varnothing S(n-i) + w(n)\right] = 0 \quad (7)$$

Variance:

$$\sigma^2{}_s = E[S(n) * S(n)] = \sum_{i=1}^{N} \varnothing i\, R_{ss}(i) + \sigma \quad (8)$$

Auto correlation:

$$R_{ss}(m) = \sum_{i=1}^{N} \varnothing i\, R_{ss}(m-i) \quad (9)$$

The autocorrelation: Coefficient is:

$$r_{ss}(m) = R_{ss}(m)/_{S^2} = \sum_{i=0}^{N} \varnothing i\, r_{ss}(m-1); m >= 1 \quad (10)$$

Above equation $r_{SS}(m)$ is an N^{th} order difference equation that can be solved for the desired AR coefficients. Expressing the difference Equation in the form of a matrix, this matrix form obtain the Yule-Walker Equation. From matrix we obtain as

$$R^*\phi = r^*_{SS}$$

Since R is invertible, we can obtain

$$\varnothing = R^{-1} * r^*_{ss} \quad (11)$$

This is the matrix of AR coefficients that models the complex Gaussian process. If we are given the autocorrelation of the process,using the Yule-Walker elations, we can calculate the appropriate AR coefficients.This will allow us to express the underlying process model in a state from allowing us to formulate a Kalman filter to track-estimate the process.

Data based channel estimation formulation: The data based estimator uses training sequences sent over the channel to estimate the impulse response of the channel. We now describe how channel estimation is performed using the correlation method. A training sequence of length M, known to the receiver is sent over the channel. It is assumed that the channel does not change over the span of the data sent. Its vector form is:

$$X^* = [x_0\ x_1\ x_2\ x_4 \ldots\ldots\ldots\ldots\ldots\ldots x_{M-1}]^T \quad (12)$$

where T is the transpose operator.

This bit sequence has been mapped to unit energy symbol is follow as (0→+1) and (1→-1) to simulate a BPSK modulation. Let the channel impulse response at the snapshot when the training sequence is sent over the channel be:

$$h^* = [\hat{h}_0\ \hat{h}_1\ \hat{h}_2\ \hat{h}_3 \ldots\ldots\ldots\ldots\ldots\ldots \hat{h}_{L-1}] \quad (13)$$

L implies that channel impulse response length Or the number of process to be tracked.

The signal received will then be the convolution sum of the signal sent and the impulse response received in the presence of channel noise:

$$\hat{Y} = X^* \text{ x } \hat{h} + n_c \quad (14)$$

The convolution sum is given by:

$$Y(n) = \sum_{n=1}^{N} h(m)X(n-m) + n_c^* \quad (15)$$

This can be written in vector form as

$$\hat{Y} = X^*\hat{h} + n_c \quad (16)$$

Where $\hat{Y}$ is an (L+N-2×1) received signal vector, X* is an (L+N-1×L) Toeplitz matrix containing delayed versions of the training sequence sent, h is the channel impulse response, n_c is the Zero mean additive white Gaussian channel noise of variance. By anchoring Eb = 1, the SNR of the channel is then given by:

$$Eb/No = 1/(2\sigma^2_c \quad (17)$$

The general linear regression method given in, the estimate of the channel is given by:

$$\hat{h} = (X^T X)^{*1}(X^T\hat{Y}) \quad (18)$$

We now calculate the error of the data based estimator. Since the received signal is given by:

$$\hat{Y} = X^*\hat{h} + n_c^* \quad (19)$$

Its performance will vary depending on the current channel noise. Using the actual noisy and distorted received signal we can see the error in estimation is,

$$h = \hat{h} + (X^T X)^{*1}(X^T n_c) \quad (20)$$

Thus the error is:

$$\hat{E} = h\text{-}h$$

SIMULATION RESULTS

For simulation assigned parameters are N = 5 is the number of taps in model, Doppler band width is 500 MHz, frames rate is $5*10^4$ Frames/sec, M = 8 is the length of the training sequence and Eb = 1. Below Figure shows the different stage of OFDM model and the kalman filter based estimated signal (Fig. 3 and 4).

Where magenta represents the original message signal, green is for transmitted signal, yellow for represents the Kalman prediction and the red is for the threshold value Kalman prediction.

Where blue as the frequency estimation and Green as for the frequency filtered by kalman based LMMSE.

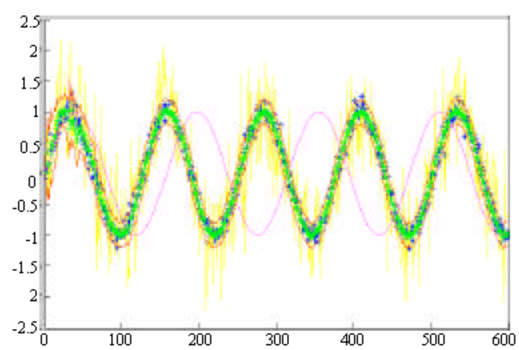

Fig. 3: Behavior of the signal at OFDM model and estimated signal

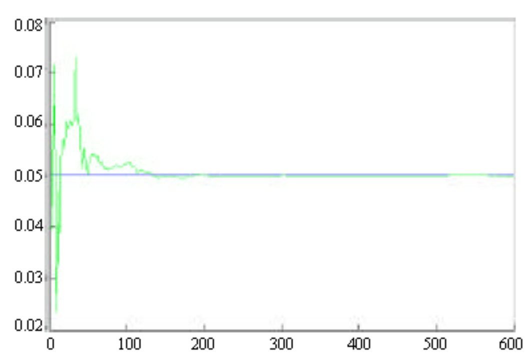

Fig. 4: It's for frequency response of estimated and filtered by kalman filter

CONCLUSION

The main contribution of this paper is estimation of OFDM channel estimation based on the development of a Kalman filter a novel LMMSE estimation. This in conjunction with the data estimator allowed us to formulate the Kalman algorithm. The Kalman estimator improved upon the performance of the data estimator by almost thirty percent on each path. Since the only way to increase the accuracy of the data estimate is to increase the length of the training sequences, the Kalman estimate provides an efficient technique for improving the channel estimate without wasting anymore bandwidth. In this paper result of improved LMMSE based Kalman estimated improved shown in Fig 3.

REFERENCES

Biglieri, E. and P.J. McLane, 1991. Uniform distance and error probability properties for TCM schemes, IEEE Trans. Commun., 39: 41-52.

Brajal, A. and A. Chouly, 1994. Optimal trellis-coded 8-PSK and 4-AM modulations for the Rayleigh channel, Proc. ICC' New Orleans, pp: 28-33.

Cimini, L.J., 1985. Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing, IEEE Trans. Commun., COM-33: 665-675,

COST, 207 Management Committee, 1989. 207: Digital land mobile radio communications, Commission of the European. Communities, Luxembourg.

Du, J.U., Y. Kamio, H. Sasaoka and B. Vucetic, 1993. Trellis coded M-QAM for efficient data transmission over land mobile radio channels, Proc. PIMRC 93, C1.6.1C1.6.5, Yokohama,.

Helard, J.F. and B. Le Floch, 1991. Trellis coded orthogonal frequency division multiplexing for digital video transmission, Proc. GLOBECOM 91, Phoenix, pp: 785-91.

Hoeher, P., 1991. TCM on frequency-selective land-mobile fading channels. Proc. Of Fifth Tirrenia International Workshop. Tirrenia, pp: 317-28.

Proakia, J.G., 1983. Digital Communications, McGraw-Hill: New York.

George, E.P., Box, M. Gwilym, Jenkins and C. Gregory Reinsel, 1994. Time series analysis: Forecasting and control. Publisher; Prentice Hall.