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# Kharitonov Based Robust Stability for a Flight Controller

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**Abstract:** In this study, an extended SIMC PID controller is designed for an unstable angle of attack of a FOXTROT aircraft and then its stability is tested for a particular range of perturbation values. The robust stability for the above system is tested analytically and graphically using Kharitonov Stability Criterion. Further, it was established that not only the designed controller along with the plant is stable but also robust stable while the aircraft flies with different speed.

**Key words:** Robust stability, kharitonov interval polynomials, frequency sweeping function, extended, SIMC

## INTRODUCTION

The major problems of flight control system are due to the non-linear dynamics, modeling uncertainties and parametric variation in characterizing the aircraft and its unpredicted environments. The aircraft motion in free flight is complicated. In general, an aircraft flies in a three dimensional plane by controlling its control surfaces aileron, rudder and elevator. These control surfaces control and change the motions of the aircraft about the roll, pitch and yaw axes. Elevators are flight control surfaces usually at the rear of an aircraft which control the orientation of the aircraft by changing the pitch and angle of attack of the aircraft. Though, a lot of researches have been done to control the angle of attack, still it is an open issue which is discussed in the present research. Not only the designed controller is required to offer satisfactory performance in terms of controlling the angle of attack, it also has to be robust stable for a wide range of change in parametric values of closed loop transfer function (of the angle of attack control system). Because the parametric changes occur due to different speed of the aircraft in different flight conditions and due to other environmental changes. Kharitonov (1978) found out asymptotic stability of a family of systems for an equilibrium position with help linear differential equations. Kharitonov theorem also provides the necessary and sufficient conditions for checking the robust stability of the dynamic system with fractional order interval systems (Chapellat and Bhattacharyya, 1989; Hote *et al*., 2010; Moornani and Haeri, 2010). Fu (1991) developed a simple approach which unifies and generalizes a class of weak Kharitonov regions for robust stability of linear uncertain systems. Chen *et al*. (2008) considered robust stability problem for interval plants in the case of single input (multi-output) or single output (multi-input) systems using a generalization of Kharitonov's theorem. Bevrani and Shokoohi (2010) designed a robust Proportional Integral Derivative (PID) feedback compensator for better stability and robust performance of a radio-frequency amplifier with wide range parameter variation. The robust stability feedback controller synthesis can be tested using Kharitonov's theorem for fuzzy parametric uncertain systems (Bhiwani and Patre, 2011). Toscano and Lyonnet (2010) synthesized a feedback controller to obtain robust static feedback using evolutionary algorithm.

Skogestad Internal Model Control (SIMC) tuning rules may be extended to cover for a 2nd order delay transfer function. SIMC for integrating process (damping ratio, $\zeta>1$) and double integrating process ($\zeta = 0$) can be applied to the process with real poles but it is not applicable for process with complex poles (Manum, 2005; Skogestad, 2003; Di Ruscio, 2010). Therefore, a new set of tuning rules called the interpolation rule is derived by interpolating between the SIMC for integrating process and SIMC for double integrating process.

In this study, a PID controller is designed using Skogestad Internal Model Control (SIMC) for different flight conditions in the presence of disturbance. In this research, the parametric perturbation ($\mu$) is allowed to increase up to a particular value below which the controller is robust stable by establishing the Kharitonov polynomials to be Hurwitz. Increasing beyond this value of $\mu$, the controller is not robust stable resulting non Hurwitz Khritonov polynomials. It is shown that the Kharitonov rectangle does not include zero within it. An interval polynomial family is shown to be robust stable for all frequencies $\omega \geq 0$ resulting H ($\omega$) (Bhattacharya *et al*., 1995).

Thus, the designed controller not only offers the desired angle of attack but also it is robust stable up to particular value of parametric perturbation $\mu$.

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## KHARITONOV POLYNOMIALS

Consider an nth order polynomial (Skogestad, 2003) of the form given by p (s) = $s^n+a_{n-1}s^{n-1}+...+a_0$ for all $a_0$, ..., $a_{n-1}$ such that $\underline{a}_k \le a_k \le \overline{a}_k$, k = 0, ..., n-1. Where $\underline{a}_k = a_k-\mu$, $\overline{a}_k = a_k+\mu$ and μ = The perturbation in parametric values which is also a positive real. Let, the polynomials be defined as:

$$g_1(s)=\underline{a}_0+\overline{a}_2s^2+\underline{a}_4s^4+...=\sum_{k=0,\,even}^{n} j^k.\min\left\{j^k\underline{a}_k, j^k\overline{a}_k\right\}.s^k$$

$$g_2(s)=\overline{a}_0+\underline{a}_2s^2+\overline{a}_4s^4+...=\sum_{k=0,\,even}^{n} j^k.\max\left\{j^k\underline{a}_k, j^k\overline{a}_k\right\}.s^k$$

$$h_1(s)=\underline{a}_1s+\overline{a}_3s^3+\underline{a}_5s^5+...=\sum_{k=1,\,odd}^{n} j^{k-1}.\min\left\{j^{k-1}\underline{a}_k, j^{k-1}\overline{a}_k\right\}.s^k$$

$$h_2(s)=\overline{a}_1s+\underline{a}_3s^3+\overline{a}_5s^5+...=\sum_{k=1,\,odd}^{n} j^{k-1}.\max\left\{j^{k-1}\underline{a}_k, j^{k-1}\overline{a}_k\right\}.s^k$$

The Kharitonov polynomials are given by:

$$K_{kl}(s)=g_k(s)+h_l(s)$$

Where, k, l = 1, 2; k =1 and l = 1.

$$k_{11}(s)=g_1(s)+h_1(s) \quad (1)$$

For k = 1 and l = 2:

$$k_{12}(s)=g_1(s)+h_2(s) \quad (2)$$

For k = 2 and l = 1:

$$k_{21}(s)=g_2(s)+h_1(s) \quad (3)$$

For k = 2 and l = 2:

$$k_{22}(s)=g_2(s)+h_2(s) \quad (4)$$

The set of polynomials $k_{11}$ (s), $k_{12}$ (s), $k_{21}$ (s) and $k_{22}$ (s) are said to be Hurwitz if and only if its every member is Hurwitz.

## ANGLE OF ATTACK

Angle of attack specifies the angle between the chord line of the wing of afixed-wing aircraft and the vector representing the relative motion between the aircraft and the atmosphere (McLean, 1990). The angle of attack is controlled by the deflection in control surface (elevator) (Fig. 1).

**Block diagram of angle of attack:** The block diagram for angle of attack is shown in Fig. 2, in which the input is the deflection of elevator ($\delta_E$) as commanded by the pilot and the output is the desired angle of attack (α).

In Fig. 2, $\delta_E$ = Deflection of elevator as commanded by the pilot; α = The desired angle of attack of the aircraft; G (s) = Open loop transfer function between $\delta_E$ and α; C (s) = PID controller to be designed (tuned); $G_d$ (s) = The transfer function of the disturbance = G (s).

**Transfer functions between $\delta_E$ and α:** The short period approximation (McLean, 1990), consists of assuming that any variations in speed of the aircraft (u) which arise in air speed as a result of control surface deflection, atmospheric turbulence or just aircraft motion are so small that any terms in the equation of motion involving u are negligible. In other words, the approximation assumes that short period transients are of sufficiently short duration that speed of the aircraft $U_0$ remain essentially constant, i.e., u = 0. Thus, the equations of longitudinal motion in terms of stability may now be written as:

$$\dot{w}=Z_w w+U_0 q+Z_{\delta_E}\delta_E \quad (5)$$

$$\dot{q}=wM_w+M_{\dot{w}}\dot{w}+M_q q+M_{\delta_E}\delta_E=\left(M_w+M_{\dot{w}}Z_w\right) w+\left(M_q+U_0M_{\dot{w}}\right)q+\left(M_{\delta_E}+Z_{\delta_E}M_{\dot{w}}\right)\delta_E \quad (6)$$

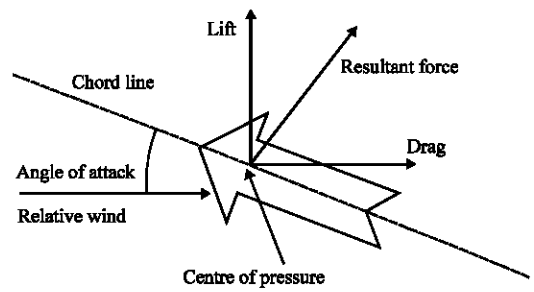


Fig. 1: Description of angle of attack

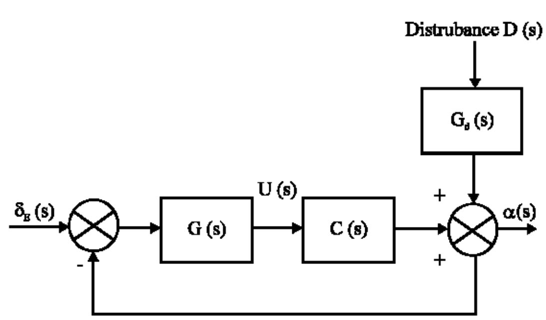


Fig. 2: Block diagram of angle of attack control system

If the state vector for short period motion is:

$$x \triangleq \begin{bmatrix} w \\ q \end{bmatrix}$$

The control vector u is taken as the elevator deflection $\delta_E$ then Eq. 5 and 6 may be written as a state equation:

$$\dot{x} = Ax + Bu \quad (7)$$

In Eq. 7, the values of A and B are:

$$A = \begin{bmatrix} Z_w & U_0 \\ (M_w + M_{\dot{w}} Z_w) & (M_q + U_0 M_{\dot{w}}) \end{bmatrix}$$

$$B = \begin{bmatrix} Z_{\delta_E} \\ (M_{\delta_E} + Z_{\delta_E} M_{\dot{w}}) \end{bmatrix}$$

$$\therefore [sI - A] = \begin{bmatrix} s - Z_w & -U_0 \\ -(M_w + Z_w M_{\dot{w}}) & [s - (M_q + U_0 M_{\dot{w}})] \end{bmatrix}$$

$$\Delta_{sp}(s) = \det[sI - A] = s^2 - [Z_w + M_q + U_0 M_{\dot{w}}]s + [Z_w M_q - U_0 M_w] = s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2 \quad (8)$$

In Eq. 8:

$$2\zeta_{sp}\omega_{sp} = Z_w + M_q + U_0 M_{\dot{w}}$$

$$\omega_{sp} = [Z_w M_q - U_0 M_w]^{\frac{1}{2}} \quad (9)$$

On simplifying the above earlier equations, the transfer function is given by:

$$= \frac{(U_0 M_{\delta E} - M_q Z_{\delta E})\left\{1 + \frac{sZ_{\delta E}}{U_0 M_{\delta E} - M_q Z_{\delta E}}\right\}}{\Delta_{sp}(s)}$$

$$\therefore \frac{w(s)}{\delta_E(s)} = \frac{K_w(1 + sT_1)}{\Delta_{sp}(s)} \quad (10)$$

In Eq. 10:

$$T_1 = \frac{Z_{\delta_E}}{K_w}, K_w = U_0 M_{\delta_E} - M_q Z_{\delta_E}$$

Again:

$$\alpha = \frac{w}{U_0}, \alpha(s) = \frac{w(s)}{U_0}$$

And w (s) = $U_0$ α (s):

$$\frac{\alpha(s)}{\delta_E(s)} = \frac{K_w(1 + sT_1)}{U_0 \Delta_{sp}(s)} \quad (11)$$

Using the values of the stability derivatives (Bhattacharya *et al*., 1995) as shown in Appendix and substituting these values in Eq. 11, the transfer function $G_1$ (s) between $\delta_E$ and α for the flight condition-1 is given by:

$$G_1(s) = \frac{2.0302s + 102.8}{s^2 + 0.901s + 0.5633} = \frac{3.604s + 182.5}{1.775s^2 + 1.598s + 1} \quad (12)$$

Similarly, the transfer function $G_2$ (s) between $\delta_E$ and α for the flight condition-2 is given by:

$$G_2(s) = \frac{15.11s + 0.003027}{s^2 + 1.2989s + 8.216} = \frac{1.84s + 368.5}{0.1217s^2 + 0.1581s + 1} \quad (13)$$

Again, the transfer function $G_3$ (s) between $\delta_E$ and α for the flight condition-3 is given by:

$$G_3(s) = \frac{27.54s + 7266}{s^2 + 1.82s + 28.54} = \frac{0.9653s + 254.6}{0.0350s^2 + 0.0638s + 1} \quad (14)$$

## DESIGN OF EXTENDED SIMC PID CONTROLLER

Let, the extended SIMC PID controller be:

$$C(s) = \frac{K_c + K_I}{s + K_D s} \quad (15)$$

Where:
$K_C$ = Proprotional constant
$K_I$ = Integral constant
$K_D$ = Derivative constant

The tuning parameters for an under damped second order process are given by (Fu, 1991):

$$K_C = \max\{A, X\}, \text{ where } X = B \text{ for } \zeta \geq 1 \text{ and } X = \zeta B + (1 - \zeta) C \text{ for } \zeta < 1$$

$$K_I = \max\{A, X\}, \text{ where } X = B \text{ for } \zeta \geq 1 \text{ and } X = \zeta B + (1 - \zeta) C \text{ for } \zeta < 1$$

$K_D$ is either A, B or C where:

$$A = \frac{2\zeta}{k''(\tau_C + \theta)\tau_0} \quad (16)$$

$$B = \frac{1 + 4(\tau_C + \theta) + \frac{\zeta}{\tau_0}}{k''(\tau_C + \theta)^2} \quad (17)$$

$$C = \frac{1}{2k''(\tau_C + \theta)^2} \quad (18)$$

Where:

$k' = k/\tau_0^2$
k = The gain
$\tau_0 = 1/\omega_n$
$\omega_n$ = Natural frequency of oscillation
$\zeta$ = Damping ratio
$\tau_C$ = The controller tuning parameter
$\theta = \tau_0\,(1.5+0.5\,\zeta)\,(0.6)^a$ is the delay angle
$a = \tau_0^2$

**Calculation of $K_C$, $K_1$ and $K_D$for FC-1:** Comparing the denominator part of $G_1$ (s) with the standard form of a 2nd order system, $s^2+2\zeta\omega_n s+\omega_n^2$, the value of $\zeta$ and $\tau_0$ for FC-1 is obtained as 0.5996 and 1.3324, respectively. In this case, $\theta = 0.9683$, $\tau_c = 5\theta = 4.8415$, k = 1 and $k'' = k/\tau_0^2 = 0.5633$. Calculation of $K_{C1}$ is the constants A, B and C mentioned in Eq. 16-18 are denoted here as $A_{C1}$-$C_{C1}$ which are calculated for FC-1 as follows:

$$A_{C1} = \frac{2\zeta}{k''(\tau_C + \theta)\tau_0} = 0.2750$$

$$B_{C1} = \frac{1 + 4(\tau_C + \theta) + \zeta / \tau_0}{k''(\tau_C + \theta)^2} = 1.2985$$

$$C_{C1} = \frac{1}{2k''(\tau_C + \theta)^2} = 0.0263$$

$$X_{C1} = \zeta B_{C1} + (1-\zeta)C_{C1} = 0.7891$$
$$K_{C1} = \text{Max}\{A_{C1}, X_{C1}\} = 0.7891 \quad (19)$$

**Calculation of $K_{I1}$:** Similiarly from Eq. 16-18, the constants $A_{I1}$-$C_{I1}$ are calculated for FC-1 as:

$$A_{I1} = \frac{1}{k''(\tau_C + \theta)\tau_0^2} = 0.1721$$

$$B_{I1} = \frac{\zeta}{k''(\tau_C + \theta)^2\,\tau_0} = 0.0237$$

$$C_{I1} = \frac{1}{16k''(\tau_C + \theta)^3} = 0.00051578$$

$$X_{I1} = \zeta B_{I1} + (1-\zeta)C_{I1} = 0.0144$$
$$K_{I1} = \text{Max}\{A_{I1}, X_{I1}\} = 0.1721 \quad (20)$$

**Calculation of $K_{D1}$:** Again from Eq. 16-18, the constants $A_{D1}$-$C_{D1}$ are calculated for FC-1 as:

$$A_{D1} = B_{D1} = C_{D1} = \frac{1}{k''(\tau_C + \theta)} = 0.3056$$
$$K_{D1} = \text{Either } A_{D1}, B_{D1} \text{ or } C_{D1} = 0.3056 \quad (21)$$

**Calculation of $K_C$, $K_1$ and $K_D$ for FC-2:** The value of $\zeta$ and $\tau_0$ for FC-2 is obtained as $\zeta$ = 0.2266 and $\tau_0$ = 0.3489. In this case, $\theta$ = 0.5289, $\tau_C$ = 0.02, $\theta$ = 0.0106 and k = 1 and $k'' = k/\tau_0^2 = 8.216$.

As, researchers have obtained in FC-1 similarly the values of $K_{C2}$, $K_{I2}$ and $K_{D2}$ are obtained as follows: $K_{C2}$ = 0.5225, $K_{I2}$ = 1.8536 and $K_{D2}$ = 0.2256.

**Calculation of $K_C$, $K_1$ and $K_D$ for FC-3:** The value of $\zeta$ and $\tau_0$ for FC-3 is obtained as: $\zeta$ = 0.1705 and $\tau_0$ = 0.1872. In this case, $\theta$ = 0.1995, $\tau_C$ = 0.0008, $\theta$ = 0.000159, k = 1 and $k'' = k/\tau_0^2 = 28.54$. As researchers have obtained in FC-1, similarly the values of $K_{C3}$, $K_{I3}$ and $K_{D3}$ are obtained as follows: $K_{C3}$= 0.7704, $K_{I3}$ = 5.0075 and $K_{D3}$ = 0.1755.

**Derivation of PID controller for different Flight Conditions (FCs):** The PID controller transfer function for flight condition-1 is $C_1$ (s) = $K_{C1}$ $K_{I1}$/s+$K_{D1}$s. The values of $K_{C1}$, $K_{I1}$ and $K_{D1}$ is obtained earlier from previous Eq. 19-21, respectively. Therefore, substituting these values in Eq. 15, $C_1$ (s) for flight condition-1 is given by:

$$C_1(s) = \frac{0.3056s^2 + 0.7891s + 0.1721}{s} \quad (22)$$

Similarly, PID controller TF for FC-2 and FC-3 are calculated, respectively after substituting the values of $K_{C2}$, $K_{I2}$ and $K_{D2}$ and $K_{I3}$, $K_{D3}$ and $K_{D3}$:

$$C_2(s) = \frac{0.2256s^2 + 0.5225s + 1.854}{s} \quad (23)$$

$$C_3(s) = \frac{0.1755s^2 + 0.7704s + 5.007}{s} \quad (24)$$

Now, the loop transfer functions for FC-1-FC-3 are obtained as follows:

$$T_1(s)=\frac{G_1(s)C_1(s)}{1+G_1(s)C_1(s)} =\frac{1.101s^3+58.61s^2+144.6s+31.41}{2.876s^3+60.2s^2+145.6s+31.41} \quad (25)$$

$$T_2(s)=\frac{G_2(s)C_2(s)}{1+G_2(s)C_2(s)} =\frac{0.4152s^3+84.11s^2+196s+683.1}{0.5369s^3+84.27s^2+197s+683.1} \quad (26)$$

$$T_3(s)=\frac{G_3(s)C_3(s)}{1+G_3(s)C_3(s)} =\frac{0.1694s^3+45.41s^2+201s+1275}{0.2044s^3+45.48s^2+202s+1275} \quad (27)$$

## ROBUST STABILITY OF PID CONTROLLER

The characteristic equation for FC-1 is obtained from Eq. 25 as:

$$p(s)=1+G_1(s)C_1(s)=2.876s^3+60.2s^2+145.6s+31.41$$

The perturbation in parametric value of p (s), i.e., μ is allowed to increase from upto 81.56% and the Kharitonov polynomials for FC-1 are found out using Eq. 1-4 are as follows:

$$\begin{aligned}K_{11}(s)&=5.8007s^3+26.8632s^2+109.2931s+5.224\\K_{12}(s)&=5.8007s^3+264.3368s^2+109.2931s+0.5306\\K_{21}(s)&=57.0793s^3+26.8632s^2+11.1069s+5.2214\\K_{22}(s)&=57.0793s^3+264.3368s^2+11.1069s+0.5306\end{aligned} \quad (28)$$

These above polynomials are tested for Hurwitz using Routh Hurwitz criteria and found out to be Hurwitz Polynomials by establishing the coefficients in first column are positive. If the perturbation is further allowed beyond the above value of μ the polynomials are found not to be Hurwitz resulting the coefficients to be are negative. Thus, it is concluded that the designed controller along with the plant transfer function (angle of attack) discussed here is robust stable up to the perturbation range of 81.56%. Similarly, the characteristic equation for FC-2 is obtained from Eq. 26 as:

$$p(s)=0.5369s^3+84.27s^2+197s+683.1$$

In this case, μ is found out to be 74.11%. Therefore, the designed controller is robust stable up to the perturbation range of 74.11%. The Kharitonov polynomials for FC-2 are calculated as:

$$\begin{aligned}K_{11}(s)&=176.8546s^3+51.0033s^2+146.7225s+0.9348\\K_{12}(s)&=176.8546s^3+342.9967s^2+146.7225s+0.1390\\K_{21}(s)&=1189.3s^3+51.0s^2+21.8s+0.9\\K_{22}(s)&=1189.3s^3+343.0s^2+21.8s+0.1\end{aligned} \quad (29)$$

Again for FC-3, the characteristic equation for FC-2 is obtained from Eq. 27 as:

$$p(s)=0.2044s^3+45.48s^2+202s+1275$$

And the perturbation range is 71.17%. Therefore, the parametric value μ is allowed up to 71.17%. The Kharitonov polynomials for FC-3 are calculated as:

$$\begin{aligned}K_{11}(s)&=367.5825s^3+58.2366s^2+77.8481s+0.3499\\K_{12}(s)&=367.5825s^3+345.7634s^2+77.8481s+0.0589\\K_{21}(s)&=2182.4s^3+58.2s^2+13.1s+0.3\\K_{22}(s)&=2182.4s^3+345.8s^2+13.1s+0.1\end{aligned} \quad (30)$$

**Kharitonov rectangle and zero exclusion for interval families (graphical testing of robust stability):** An interval polynomial family having invariant degree and at least one stable variable is robustly stable if and only if the origin of the complex plane is excluded from the Kharitonov rectangle at all non-negative frequencies, i.e., for all frequencies $\omega \geq 0$.

The four vertices of Kharitonov rectangle $K_{11}$ ($j\omega_0$), $K_{21}$ ($j\omega_0$) and $K_{22}$ ($j\omega_0$) are obtained by substituting $s = j\omega_0$ in Eq. 28 for FC-1, Eq. 29 for FC-2 and Eq. 30 for FC-3 at a fixed frequency $\omega_0$. The Kharitonov rectangles for FC-1 (at $\omega_0 = 8$), FC-2 (at $\omega_0 = 3$) and FC-3 (at $\omega_0 = 5$) are shown in Fig. 3-5, respectively.

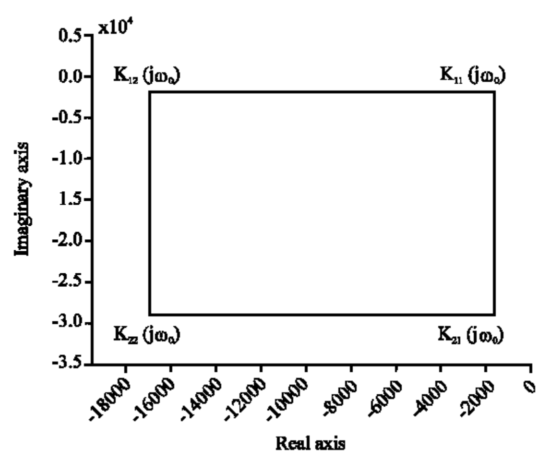


Fig. 3: Kharitonov rectangle for FC-1 at $\omega_0 = 8$

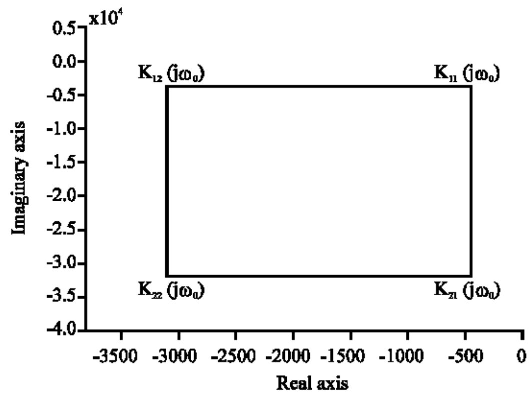


Fig. 4: Kharitonov rectangle for FC-2 at $\omega_0 = 3$

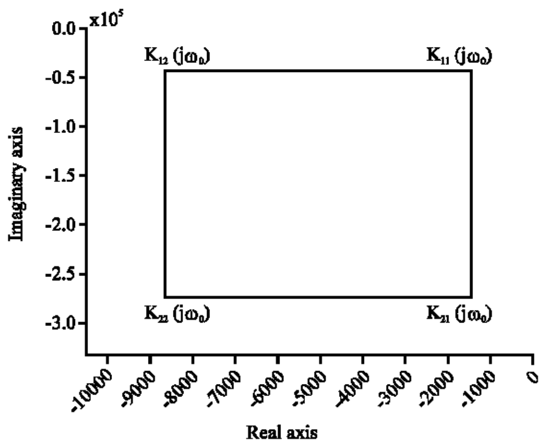


Fig. 5: Kharitonov rectangle for FC-2 at $\omega_0 = 5$

However, the size and the position of the Kharitonov rectangle may change with ω but the sides of the rectangle remain parallel to the respective real and imaginary axis.

**Frequency sweeping function for robust stability:** An interval polynomial family is robustly stable if and only if H (ω)>0 for all frequencies $\omega \geq 0$.

$$H(\omega) = \max \begin{Bmatrix} \operatorname{Re} K_{11}(j\omega) \\ -\operatorname{Re} K_{12}(j\omega) \\ \operatorname{Im} K_{21}(j\omega) \\ -\operatorname{Im} K_{22}(j\omega) \end{Bmatrix}$$

**For FC-1:** Substituting s = jω in Eq. 28, researchers get:

$$K_{11}(j\omega) = -5.8007j\omega^3 - 26.8632j\omega^2 + 109.2931j\omega + 5.2214$$
$$K_{12}(j\omega) = -5.8007j\omega^3 - 264.3368j\omega^2 + 109.2931j\omega + 0.5306$$
$$K_{21}(j\omega) = -57.0793j\omega^3 - 26.8632j\omega^2 + 11.1069j\omega + 5.2214$$
$$K_{22}(j\omega) = -57.0793j\omega^3 - 264.3368j\omega^2 + 11.1069j\omega + 0.5306$$

Again:

$$\operatorname{Re} K_{11}(j\omega) = -26.8632\omega^2 + 5.2214$$
$$\operatorname{Re} K_{12}(j\omega) = -264.3368\omega^2 + 0.5306$$
$$\operatorname{Im} K_{21}(j\omega) = -57.0793\omega^3 + 11.1069\omega$$
$$\operatorname{Im} K_{22}(j\omega) = -57.0793\omega^3 + 11.1069\omega$$

**For FC-2:** Similarly substituting s = jω in Eq. 29, researchers get:

$$K_{11}(j\omega) = -176.8546j\omega^3 - 51.0033j\omega^2 + 146.7225j\omega + 0.9348$$
$$K_{12}(j\omega) = -176.8546j\omega^3 - 342.9967j\omega^2 + 146.7225j\omega + 0.1390$$
$$K_{21}(j\omega) = -1189.3j\omega^3 - 51.0j\omega^2 + 21.8j\omega + 0.9$$
$$K_{22}(j\omega) = -1189.3j\omega^3 - 343.0j\omega^2 + 21.8j\omega + 0.1$$

Again:

$$\operatorname{Re} K_{11}(j\omega) = -51.0033\omega^2 + 0.9348$$
$$\operatorname{Re} K_{12}(j\omega) = -342.9967\omega^2 + 0.1390$$
$$\operatorname{Im} K_{21}(j\omega) = -1189.3\omega^3 + 21.8\omega$$
$$\operatorname{Im} K_{22}(j\omega) = -1189.3\omega^3 + 21.8\omega$$

**For FC-3:** Substituting s = jω in Eq. 30, researchers get:

$$K_{11}(j\omega) = -367.5825j\omega^3 - 58.2366j\omega^2 + 77.8481j\omega + 0.3499$$
$$K_{12}(j\omega) = -367.5825j\omega^3 - 345.7634j\omega^2 + 77.8481j\omega + 0.0589$$
$$K_{21}(j\omega) = -2182.4j\omega^3 - 58.2j\omega^2 + 13.1j\omega + 0.3$$
$$K_{22}(j\omega) = -2182.4j\omega^3 - 345.8j\omega^2 + 13.1j\omega + 0.1$$

Again:

$$\operatorname{Re} K_{11}(j\omega) = -58.2366\omega^2 + 0.3499$$
$$\operatorname{Re} K_{12}(j\omega) = -345.7634\omega^2 + 0.0589$$
$$\operatorname{Im} K_{21}(j\omega) = -2182.4\omega^3 + 13.1\omega$$
$$\operatorname{Im} K_{22}(j\omega) = -2182.4\omega^3 + 13.1\omega$$

It is clear from the equations that for any frequencies $\omega \geq 0$, the value of H (ω)>0 and the family of interval polynomial is robustly stable.

## CONCLUSION

In this study, μ is allowed to increase up to a particular value below which the controller is robust stable by establishing the Kharitonov polynomials to be Hurwitz. These values of parametric perturbation μ for different flight conditions are 81.56% (FC-1), 74.11% (FC-2) and 71.17% (FC-3), respectively. Increasing beyond this value of μ the controller is not robust stable resulting

Non-Hurwitz Khritonov polynomials. Thus it is shown in this study, the controller designed here not only offers the desired angle of attack but also it is robust stable up to particular value of parametric change, $\mu$. The earlier result analysis also shows that the aircraft is less robust stable with increasing the speed of the aircraft. However, this is not surprising as aircraft becomes less stable and becomes more unstable with increase in its speed. It is shown that the Kharitonov rectangle does not include zero within it. The interval polynomial family is shown to be robust stable for all frequencies $\geq 0$ resulting $H(\omega)$.

## APPENDIX

Stability derivatives of longitudinal dynamics of foxtrot aircraft

| Stability derivatives | Flight Conditions (FC) | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| $U_0$ (m sec$^{-1}$) | 70 | 265 | 350 |
| $X_u$ | -0.012 | -0.009 | -0.0135 |
| $X_w$ | 0.14 | 0.016 | 0.006 |
| $Z_u$ | -0.117 | -0.088 | 0.0125 |
| $Z_w$ | -0.452 | -0.547 | -0.727 |
| $Z_q$ | -0.76 | -0.88 | -0.125 |
| $M_u$ | 0.0024 | -0.008 | 0.009 |
| $M_w$ | -0.006 | -0.03 | -0.08 |
| $M_w$ | -0.002 | -0.001 | -0.001 |
| $M_q$ | -0.317 | -0.487 | -0.745 |
| $X_{\delta E}$ | 1.83 | 0.69 | 0.77 |
| $Z_{\delta E}$ | -2.03 | -15.12 | -27.55 |
| $M_{\delta E}$ | -1.46 | -11.14 | -20.07 |

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