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Anti-Synchronization of Hyperchaotic Lorenz and Hyperchaotic Chen Systems by Adaptive Control

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Abstract: In this study, researchers apply Adaptive control method to derive new results for the global chaos anti-synchronization of identical hyperchaotic Lorenz systems in 2007, identical hyperchaotic Chen systems in 2010 and non-identical hyperchaotic Lorenz and hyperchaotic Chen systems. In this study, we shall assume that the parameters of both master and slave systems are unknown and we devise adaptive anti-synchronization schemes using the estimates of parameters for both master and slave systems. The adaptive anti-synchronization results derived in this study are established using Lyapunov stability theory. Since, the Lyapunov exponents are not required for these calculations, the proposed Adaptive control method is very effective and convenient to achieve anti-synchronization of identical and non-identical hyperchaotic Lorenz and hyperchaotic Chen systems. Numerical simulations are shown to demonstrate the effectiveness of the proposed adaptive anti-synchronization schemes for the hyperchaotic systems addressed in this study.

Key words: Anti-synchronization, adaptive control, hyperchaotic Lorenz system, hyperchaotic Chen system, non-identical, India

INTRODUCTION

Chaotic systems are non-linear dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect (Alligood *et al*., 1997). Since, the seminal research by Pecora and Carroll (1990), chaos synchronization has been studied intensively and extensively in the last two decades (Pecora and Carroll, 1990; Lakshmanan and Murali, 1996; Han *et al*., 1995; Blasius *et al*., 1999; Cuomo and Oppenheim, 1993; Kocarev and Parlitz, 1995; Tao, 1999; Ott *et al*., 1990; Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian and Karthikeyan, 2011a, b; Lu *et al*., 2004; Chen and Lu, 2002; Park and Kwon, 2003). Chaos theory has been applied to a variety of fields like physical systems (Lakshmanan and Murali, 1996), chemical systems (Han *et al*., 1995), ecological systems (Blasius *et al*., 1999), secure communications (Cuomo and Oppenheim, 1993; Kocarev and Parlitz, 1995; Tao, 1999), etc. In the recent years, various schemes such as PC method (Pecora and Carroll, 1990), OGY method (Ott *et al*., 1990), Active control method (Ho and Hung, 2002; Huang *et al*., 2004; Chen, 2005; Sundarapandian and Karthikeyan, 2011a, b; Lu *et al*., 2004), Adaptive control method (Lu *et al*., 2004; Chen and Lu, 2002), time-delay feedback approach (Park and Kwon, 2003), Backstepping design method (Yu and Zhang, 2006), Sampled-data feedback synchronization method (Zhao and Lu, 2008), sliding mode control (Konishi *et al*., 1998; Yau, 2004), etc. have been successfully applied for chaos synchronization.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system then the idea of the anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically.

In this study, researchers apply Adaptive control method to derive new results for the global chaos anti-synchronization of identical hyperchaotic Lorenz systems (Gao *et al*., 2007), identical hyperchaotic Chen systems (Li-Xin *et al*., 2010) and non-identical hyperchaotic

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Lorenz and hyperchaotic Chen systems. It is assume that the parameters of the master and slave systems are unknown.

ADAPTIVE ANTI-SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC LORENZ SYSTEMS

Theoretical results: In this study, researchers apply the Adaptive control method to derive new results for the anti-synchronization of identical uncertain hyperchaotic Lorenz systems (Gao *et al*., 2007). Thus, the master system is described by the hyperchaotic Lorenz dynamics:

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= \rho x_1 - x_2 - x_4 - x_1 x_3 \\ \dot{x}_3 &= x_1 x_2 - \beta x_3 \\ \dot{x}_4 &= r x_2 x_3 \end{aligned} \tag{1}$$

where, x_1-x_4 are the state variables and σ, β, ρ and r are unknown parameters of the system. The slave system is described by the controlled hyperchaotic Lorenz dynamics:

$$\begin{aligned} \dot{y}_1 &= \sigma(y_2 - y_1) + u_1 \\ \dot{y}_2 &= \rho y_1 - y_2 - y_4 - y_1 y_3 + u_2 \\ \dot{y}_3 &= y_1 y_2 - \beta y_3 + u_3 \\ \dot{y}_4 &= r y_2 y_3 + u_4 \end{aligned} \tag{2}$$

where, y_1-y_4 are the state variables and μ_1-μ_4 are the non-linear controllers to be designed. The hyperchaotic Lorenz system (Eq. 1) is a new hyperchaotic system derived from the Lorenz system by Gao *et al*. (2007). The system (Eq. 1) is hyperchaotic when:

$$\sigma = 10,\ \beta = 8/3,\quad \rho = 28 \text{ and } r = 0.1$$

The state portrait of the hyperchaotic Lorenz system (Eq. 1) is shown in Fig. 1. The anti-synchronization error is defined as:

$$e_i = y_i + x_i,\ \ (i = 1, 2, 3, 4) \tag{3}$$

A simple calculation gives the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= \sigma(e_2 - e_1) + u_1 \\ \dot{e}_2 &= \rho e_1 - e_2 - e_4 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -\beta e_3 + y_1 y_2 + x_1 x_2 + u_3 \\ \dot{e}_4 &= r(y_2 y_3 + x_2 x_3) + u_4 \end{aligned} \tag{4}$$

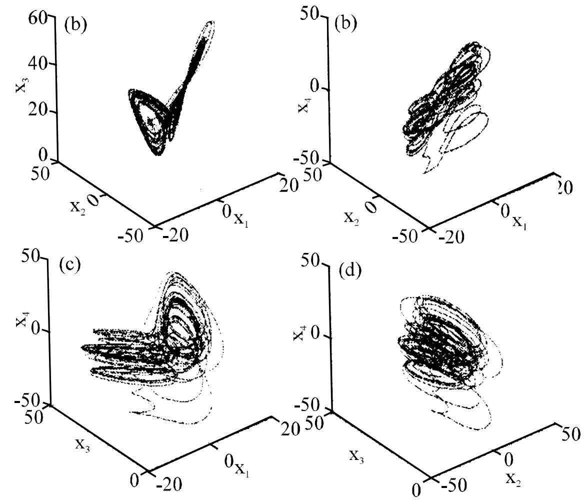

Fig. 1: State orbits of the hyperchaotic Lorenz system

Let us now define the adaptive functions $u_1(t)$,-$u_4(t)$ as:

$$\begin{aligned} u_1(t) &= -\hat{\sigma}(e_2 - e_1) - k_1 e_1 \\ u_2(t) &= -\hat{\rho} e_1 + e_2 + e_4 + y_1 y_3 + x_1 x_3 - k_2 e_2 \\ u_3(t) &= \hat{\beta} e_3 - y_1 y_2 - x_1 x_2 - k_3 e_3 \\ u_4(t) &= -\hat{r}(y_2 y_3 + x_2 x_3) - k_4 e_4 \end{aligned} \tag{5}$$

where, $\hat{\sigma}, \hat{\beta}, \hat{\rho}$ and $\hat{r}$ are estimates of σ, β, ρ and r, respectively and k_i (i = 1, 2, 3, 4) are positive constants. Substituting Eq. 5 into 4, the error dynamics simplifies to:

$$\begin{aligned} \dot{e}_1 &= (\sigma - \hat{\sigma})(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= (\rho - \hat{\rho}) e_1 - k_2 e_2 \\ \dot{e}_3 &= -(\beta - \hat{\beta}) e_3 - k_3 e_3 \\ \dot{e}_4 &= (r - \hat{r})(y_2 y_3 + x_2 x_3) - k_4 e_4 \end{aligned} \tag{6}$$

Let us now define the parameter estimation error as:

$$\begin{aligned} e_\sigma &= \sigma - \hat{\sigma} \\ e_\beta &= \beta - \hat{\beta} \\ e_\rho &= \rho - \hat{\rho} \\ e_r &= r - \hat{r} \end{aligned} \tag{7}$$

Substituting Eq. 7 into 6, researchers obtain the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= e_\sigma(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 &= e_\rho e_1 - k_2 e_2 \\ \dot{e}_3 &= -e_\beta e_3 - k_3 e_3 \\ \dot{e}_4 &= e_r(y_2 y_3 + x_2 x_3) - k_4 e_4 \end{aligned} \tag{8}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. Researchers consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_\sigma^2 + e_\beta^2 + e_\rho^2 + e_r^2) \qquad (9)$$

which is a positive definite function on R^8. We also note that:

$$\dot{e}_\sigma = -\dot{\hat{\sigma}},\ \dot{e}_\beta = -\dot{\hat{\beta}},\ \dot{e}_\rho = -\dot{\hat{\rho}},\ \dot{e}_r = -\dot{\hat{r}} \qquad (10)$$

Differentiating Eq. 9 along the trajectories of Eq. 8 and noting Eq. 10, they find that:

$$\begin{aligned}\dot{V} = &-k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + \\ &e_\sigma\left[e_1(e_2 - e_1) - \dot{\hat{\sigma}}\right] + e_\beta\left[-e_3^2 - \dot{\hat{\beta}}\right] + \\ &e_\rho\left[e_1e_2 - \dot{\hat{\rho}}\right] + e_r\left[e_4(y_2y_3 + x_2x_3) - \dot{\hat{r}}\right]\end{aligned} \qquad (11)$$

In view of Eq. 11, the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{\sigma}} &= e_1(e_2 - e_1) + k_5e_\sigma \\ \dot{\hat{\beta}} &= -e_3^2 + k_6e_\beta \\ \dot{\hat{\rho}} &= e_1e_2 + k_7e_\rho \\ \dot{\hat{r}} &= e_4(y_2y_3 + x_2x_3) + k_8e_r\end{aligned} \qquad (12)$$

where, k_5-k_8 are positive constants. Substituting Eq. 12 into 11, they obtain:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_\sigma^2 - k_6e_\beta^2 - k_7e_\rho^2 - k_8e_r^2 \qquad (13)$$

which is a negative definite function on R^8. Thus by Lyapunov stability theory (Hahn, 1967), it is immediate that the anti-synchronization error e_i (1, 2, 3, 4) and the parameter estimation error e_σ, e_β, e_ρ and e_r decay to zero exponentially with time. Hence, they have proved the following result.

Theorem 1:. The identical uncertain hyperchaotic Lorenz systems (Eq. 1 and 2) are globally and exponentially anti-synchronized by the adaptive control law (Eq. 5), where the update law for the parameter estimates is given by Eq. 12 and k_i (i = 1, ..., 8) are positive constants.

Numerical results: For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential Eq. 1 and 2 with the adaptive non-linear controller (Eq. 5). We take k_i = 2 for i = 1, 2, ..., 8. The parameters of the

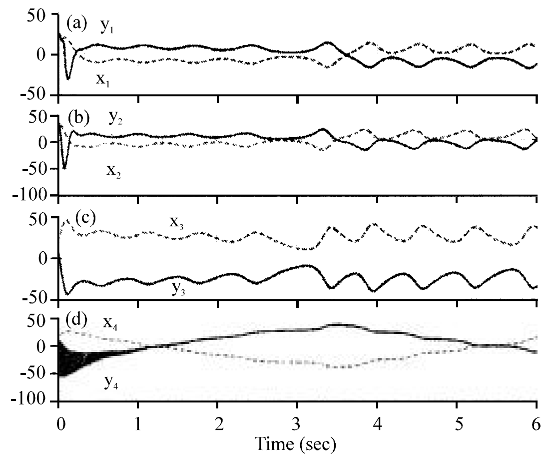

Fig. 2: Anti-synchronization of hyperchaotic Lorenz systems

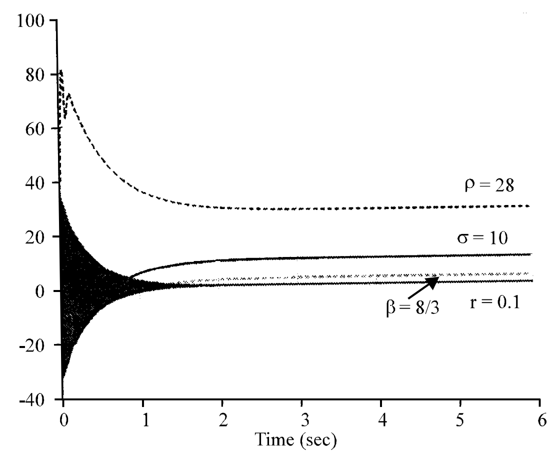

Fig. 3. Parameter estimates $\hat{\sigma}(t), \hat{\beta}(t), \hat{\rho}(t), \hat{r}(t)$

hyperchaotic Lorenz systems (Eq. 1 and 2) are chosen so that the systems are hyperchaotic, i.e.:

$$\sigma = 10,\ \beta = 8/3,\ \rho = 28 \text{ and } r = 0.1$$

The initial values of the parameter estimates are taken as:

$$\hat{\sigma}(0) = 2,\ \hat{\beta}(0) = 10,\ \hat{\rho}(0) = 4 \text{ and } \hat{r}(0) = 6$$

The initial values of the master system (Eq. 1) are:

$$x_1(0) = 12,\ x_2(0) = 25,\ x_3(0) = 16 \text{ and } x_4(0) = 20$$

The initial values of the slave system (Eq. 2) are:

$$y_1(0) = 25,\ y_2(0) = 30,\ y_3(0) = 10 \text{ and } y_4(0) = 14$$

Figure 2 shows anti-synchronization of the hyperchaotic Lorenz systems (Eq. 1 and 2). Figure 3 shows that the estimated values of the parameters, viz., $\hat{\sigma}, \hat{\beta}, \hat{\rho}$ and $\hat{r}$ converge to the system parameters $\sigma = 10$, $\beta = 8/3$, $\rho = 28$ and $r = 0.1$, respectively.

ADAPTIVE ANTI-SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC CHEN SYSTEMS

Theoretical results: In this study, researchers apply the Adaptive control method to derive new results for the anti-synchronization of identical uncertain hyperchaotic Chen systems (Li-Xin *et al.*, 2010). Thus, the master system is described by the hyperchaotic Chen dynamics:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) \\ \dot{x}_2 &= 4x_1 - 10x_1x_3 + cx_2 + 4x_4 \\ \dot{x}_3 &= x_2^2 - bx_3 \\ \dot{x}_4 &= -dx_1 \end{aligned} \tag{14}$$

where, x_1-x_4 are the state variables and a-d are unknown parameters of the system. The slave system is described by the controlled hyperchaotic Chen dynamics:

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + u_1 \\ \dot{y}_2 &= 4y_1 - 10y_1y_3 + cy_2 + 4y_4 + u_2 \\ \dot{y}_3 &= y_2^2 - by_3 + u_3 \\ \dot{y}_4 &= -dy_1 + u_4 \end{aligned} \tag{15}$$

where, y_1-y_4 are the state variables and u_1-u_4 are the non-linear controllers to be designed. The hyperchaotic Chen system (Eq. 14) is a new hyperchaotic system derived from the Chen system by Li-Xin *et al.* (2010). The system (Eq. 1) is hyperchaotic when:

$$a = 35,\ b = 3,\ c = 21 \text{ and } d = 2$$

The state portrait of the hyperchaotic Chen system (Eq. 14) is shown in Fig. 4. The anti-synchronization error is defined as:

$$e_i = y_i + x_i,\ (i = 1, 2, 3, 4) \tag{16}$$

A simple calculation gives the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + u_1 \\ \dot{e}_2 &= 4e_1 - 10(y_1y_3 + x_1x_3) + ce_2 + 4e_2 + u_2 \\ \dot{e}_3 &= -be_3 + y_2^2 + x_2^2 + u_3 \\ \dot{e}_4 &= -de_1 + u_4 \end{aligned} \tag{17}$$

Let us now define the adaptive functions $u_1(t)$-$u_4(t)$ as:

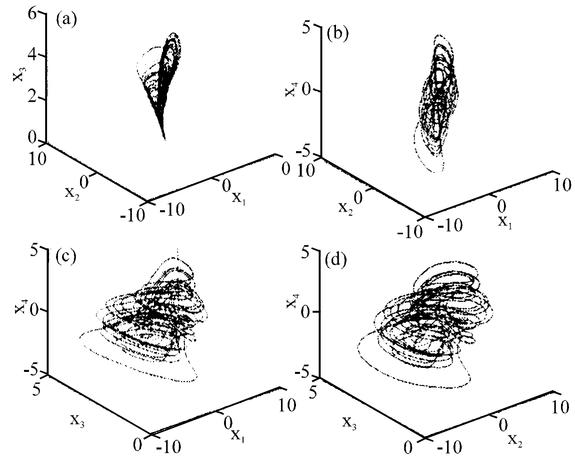

Fig. 4: State orbits of the hyperchaotic Chen system

$$\begin{aligned} u_1(t) &= -\hat{a}(e_2 - e_1) - k_1e_1 \\ u_2(t) &= -4e_1 + 10(y_1y_3 + x_1x_3) - \hat{c}e_2 - 4e_4 - k_2e_2 \\ u_3(t) &= \hat{b}e_3 - y_2^2 - x_2^2 - k_3e_3 \\ u_4(t) &= \hat{d}e_1 - k_4e_4 \end{aligned} \tag{18}$$

where, $\hat{a}$-$\hat{d}$ are estimates of a-d, respectively and k_i (1, 2, 3, 4) are positive constants. Substituting Eq. 18 into 17, the error dynamics simplifies to:

$$\begin{aligned} \dot{e}_1 &= (a - \hat{a})(e_2 - e_1) - k_1e_1 \\ \dot{e}_2 &= (c - \hat{c})e_2 - k_2e_2 \\ \dot{e}_3 &= -(b - \hat{b})e_3 - k_3e_3 \\ \dot{e}_4 &= -(d - \hat{d})e_1 - k_4e_4 \end{aligned} \tag{19}$$

Let us now define the parameter estimation error:

$$e_a = a - \hat{a},\ e_b = b - \hat{b},\ e_c = c - \hat{c},\ e_d = d - \hat{d} \tag{20}$$

Substituting Eq. 20 into 19, we obtain the error dynamics as:

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1) - k_1e_1 \\ \dot{e}_2 &= e_ce_2 - k_2e_2 \\ \dot{e}_3 &= -e_be_3 - k_3e_3 \\ \dot{e}_4 &= -e_de_1 - k_4e_4 \end{aligned} \tag{21}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. We consider the quadratic Lyapunov function defined:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2) \tag{22}$$

which is a positive definite function on R^8. We also note that:

$$\dot{e}_a = -\dot{\hat{a}},\ \dot{e}_b = -\dot{\hat{b}},\ \dot{e}_c = -\dot{\hat{c}},\ \dot{e}_d = -\dot{\hat{d}} \tag{23}$$

Differentiating Eq. 22 along the trajectories of Eq. 21 and noting Eq. 23, we find that:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 + e_a[e_1(e_2 - e_1) - \dot{\hat{a}}] + e_b[-e_3^2 - \dot{\hat{b}}] + e_c[e_2^2 - \dot{\hat{c}}] + e_d[-e_1e_4 - \dot{\hat{d}}] \tag{24}$$

In view of Eq. 11, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(e_2 - e_1) + k_5e_a \\ \dot{\hat{b}} &= -e_3^2 + k_6e_b \\ \dot{\hat{c}} &= e_2^2 + k_7e_c \\ \dot{\hat{d}} &= -e_1e_4 + k_8e_d \end{aligned} \tag{25}$$

where, k_5-k_8 are positive constants. Substituting Eq. 25 into 24, we obtain:

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 - k_5e_a^2 - k_6e_b^2 - k_7e_c^2 - k_8e_d^2 \tag{26}$$

which is a negative definite function on R^8. Thus by Lyapunov stability theory (Hahn, 1967), it is immediate that the anti-synchronization error e_i (1, 2, 3, 4) and the parameter estimation error e_a-e_d decay to zero exponentially with time. Hence, we have proved the following result.

Theorem 2: The identical uncertain hyperchaotic Chen systems (Eq. 14 and 15) are globally and exponentially anti-synchronized by the adaptive control law (Eq. 18) where the update law for the parameter estimates is given by Eq. 25 and k_i (i = 1,..., 8) are positive constants.

Numerical results: For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential Eq. 14 and 15 with the adaptive non-linear controller (Eq. 18). Researchers take $k_i = 2$ for i = 1, 2, ..., 8.

The parameters of the hyperchaotic Chen systems (Eq. 14 and 15) are chosen so that the systems are hyperchaotic, i.e.:

$$a = 35,\ b = 3,\ c = 21 \text{ and } d = 2$$

The initial values of the parameter estimates are taken as:

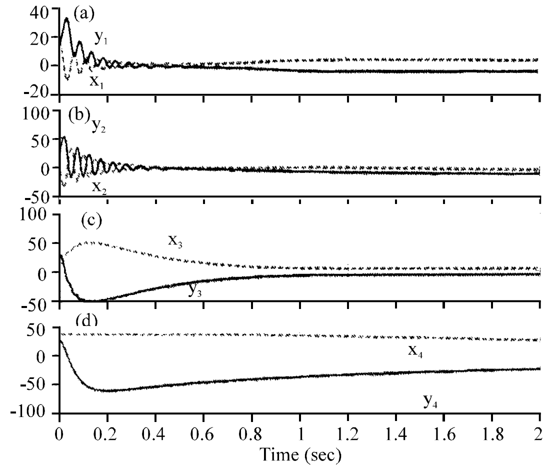

Fig. 5: Anti-synchronization of hyperchaotic Chen systems

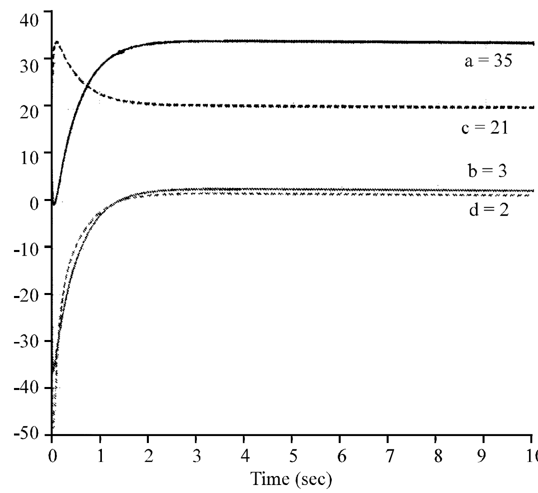

Fig. 6: Parameter estimates $\hat{a}(t)$-$\hat{d}(t)$

$$\hat{a}(0) = 10,\ \hat{b}(0) = 7,\ \hat{c}(0) = 20 \text{ and } \hat{d}(0) = 4$$

The initial values of the master system (Eq. 14) are chosen as:

$$x_1(0) = 21,\ x_2(0) = 8,\ x_3(0) = 15 \text{ and } x_4(0) = 34$$

The initial values of the slave system (Eq. 15) are chosen as:

$$y_1(0) = 15,\ y_2(0) = 10,\ y_3(0) = 30 \text{ and } y_4(0) = 24$$

Figure 5 shows anti-synchronization of the hyperchaotic Chen systems (Eq. 14 and 15). Figure 6 shows that the estimated values of the parameters, viz., $\hat{a}$-$\hat{d}$ converge to the system parameters a = 35, b = 3, c = 21 and d = 2, respectively.

ADAPTIVE ANTI-SYNCHRONIZATION OF HYPERCHAOTIC LORENZ AND HYPERCHAOTIC CHEN SYSTEMS

Theoretical results: In this study, researchers apply the Adaptive control method to derive new results for the anti-synchronization of uncertain hyperchaotic Lorenz system (Gao *et al*., 2007) and hyperchaotic Chen system (Li-Xin *et al*., 2010). Thus, the master system is described by the hyperchaotic Lorenz dynamic:

$$\begin{aligned}\dot{x}_1 &= \sigma(x_2 - x_1)\\ \dot{x}_2 &= \rho x_1 - x_2 - x_4 - x_1x_3\\ \dot{x}_3 &= x_1x_2 - \beta x_3\\ \dot{x}_4 &= r x_2 x_3\end{aligned} \tag{27}$$

where x_1-x_4 are the state variables and σ, β, ρ and r are unknown parameters of the system. The slave system is described by the controlled hyperchaotic Chen dynamics:

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + u_1\\ \dot{y}_2 &= 4y_1 - 10y_1y_3 + cy_2 + 4y_4 + u_2\\ \dot{y}_3 &= y_2^2 - by_3 + u_3\\ \dot{y}_4 &= -dy_1 + u_4\end{aligned} \tag{28}$$

where y_1-y_4 are the state variables, a-d are unknown parameters of the system and u_1-u_4 are the nonlinear controllers to be designed. The anti-synchronization error is defined as:

$$e_i = y_i + x_i,\quad (i = 1, 2, 3, 4) \tag{29}$$

A simple calculation gives the error dynamics as:

$$\begin{aligned}\dot{e}_1 &= a(y_2 - y_1) + \sigma(x_2 - x_1) + u_1\\ \dot{e}_2 &= 4y_1 - 10y_1y_3 + cy_2 + 4y_4 +\\ &\quad \rho x_1 - x_2 - x_4 - x_1x_3 + u_2\\ \dot{e}_3 &= y_2^2 - by_3 + x_1x_2 - \beta x_3 + u_3\\ \dot{e}_4 &= -dy_1 + rx_2x_3 + u_4\end{aligned} \tag{30}$$

Let us now define the adaptive functions $u_1(t)$-$u_4(t)$ as:

$$\begin{aligned}u_1(t) &= -\hat{a}(y_2 - y_1) - \hat{\sigma}(x_2 - x_1) - k_1e_1\\ u_2(t) &= -4y_1 + 10y_1y_3 - \hat{c}y_2 - 4y_4 -\\ &\quad \hat{\rho}x_1 + x_2 + x_4 + x_1x_3 + k_2e_2\\ u_3(t) &= -y_2^2 + \hat{b}y_3 - x_1x_2 - \hat{\beta}x_3 - k_3e_3\\ u_4(t) &= -\hat{d}y_1 - \hat{r}x_2x_3 - k_4e_4\end{aligned} \tag{31}$$

where, $\hat{\sigma}, \hat{\beta}, \hat{\rho}, \hat{r}$ and $\hat{a}$-$\hat{d}$ are estimates of σ, β, ρ and r, a-d, respectively and k_1-k_4 are positive constants. Substituting Eq. 31 into Eq. 30, the error dynamics simplifies to:

$$\begin{aligned}\dot{e}_1 &= (a - \hat{a})(y_2 - y_1) + (\sigma - \hat{\sigma})(x_2 - x_1) - k_1e_1\\ \dot{e}_2 &= (c - \hat{c})y_2 + (\rho - \hat{\rho})x_1 - k_2e_2\\ \dot{e}_3 &= -(b - \hat{b})y_3 - (\beta - \hat{\beta})x_3 - k_3e_3\\ \dot{e}_4 &= -(d - \hat{d})y_1 + (r - \hat{r})x_2x_3 - k_4e_4\end{aligned} \tag{32}$$

Let us now define the parameter estimation error as:

$$\begin{aligned}&e_\sigma = \sigma - \hat{\sigma},\ e_\beta = \beta - \hat{\beta},\ e_\rho = \rho - \hat{\rho}\\ &e_r = r - \hat{r},\ e_a = a - \hat{a},\ e_b = b - \hat{b}\\ &e_c = c - \hat{c},\ e_d = d - \hat{d}\end{aligned} \tag{33}$$

Substituting Eq. 33 into Eq. 32, we obtain the error dynamics as:

$$\begin{aligned}\dot{e}_1 &= e_a(y_2 - y_1) + e_\sigma(x_2 - x_1) - k_1e_1\\ \dot{e}_2 &= e_cy_2 + e_\rho x_1 - k_2e_2\\ \dot{e}_3 &= -e_by_3 - e_\beta x_3 - k_3e_3\\ \dot{e}_4 &= -e_dy_1 + e_rx_2x_3 - k_4e_4\end{aligned} \tag{34}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used. Researchers consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\begin{pmatrix} e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_\sigma^2 + \\ e_\beta^2 + e_\rho^2 + e_r^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\end{pmatrix} \tag{35}$$

which is a positive function on R^{12}. We also note that:

$$\begin{aligned}&\dot{e}_\sigma = -\dot{\hat{\sigma}},\ \dot{e}_\beta = -\dot{\hat{\beta}},\ \dot{e}_\rho = -\dot{\hat{\rho}},\ \dot{e}_r = -\dot{\hat{r}},\\ &\dot{e}_a = -\dot{\hat{a}},\ \dot{e}_b = -\dot{\hat{b}},\ \dot{e}_c = -\dot{\hat{c}},\ \dot{e}_d = -\dot{\hat{d}}\end{aligned} \tag{36}$$

Differentiating Eq. 22 along the trajectories of Eq. 21 and noting Eq. 23, we find that:

$$\begin{aligned}\dot{V} &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 +\\ &\quad e_\sigma[e_1(x_2 - x_1) - \dot{\hat{\sigma}}] + e_\beta[-e_3x_3 - \dot{\hat{\beta}}] +\\ &\quad e_\rho[e_2x_1 - \dot{\hat{\rho}}] + e_r[e_4x_2x_3 - \dot{\hat{r}}] +\\ &\quad e_a[e_1(y_2 - y_1) - \dot{\hat{a}}] + e_b[-e_3y_3 - \dot{\hat{b}}] +\\ &\quad e_c[e_2y_2 - \dot{\hat{c}}] + e_d[-e_4y_1 - \dot{\hat{d}}]\end{aligned} \tag{37}$$

In view of Eq. 37, the estimated parameters are updated by the following law:

$$\begin{aligned}
\dot{\hat{\sigma}} &= e_1(x_2 - x_1) + k_5 e_\sigma \\
\dot{\hat{\beta}} &= -e_3 x_3 + k_6 e_\beta \\
\dot{\hat{\rho}} &= e_2 x_1 + k_7 e_\rho \\
\dot{\hat{r}} &= e_4 x_2 x_3 + k_8 e_r \\
\dot{\hat{a}} &= e_1(y_2 - y_1) + k_9 e_a \\
\dot{\hat{b}} &= -e_3 y_3 + k_{10} e_b \\
\dot{\hat{c}} &= e_2 y_2 + k_{11} e_c \\
\dot{\hat{d}} &= -e_4 y_1 + k_{12} e_d
\end{aligned} \tag{38}$$

where, k_5-k_8 are positive constants. Substituting Eq. 38 into Eq. 37, we obtain:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_\sigma^2 - k_6 e_\beta^2 - k_7 e_\rho^2 - k_8 e_r^2 - k_9 e_a^2 - k_{10} e_b^2 - k_{11} e_c^2 - k_{12} e_d^2 \tag{39}$$

which is a negative definite function on R^{12}. Thus by Lyapunov stability theory (Hahn, 1967), it is immediate that the anti-synchronization error e_i (1, 2, 3, 4) and the parameter estimation error decay to zero exponentially with time. Hence, we have proved the following result.

Theorem 3: The hyperchaotic Lorenz system (Eq. 27) and hyperchaotic Chen system (Eq. 28) with unknown parameters are globally and exponentially anti-synchronized by the adaptive control law (Eq. 31) where the update law for the parameter estimates is given by Eq. 38 and k_i (i = 1, ..., 12) are positive constants.

Numerical results: For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the two systems of differential Eq. 27 and 28 with the adaptive non-linear controller (Eq. 31). Researchers take:

$$k_i = 2 \quad \text{for} \quad I = 1, 2, ..., 12$$

The parameters of the hyperchaotic Lorenz system (Eq. 27) and hyperchaotic Chen system (Eq. 28) are chosen so that the systems are hyperchaotic, i.e.:

$$\sigma = 10,\ \beta = 8/3,\ \rho = 28,\ r = 0.1$$

$$a = 35,\ b = 3,\quad c = 21,\ d = 2$$

The initial values of the parameter estimates are:

$$\hat{\sigma}(0) = 12, \hat{\beta}(0) = 10, \hat{\rho}(0) = 8 \text{ and } \hat{r}(0) = 2$$

$$\hat{a}(0) = 6, \hat{b}(0) = 5, \hat{c}(0) = 4 \text{ and } \hat{d}(0) = 11$$

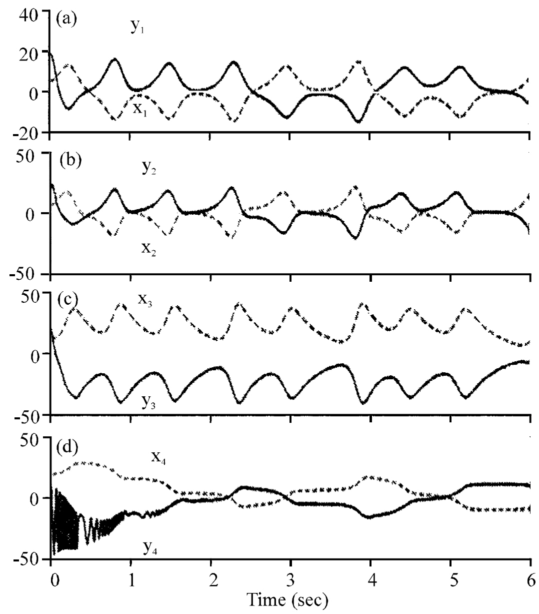

Fig. 7: Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems

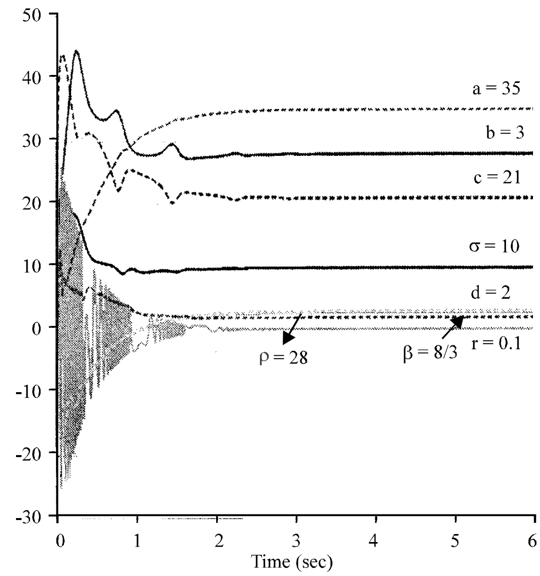

Fig. 8: Parameter estimates $\hat{\sigma}(t), \hat{\beta}(t), \hat{\rho}(t), \hat{r}(t), \hat{a}(t)$-$\hat{d}(t)$

The initial values of the master system (Eq. 27) are:

$$x_1(0) = 6,\ x_2(0) = 4,\ x_3(0) = 12 \text{ and } x_4(0) = 18$$

The initial values of the slave system (Eq. 28) are:

$$y_1(0) = 21,\ y_2(0) = 18,\ y_3(0) = 20 \text{ and } y_4(0) = 12$$

Figure 7 shows anti-synchronization of the hyperchaotic Chen systems (Eq. 27 and 28). Figure 8 shows that the estimated values of the parameters, $\hat{\sigma}, \hat{\beta}, \hat{\rho}, \hat{r}$ and $\hat{a}$-$\hat{d}$ converge to the system parameters σ = 10, β = 8/3, ρ = 28, r = 0.1, a = 35, b = 3, c = 21 and d = 2, respectively.

CONCLUSION

In this study, researchers have applied Adaptive control method for the global chaos anti-synchronization of identical hyperchaotic Lorenz systems in 2007, identical hyperchaotic Chen systems in 2010 and non-identical hyperchaotic Lorenz and Chen systems with unknown parameters. The adaptive anti-synchronization results derived in this study are established using Lyapunov stability theory. Since, the Lyapunov exponents are not required for these calculations, the Adaptive non-linear control method is very effective and convenient to achieve global chaos anti-synchronization for the uncertain hyperchaotic systems discussed in this study. Numerical simulations are also shown for the anti-synchronization of identical and non-identical uncertain hyperchaotic Lorenz and hyperchaotic Chen systems to demonstrate the effectiveness of the adaptive anti-synchronization schemes derived in this study.

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