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Comparison of a Triple Inverted Pendulum Stabilization using Optimal Control Technique

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Key words: Inverted pendulum, linear quadratic regulator, pole placement, strongly, controllers

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Abstract: In this study, modelling design and analysis of a triple inverted pendulum have been done using MATLAB/Script toolbox. Since, a triple inverted pendulum is highly nonlinear, strongly unstable without using feedback control system. In this study, an optimal control method means a linear quadratic regulator and pole placement controllers are used to stabilize the triple inverted pendulum upside. The impulse response simulation of the open loop system shows us that the pendulum is unstable. The comparison of the closed loop impulse response simulation of the pendulum with LQR and pole placement controllers results that both controllers have stabilized the system but the pendulum with LQR controllers have a high overshoot with long settling time than the pendulum with pole placement controller. Finally, the comparison results prove that the pendulum with pole placement controller improve the stability of the system.

INTRODUCTION

An inverted pendulum is a pendulum that has its center of mass above its pivot point. It is unstable and without additional assist will fall over. It may be suspended stably in this inverted position by means of the usage of a feedback control system to reveal the angle of the pole and flow the pivot factor horizontally returned beneath the center of mass while it begins to fall over, retaining it balanced[1]. The inverted pendulum is a classic problem in dynamics and manage idea and is used as a benchmark for testing control techniques. An inverted pendulum is inherently unstable, and have to be actively balanced with a view to stay upright; this could be accomplished either by applying a torque at the pivot factor with the aid of transferring the pivot point horizontally as a part of a feedback system, changing the state of rotation of a mass installed at the pendulum on an axis parallel to the pivot axis and thereby generating an internal torque at the pendulum or with the aid of oscillating the pivot factor vertically[2,3]. In order to stabilize a pendulum in this inverted position, a feedback control system may be used which monitors the pendulum's attitude and actions the position of the pivot point sideways while the pendulum starts off evolved to fall over, to hold it balanced.

MATERIALS AND METHODS

Mathematical modeling: The pendulum consists of three arms that are hinged by ball bearings and can rotate in the vertical plane. The torques T1 and T2 are the inputs to the

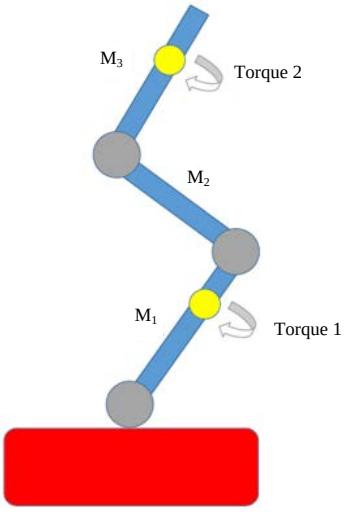

Fig. 1: The triple pendulum

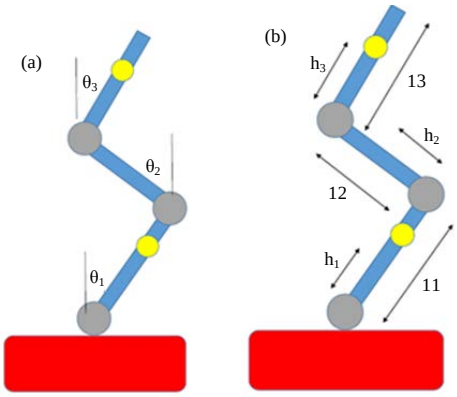

Fig. 2(a, b): System configuration

pendulum with the middle hinge made free for rotation. By controlling the angles of the arms around specified values, the pendulum can be stabilized inversely with the desired angle attitudes. The triple inverted pendulum is shown in Fig. 1.

Let Ti denote the angle of the ith arm measured from the vertical axis as shown in Fig. 2. The mathematical modelling of the triple inverted pendulum is derived under the assumption that each arm is a rigid body. Lagrange differential equations is the method used to construct the triple pendulum with a nonlinear vector-matrix differential (Eq. 1) of the form:

$$M(\theta)\ddot{\theta}_i+N\dot{\theta}_i+q_i=GT_j \quad i=1,2,3 \quad j=1,2 \tag{1}$$

Where:

$$M(\theta)=\begin{pmatrix} J_1 & l_1M_2\cos(\theta_1-\theta_2) & l_1M_3\cos(\theta_1-\theta_3) \\ l_1M_2\cos(\theta_1-\theta_2) & J_2 & l_2M_3\cos(\theta_2-\theta_3) \\ l_1M_3\cos(\theta_1-\theta_3) & l_2M_3\cos(\theta_2-\theta_3) & J_3 \end{pmatrix} \tag{2}$$

Where:

$$M_1=m_1h_1+m_2l_1+m_3l_1 \qquad J_1=I_1+m_1h_1^2+m_2l_1^2+m_3l_1^2$$
$$M_2=m_2h_2+m_3l_2 \qquad J_2=I_2+{}_2\,h_2^2+m_3l_2^2$$
$$M_3=m_3h_3 \quad \text{and} \quad J_3=I_3+m_3h_3^2$$

The N matrix become:

$$N=\begin{pmatrix} C_1+C_2 & -C_2 & 0 \\ -C_2 & C_2+C_3 & -C_3 \\ 0 & -C_3 & C_3 \end{pmatrix} \tag{3}$$

And the q matrix and G matrix are:

$$q_1=l_1M_2\sin(\theta_1-\theta_2)\dot{\theta}_2^2+l_1M_3\sin(\theta_1-\theta_3)\dot{\theta}_3^2-M_1g\sin(\theta_1)$$
$$q_2=l_1M_2\sin(\theta_1-\theta_2)\dot{\theta}_1^2+l_2M_3\sin(\theta_2-\theta_3)\dot{\theta}_3^2-M_2g\sin(\theta_2)$$
$$q_3=l_1M_3\sin(\theta_1-\theta_3)(\dot{\theta}_1^2-2\dot{\theta}_1\dot{\theta}_3)+$$
$$l_2M_3\sin(\theta_2-\theta_3)(\dot{\theta}_2^2-2\dot{\theta}_2\dot{\theta}_3)-M_3g\sin(\theta_3)$$

$$G=\begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}$$

After linearization of Eq. 2 under the assumptions of small deviations of the pendulum from the vertical position and of small velocities, one obtains the following Eq. 4:

$$M\ddot{\theta}_i+N\dot{\theta}_i+P\theta_i=GT_{m_j} \quad i=1,2,3 \quad j=1,2 \tag{4}$$

Where:

$$M=\begin{pmatrix} J_1 & l_1M_2 & l_1M_3 \\ l_1M_2 & J_2 & l_2M_3 \\ l_1M_3 & l_2M_3 & J_3 \end{pmatrix} \tag{5}$$

And:

$$P=\begin{pmatrix} M_1g & 0 & 0 \\ 0 & -M_2g & 0 \\ 0 & 0 & -M_3g \end{pmatrix} \tag{6}$$

The block-diagram of the pendulum system is shown in Fig. 3 and the nominal values of the

Table 1: The description of the system

Symbols	Descriptions
l_i	Length of the ith arm
h_i	The distance from the bottom to the centre of gravity of the ith arm
m_i	Mass of the ith arm
θ_i	Angle of the ith arm from vertical line
C_i	Coefficient of viscous friction of the ith hinge
I_i	Moment of inertia of the i-th arm around the centre of gravity
T_j	Control torque of the jth hinge

Table 2: Nominal values of the parameters

Symbols	Values
h_1	0.45 m
h_2	0.2 m
h_3	0.3 m
l_1	0.5 m
l_2	0.4 m
m_1	3.5 kg
m_2	2 kg
m_3	2.25 kg
I_1	0.55 kg m^2
I_2	0.12 kg m^2
I_3	0.65 kg m^2
C_1	0.07 N m sec
C_2	0.03 N m sec
C_3	0.009 N m sec

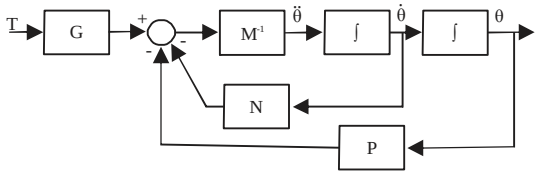

Fig. 3: Block-diagram of the pendulum system

parameters are given in Table 1 and 2. The state space representation of the triple inverted pendulum becomes:

$$\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{pmatrix}=\begin{pmatrix}-57.75 & -14.42 & -15.82 & -16.26 & -5.5 & -7\\ -25.3 & -43 & -12.7 & -11 & -13.6 & -5\\ 178 & 88.6 & 55.6 & 57.3 & 30.2 & 25\\ 22.9 & -10 & -0.14 & -0.08 & 0.04 & -0.008\\ -26 & 37.5 & -3.6 & 0.15 & -0.13 & 0.033\\ -0.83 & -8.2 & 9.25 & -0.01 & 0.04 & -0.02\end{pmatrix}\begin{pmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{pmatrix}+$$

$$\begin{pmatrix}1.559 & -0.8\\ -4 & 3.8\\ 0.65 & -2.4\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}T_1\\ T_2\end{pmatrix}y=\begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{pmatrix}$$

The proposed controllers design

LQR controller design: The principle of most reliable optimal control is involved with working a dynamic system at minimum cost. The case wherein the system dynamics are described via. a fixed of linear differential

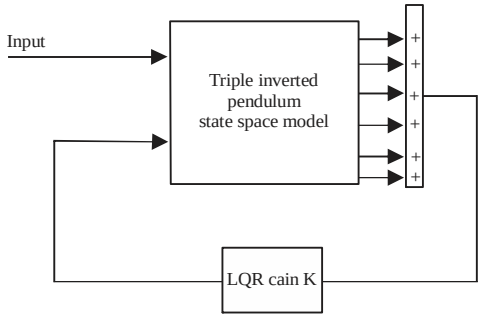

Fig. 4: Block diagram of the triple inverted pendulum with LQR controller

equations and the cost is defined through a quadratic function is referred to as the LQ problem. One of the primary outcomes within the theory is that the solution is furnished with the aid of the Linear Quadratic Regulator (LQR)[4]. The block diagram of the triple inverted pendulum with LQR controller is shown below in Fig. 4. In this study, the value of Q and R is chosen as:

$$Q=10\begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}\text{ and }R=\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$$

The value of obtained feedback gain matrix K of LQR is given by:

$$K=\begin{pmatrix}87.4053 & 32.8355 & 25.6454 & 27.1508 & 11.2981 & 11.1817\\ 97.7657 & 45.7910 & 30.0834 & 31.2118 & 15.6479 & 12.9896\end{pmatrix}$$

Pole placement controller design: Pole placement is a way employed in feedback control system principle to region the closed-loop poles of a plant in pre-decided locations in the s-plane. Placing poles is proper because the region of the poles corresponds immediately to the eigenvalues of the system which control the traits of the reaction of the system[5, 6]. The system ought to be considered controllable on the way to put into effect this technique. The block diagram of the triple inverted pendulum with pole placement controller is shown in Fig. 5. The state equations for the closed-loop system of Fig. 5 can be written by inspection as:

$$\begin{aligned}\dot{x}&=Ax+Bu=Ax+B(-Kx)=(A-BK)x\\ y&=Cx\end{aligned}\qquad(7)$$

The poles for this system is chosen as:

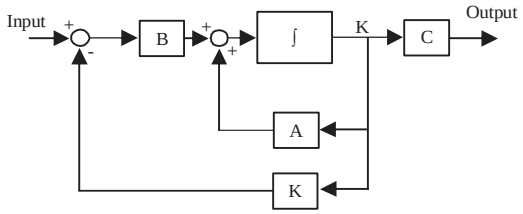

Fig. 5: Block diagram of the triple inverted pendulum with pole placement controller

$$P = [-1, -2, -3, -4, -5, -6]$$

Solving using MATLAB the robust pole placement algorithm gain will be:

$$K = \begin{bmatrix} 19329 & 8885 & 7472 & 11601 & 5861 & 6699 \\ 23483 & 10820 & 9086 & 14362 & 7268 & 8307 \end{bmatrix}$$

RESULTS AND DISCUSSION

Controllability and observability of the pendulum: A system (state space representation) is controllable iff the controllable matrix C = [B AB A2B, ..., An-1B] has rank n where n is the number of degrees of freedom of the system.

In our system, the controllable matrix C = [B AB A2B A3B A4B A5B] has rank 6 which the degree of freedom of the system. So, the system is controllable. A system (state space representation) is Observable iff the Observable matrix D = [C CA CA2….CAn-1] T has a full rank n. In our system, the Observable matrix D = [C CA CA2 CA3 CA4 CA5] T has a full rank of 6. So, the system is Observable.

Open loop impulse response of the triple inverted pendulum: The open loop simulation for a 1 Nm impulse input of torque 1 for angular displacement 1-3 and for angular velocity 1-3 is shown in Fig. 6-11 and for torque 2 input the angular displacement 1-3 and for angular velocity 1-3 is shown in Fig. 12-17, respectively. As we seen from the Figures above the angular displacements and the angular velocities are unstable[7].

Comparison of the triple inverted pendulum with LQR and pole placement controllers for impulse input signal: The comparison of the triple inverted pendulum with LQR and pole placement controller for a 1 Nm impulse input of torque 1 for angular displacement 1-3 and for angular velocity 1-3 is shown in Fig. 18-23 and for torque 2 input the angular displacement 1-3 and for angular velocity 1-3 is shown in Fig. 24-29, respectively.

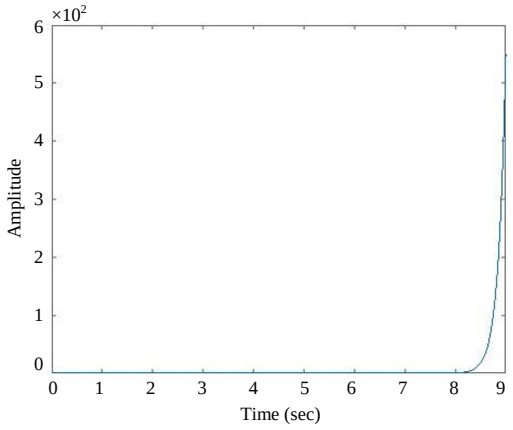

Fig. 6: Response of θ_1; for a 1 Nm impulse input of T1

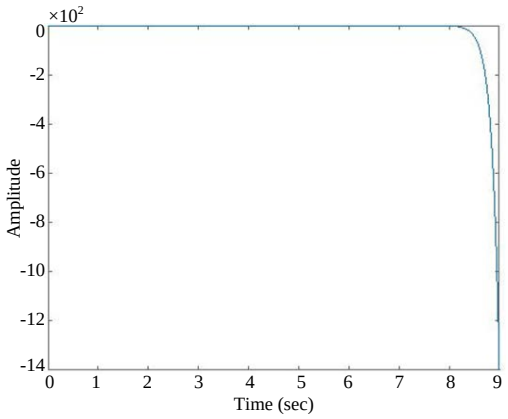

Fig. 7: Response of θ_2; θ_1 for a 1 Nm impulse input of T1

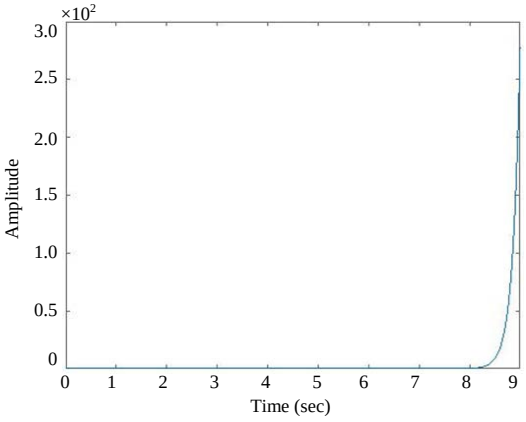

Fig. 8: Response of θ_3; θ_1 for a 1 Nm impulse input of T1

As we seen from Figure 18, 19 and 20, for the impulse signal the angles starts to increase and returns to

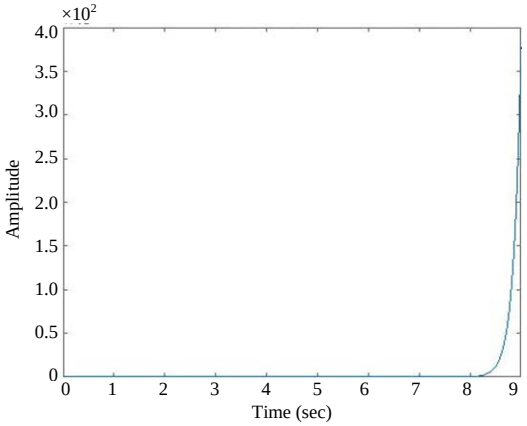

Fig. 9: Response of θ_1; θ_1 for a 1 Nm impulse input of T1

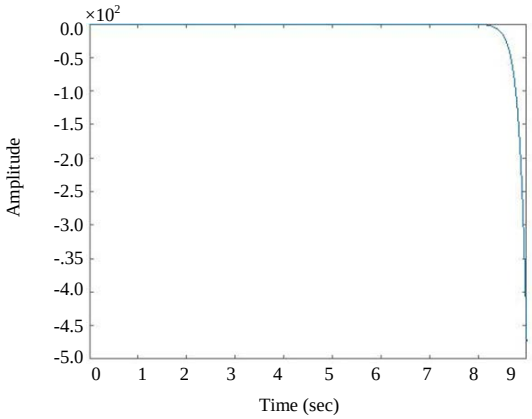

Fig. 12: Response of θ_1; θ_1 for a 1 Nm impulse input of T2

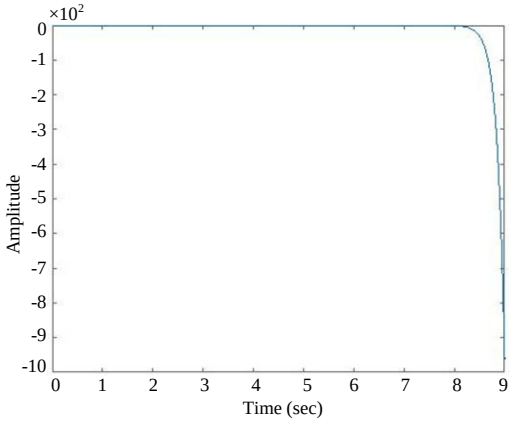

Fig. 10: Response of θ_2; θ_1 for a 1 Nm impulse input of T1

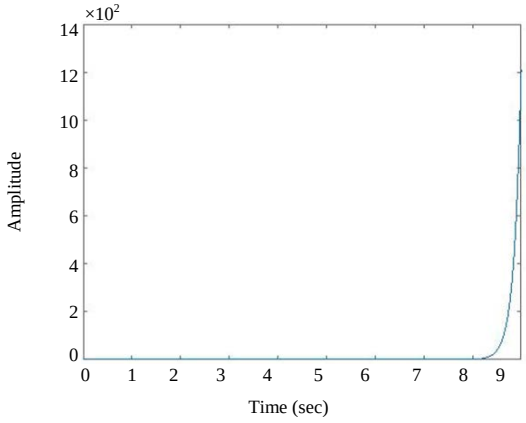

Fig. 13: Response of θ_2; θ_1 for a 1 Nm impulse input of T2

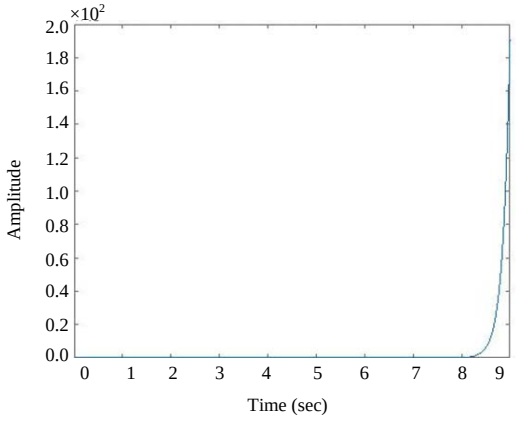

Fig. 11: Response of θ_3; θ_1 for a 1 Nm impulse input of T1

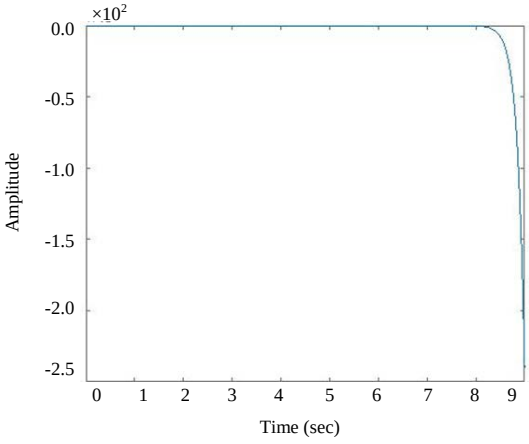

Fig. 14: Response of θ_3; θ_1 for a 1 Nm impulse input of T2

0° for the two controllers but the pendulum with LQR controller has a high overshoot with more settling time

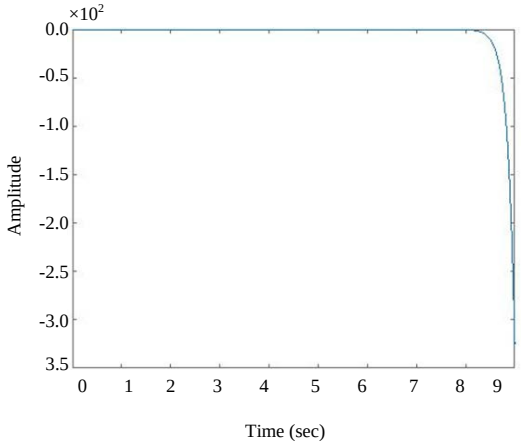

Fig. 15: Response of θ_1; θ_1 for a 1 Nm impulse input of T2

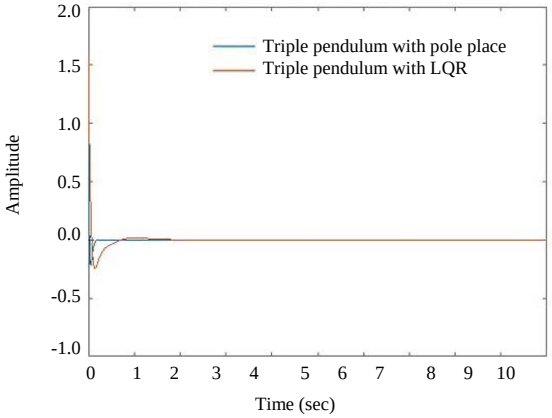

Fig. 18: Response of θ_1; θ_1 for a 1 Nm impulse input of T1

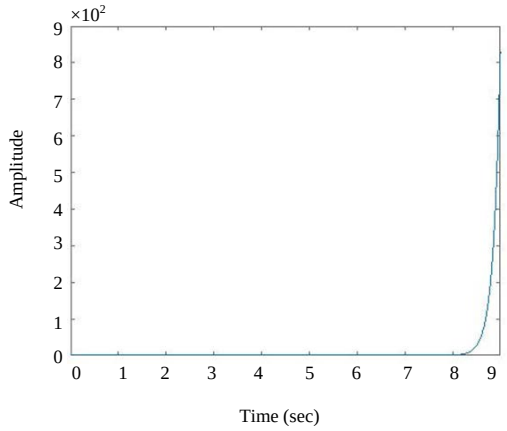

Fig. 16: Response of θ_2; θ_1 for a 1 Nm impulse input of T2

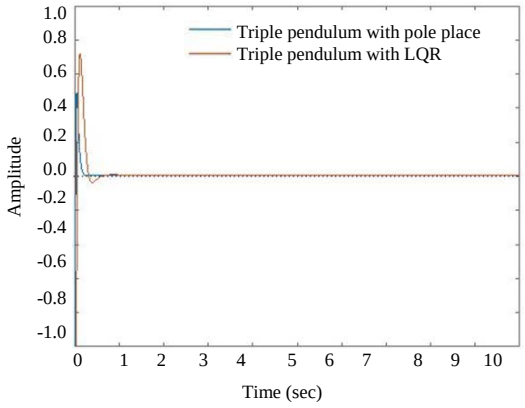

Fig. 19: Response of θ_2; θ_1 for a 1 Nm impulse input of T1

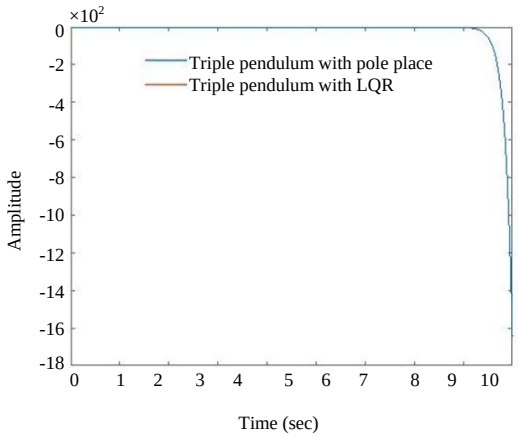

Fig. 17: Response of θ_3; θ_1 for a 1 Nm impulse input of T2

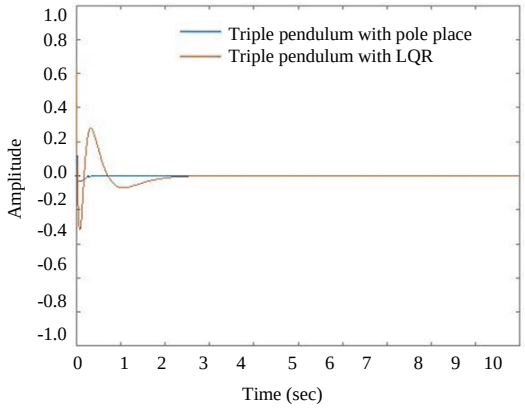

Fig. 20: Response of θ_3; θ_1 for a 1 Nm impulse input of T1

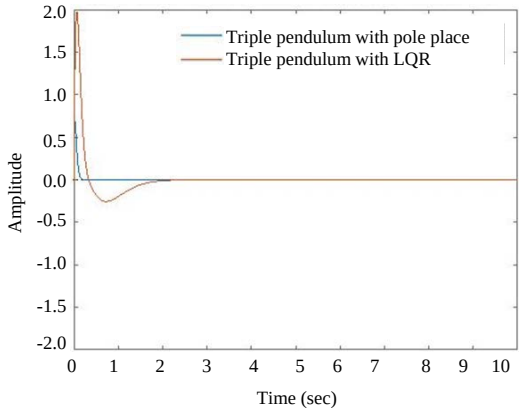

Fig. 21: Response of θ_1; θ_1 for a 1 Nm impulse input of T1

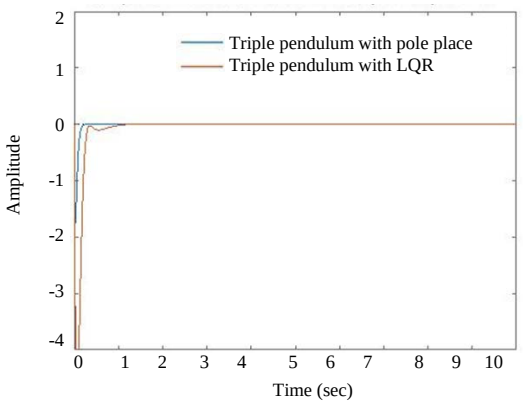

Fig. 22: Response of θ_2; θ_1 for a 1 Nm impulse input of T1

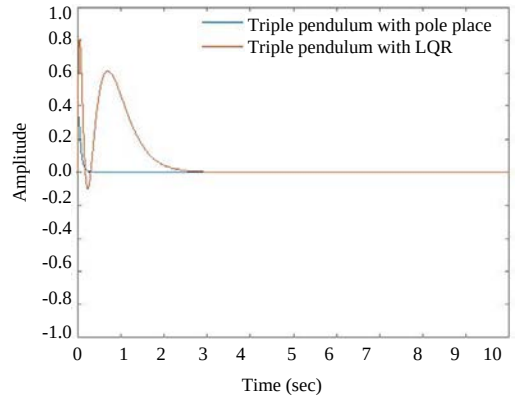

Fig. 23: Response of θ_3; θ_1 for a 1 Nm impulse input of T1

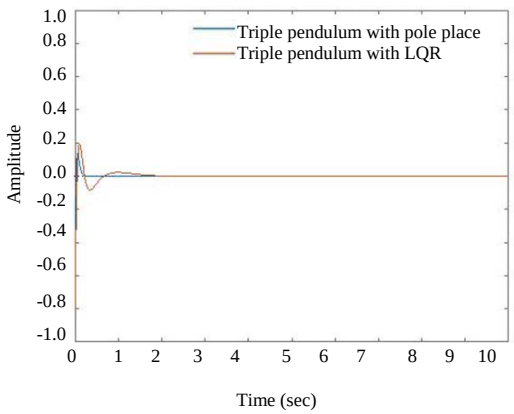

Fig. 24: Response of θ_1; θ_1 for a 1 Nm impulse input of T2

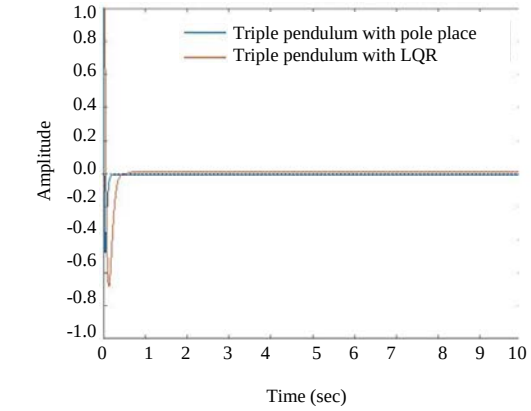

Fig. 25: Response of θ_2; θ_1 for a 1 Nm impulse input of T2

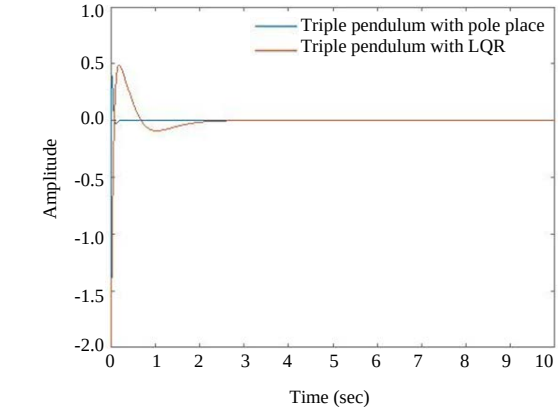

Fig. 26: Response of T θ_3; θ_1 for a 1 Nm impulse input of T2

than the pendulum with pole placement controller. Figure 21-23 for the impulse signal the angular velocities starts to increase and returns to zero for the two controllers but the pendulum with LQR controller

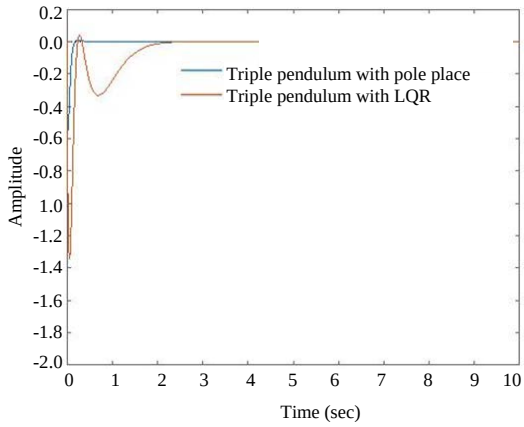

Fig. 27: Response of θ_1 Dot; θ_1 for a 1 Nm impulse input of T2

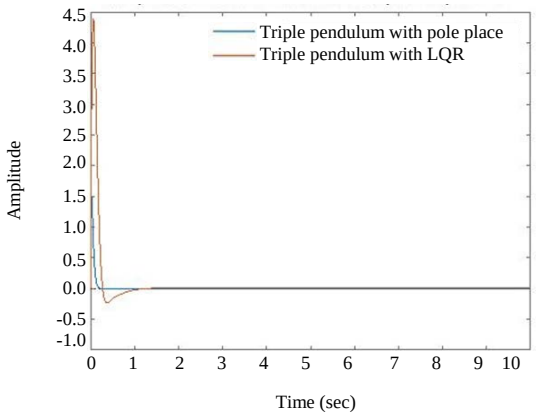

Fig. 28: Response of θ_2 Dot; θ_1 for a 1 Nm impulse input of T2

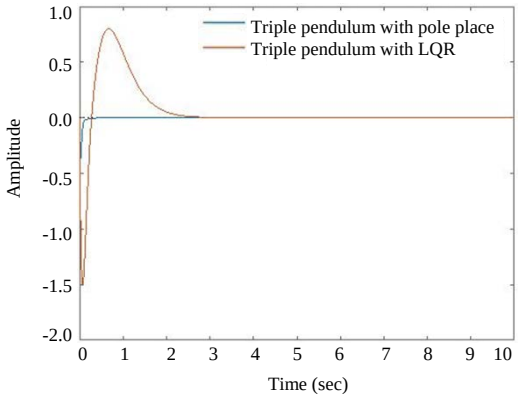

Fig. 29: Response of θ_3 Dot; θ_1 for a 1 Nm impulse input of T2

has a high overshoot with more settling time than the pendulum with pole placement controller. As we seen from Fig. 24-26, for the impulse signal the angles starts to increase and returns to zero degree for the two controllers but the pendulum with LQR controller has a high overshoot with more settling time than the pendulum with pole placement controller.

As we seen from Fig. 27-29, for the impulse signal the angular velocities starts to increase and returns to zero for the two controllers but the pendulum with LQR controller has a high overshoot with more settling time than the pendulum with pole placement controller.

CONCLUSION

In this study, stabilization of the triple inverted pendulum with LQR and pole placement controller have been analyzed simulated and compared suceesfully. The open loop simulation prove that the system is not stable without feedback control system. Comparison of the proposed controllers for an impulse input have been done and the system with pole placement controller improves the stability of the system.

REFERENCES

01. Tang, Y., D. Zhou and W. Jiang, 2016. A new fuzzy-evidential controller for stabilization of the planar inverted pendulum system. PloS One, Vol. 11, No. 8. 10.1371/journal.pone.0160416.
02. Jibril, M., 2020. Robust control theory based performance investigation of an inverted pendulum system using simulink. Int. J. Adv. Res. Innovative Ideas Educ., 6: 808-814.
03. Huang, X., F. Wen and Z. Wei, 2018. Optimization of triple inverted pendulum control process based on motion vision. EURASIP J. Image Video Process., Vol. 2018. 10.1186/s13640-018-0294-6.
04. Chen, W. and N. Theodomile, 2016. Simulation of a triple inverted pendulum based on fuzzy control. World J. Eng. Technol., 4: 267-272.
05. Pristovani, R.D., D.R. Sanggar and P. Dadet, 2018. Implementation of push recovery strategy using triple linear inverted pendulum model in T-flow humanoid robot. IOP. J. Phys. Conf. Ser., 1007: 1-13.
06. Dan, Y., P. Xu, Z. Tan and Z. Li, 2015. Multi-mode control based on HSIC for double pendulum robot. J. Vibroengineering, 17: 3683-3692.
07. Rohan, A., M. Rabah, K.H. Nam and S.H. Kim, 2018. Design of fuzzy logic based controller for gyroscopic inverted pendulum system. Int. J. Fuzzy Logic Intell. Syst., 18: 58-64.