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Stability Control of a Rotational Inverted Pendulum Using Augmentations with Weighting Functions Based Robust Control System

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Abstract: This study mainly analyzes the design and control of the rotational inverted pendulum and presents a state space expression. Since the system is highly unstable a feedback control system is used. Augmentations with weighting functions based mixed sensitivity and H2 optimal control methods are used to make the system stable for uprise position. The rotational inverted pendulum have been simulated and compared with the proposed controllers and a promising results have been analyzed sussesfuly.

INTRODUCTION

Inverted pendulum system is a typical multivariable nonlinear strong coupling unstable system. In order to control this system, the theory of controlling the system stability, controllability, robustness and tracking the system. Its control method are widly used in military, industry, robot and in the field of general industrial process control such as the balance in the process of the robot control and satellite attitude control in flight, Inverted pendulum system which is more ideal experimental apparatus control theory is often used to test the effect of the control strategy[1]. This article aims at single stage of nonlinear rotational inverted pendulum control problem, the design has realized the single inverted pendulum with robust control based theory.

MATERIALS AND METHODS

Mathematical modelling of the rotational pendulum: Figure 1 shows the structural design of the rotational pendulum.

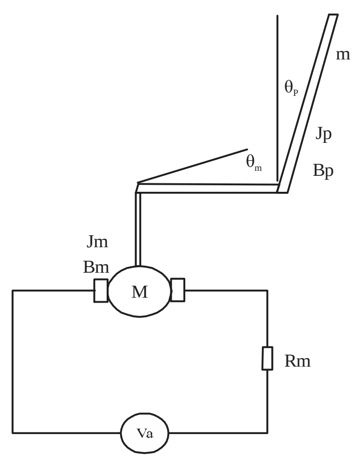

Fig. 1: Rotational pendulum

For the DC motor

Assume: The stator current is constant therefore the magnetic flux is constant:

$$\phi(t) = K_f I_f \quad (1)$$

The motor torque is proportional to the armature current and the flux:

$$T_m(t) = K_m i_a(t)\phi = K_1 i_a \quad (2)$$

The voltage vb is proportional to the angular speed of the motor:

$$V_b(t) = K_2 \dot{\theta}_m(t) \quad (3)$$

Applying KVL to the motor circuit neglecting the coil inductance:

$$R_m i_a(t) + V_b(t) = V_a(t) \quad (4)$$

Substituting Eq. 3 and 4 yields:

$$R_m i_a(t) + K_2 \dot{\theta}_m(t) = V_a(t) \quad (5)$$

The rotational equation of the motor is:

$$J_m \frac{d^2\theta_m(t)}{dt^2} + B_m \frac{d\theta_m(t)}{dt} = T_m(t) = K_1 i_a(t) \quad (6)$$

$$\frac{J_m}{K_1}\frac{d^2\theta_m(t)}{dt^2} + \frac{B_m}{K_1}\frac{d\theta_m(t)}{dt} = i_a(t) \quad (7)$$

Substitution Eq. 5-7:

$$R_m\left(\frac{J_m}{K_1}\frac{d^2\theta_m(t)}{dt^2} + \frac{B_m}{K_1}\frac{d\theta_m(t)}{dt}\right) + K_2\frac{d\theta_m(t)}{dt} = V_a(t) \quad (8)$$

Rearranging Eq. 8 becomes:

$$\frac{d^2\theta_m(t)}{dt^2} = -a_m \frac{d\theta_m(t)}{dt} + b_m V_a(t) \quad (9)$$

Where:

$$a_m = \left(\frac{K_1}{R_m J_m}\right)\left(\frac{B_m}{K_1} + K_2\right)$$

$$b_m = \left(\frac{K_1}{R_m J_m}\right)$$

For the pendulum: The rotational equation of the pendulum is:

$$J_P \frac{d^2\theta_P(t)}{dt^2} + B_P \frac{d\theta_P(t)}{dt} = T_T(t) \quad (10)$$

Where:

$$T_T(t) = T_{mP}(t) + T_P(t)$$

Where:
$T_{mP}(t)$ = Torque of the motor based on the pendulum
$T_P(t)$ = Torque of the pendulum

Torque of the motor based on the pendulum is:

$$T_{mP}(t) = K_3 \frac{d^2\theta_m(t)}{dt^2} \quad (11)$$

Note that the sign of K3 depends on whether the pendulum is in the inverted or non-inverted position. The torque of the pendulum is[2, 3]:

$$T_P(t) = mgl\sin(\theta_P) \quad (12)$$

Substituting Eq. 11 and 12 in to Eq. 10 and rearranging yields:

$$\frac{d^2\theta_P(t)}{dt^2} = -\frac{B_P}{J_P}\frac{d\theta_P(t)}{dt} + \frac{mgl\sin(\theta_P)}{J_P} + K_3\frac{d^2\theta_m(t)}{dt^2} \quad (13)$$

Linearization of these equations about the vertical position (i.e., = 0), results in the linear, time-invariant state variable model:

$$\begin{pmatrix}\dot{\theta}_m\\ \ddot{\theta}_m\\ \dot{\theta}_P\\ \ddot{\theta}_P\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0\\ 0 & -a_m & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\frac{K_3 a_m}{J_P} & \frac{mgl}{J_P} & -\frac{B_P}{J_P}\end{pmatrix}\begin{pmatrix}\theta_m\\ \dot{\theta}_m\\ \theta_P\\ \dot{\theta}_P\end{pmatrix} + \begin{pmatrix}0\\ b_m\\ 0\\ \frac{K_3 b_m}{J_P}\end{pmatrix} V_a$$

$$y = \begin{pmatrix}0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\theta_m\\ \dot{\theta}_m\\ \theta_P\\ \dot{\theta}_P\end{pmatrix}$$

The values of the parameters of the system is shown in Table 1. The state space representation becomes:

$$\begin{pmatrix}\dot{\theta}_m\\ \ddot{\theta}_m\\ \dot{\theta}_P\\ \ddot{\theta}_P\end{pmatrix} = \begin{pmatrix}0 & 1 & 0 & 0\\ 0 & -33.04 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 49.3 & 73.41 & -2.29\end{pmatrix}\begin{pmatrix}\theta_m\\ \dot{\theta}_m\\ \theta_P\\ \dot{\theta}_P\end{pmatrix} + \begin{pmatrix}0\\ 74.89\\ 0\\ -111.74\end{pmatrix} V_a$$

$$y = \begin{pmatrix}0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\theta_m\\ \dot{\theta}_m\\ \theta_P\\ \dot{\theta}_P\end{pmatrix}$$

And the transfer function of the system becomes:

Proposed controllers design

Augmentations of the model with weighting functions: In this study, we will focus on the weighted control structure shown in Fig. 2 where W1(s), W2(s) and W3(s) are weighting functions or weighting filters.

$$G(s) = \frac{\Theta_P(s)}{V_a(s)} = \frac{-111.7\,s + 0.1874}{s^3 + 35.33s^2 + 2.252s - 2425}$$

We assume that G(s), $W_1(s)$ and $W_3(s)$ G(s) are all proper, i.e., they are bounded when s→∞. It can be seen that the weighting function W3(s) is not required to be proper. One may wonder why we need to use three weighting functions. First, we note that the weighting functions are, respectively, for the three signals, namely, the error, the input, and the output[4, 5]. In the two-port state space structure, the output vector y1 = [y1a, y1b, y1c] T is not used directly to construct the control signal vector u2. We should understand that y1 is actually for the control system performance measurement. So, it is not strange to include the filtered "input signal" u(t) in the "output signal" y1 because one may need to measure the control energy to assess whether the designed controller is good or not. Clearly, Fig. 2 represents a more general picture of optimal and robust control systems. We can design an H2 optimal and mixed sensitivity controllers by using the idea of the augmented state space model. The weighting function $W_1(s)$, $W_2(s)$ and $W_3(s)$ are chosen as:

$$W_1(s) = \frac{s+3}{5s+1} \quad W_2(s) = \frac{s+5}{10s+15} \quad W_3(s) = \frac{s+2}{s+25}$$

The H2 optimal controller become:

Table 1: System parameters

Parameters	Symbols	Values
Motor torque constant	K_1	22.5 Nm/A
Motor speed constant	K_2	0.43 Vs^{-2} rad
Armature resistance	R_m	10 Ω
Motor damping friction	B_m	0.05 Nms/rad
Motor inertia	J_m	0.03 Nms^2
Pendulum inertia	J_P	0.0013 Nms^2
Pendulum damping friction	B_P	0.003 Nms/rad
Torque constant	K_3	0.0019412
Mass of the pendulum	m	0.086184 kg
Length of the pendulum	l	0.113 m
Acceleration due to gravity	g	9.8 m sec^{-2}

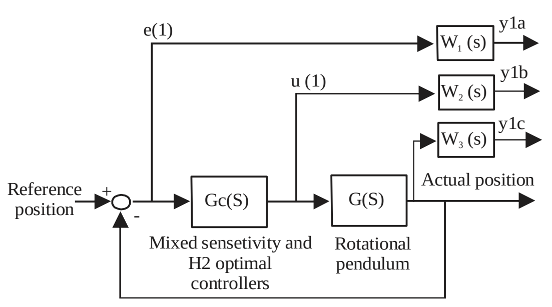

Fig. 2: Weighted control structure with the proposed controllers

$$G_{cH_2} = \frac{-325.4s^5 - 2.23e04s^4 - 4.691e05s^3 - 2.952e06s^2 - 1.894e06s - 2.662e05}{s^6 + 100.8s^5 + 4132s^4 + 4.724e04s^3 - 1.72e05s^2 - 3.395e05s - 6.066e04}$$

The mixed sensitivity controller become:

$$G_{cMix} = \frac{-736.2s^5 - 5.031e04s^4 - 1.052e06s^3 - 6.473e06s^2 - 3.024e06s - 2.179e04}{s^6 + 119s^5 + 5830s^4 + 6.344e04s^3 - 6.39e05s^2 - 1.197e06s - 2.133e05}$$

RESULTS AND DISCUSSION

Open loop response of the rotational pendulum: The open loop response for an impulse input of the rotational pendulum is shown in Fig. 3. The open loop response for a step input of the rotational pendulum is shown in Fig. 4. The simulation results of the open loop system shows that the rotational pendulum is unstable so the need of feedback control system is essential.

Comparison of the rotational pendulum with mixed sensitivity and H2 optimal controllers for an impulse input voltage: The simulation result of the rotational pendulum with mixed sensitivity and H2 optimal controllers for an impulse input voltage signal is shown in Fig. 5. The simulation result shows that the rotational pendulum with H2 optimal controller improve the settling time and the overshoot and the angular position returns to zero means the rotational pendulum is in upward stable position.

Comparison of the rotational pendulum with mixed sensitivity and H2 optimal controllers for a step input voltage: The simulation result of the rotational

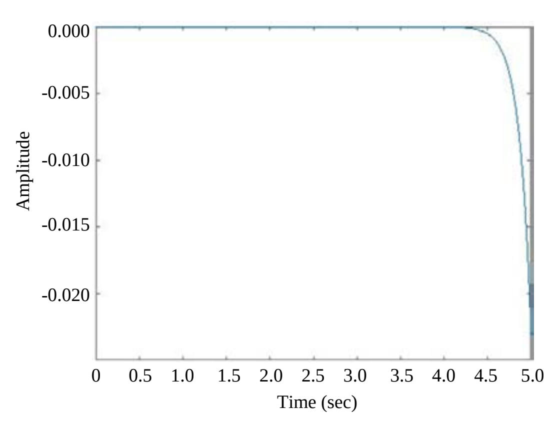

Fig. 3: Open loop impulse response of the rotational pendulum (1 volt impulse input)

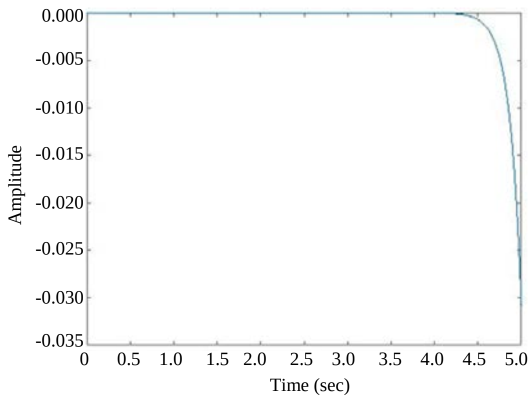

Fig. 4: Open loop step response of the rotational pendulum (1 volt step input)

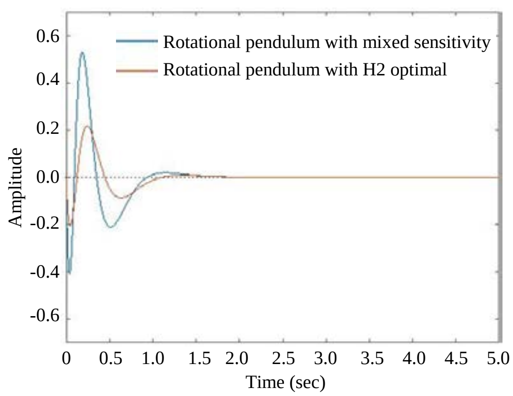

Fig. 5: Impulse response of the rotational pendulum (1 volt impulse input)

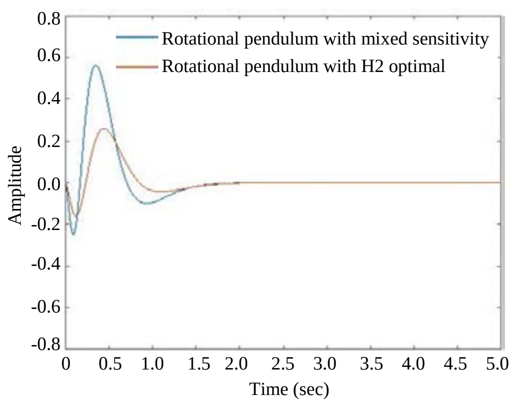

Fig. 6: Step response of the rotational pendulum (1 volt step input)

pendulum with mixed sensitivity and H2 optimal controllers for a step input voltage signal is shown in Fig. 6.

The simulation result shows that the rotational pendulum with H2 optimal controller improve the settling time and the overshoot and the angular position returns to zero means the rotational pendulum is in upward stable position[6, 7].

CONCLUSION

In this study, modeling, simulation and comparison of the rotational inverted pendulum have been done using MATLAB/Script Toolbox. Augmentations with weighting functions based mixed sensitivity and H2 optimal controllers have been used to control the system instability. The open loop response of the system for a step and impulse voltage input shows that the system is unstable. Comparison of the rotational inverted pendulum with mixed sensitivity and H2 optimal controllers have been done for a step and impulse voltage input and the simulation results prove that the rotational inverted pendulum with H2 optimal controller improves the settling time and overshoot and the angular position returns to its position successfully.

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