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Cost Minimization for Earthing Grid System Design Using Geometric Programming

[1]E.U. Ubeku, [1]S.T. Apeh and [2]J.O. Ariavie
[1]Department of Electrical/Electronic Engineering,
[2]Department of Civil Engineering, University of Benin, Benin City, Nigeria

Abstract: This study, presents a new trend methodology for the cost minimization of an earthing grid system. To optimally, design an earthing grid system researchers apply a geometric programming approach. In this approach, the cost minimization variable is positive and the constraints are expressed in a posynomial form. In forming the cost minimization variable researcher took into consideration the cost of grid conductor, excavation cost and joint welding cost. The simulated result shows the geometric programming approach was able to predict the total length of conductor, the grid configuration, the depth of burial and diameter of conductor cross section required to design an earthing grid system that meets the optimal safety criterion, as well as cost of installation.

Key words: Earthing grid, geometric programming, cost minimization, optimal safety, methodology

INTRODUCTION

Mathematical programming is a useful tool in engineering designs especially when such designs have to be optimized. The design of earthing grid system is an optimization design process. Both analytical and numerical optimization methods have been applied by several researchers in the design of earthing grid system which give accurate results (Sverak, 1976; Colominas *et al*., 2002; Taher and Shemshadi, 2008; Lee and Shen, 2009). The need to produce an efficient and effective earthing grid system has resulted in further search for a more accurate mathematical programming tool.

The fact that the expressions used in the design of earthing grid are non-linear has prompted some researchers to use non-linear programming methods, such as genetic algorithms or probabilistic methods (Lee and Shen, 2009). Nevertheless, most of these non-linear optimization techniques can not guarantee that the global optimum is attained because a local optimum can stop the searching process. In most cases, these methods fail to detect the unfeasibility of the problem (Ribes-Mallada *et al*., 2011).

Due to these observable defects in some of the non-linear mathematical tools, the Geometric Programming (GP) technique is proposed in this study. The geometric programming is a technique that is able to globally optimize a problem when the objective function and the constraint have a given form. GP ensures that the global solution is readily found or that the unfeasibility is detected very quickly (Islam and Roy, 2005; Dupacova, 2010; Sadjadi and Arabzadeh, 2008; Boyd *et al*., 2007; Jabr, 2005). The advantage of GP over other non-linear programmes like GA is the significant development of the interior point method for solving convex optimization problems. This made GP an extremely efficient and reliable programming tool (Jabr, 2005; Boyd and Vandenberghe, 2004).

MATERIALS AND METHODS

Basics on geometric programming: GP requires that the function to be minimized, the objective or cost function and all the constraints are to be expressed as monomials or posynomials functions (Liu *et al*., 2010; Nesterov and Nemirovsky, 1994). Let, $x_1, ..., x_n$ denote n real positive variable and $x = x_1, ..., x_n$ a vector with components x_i. Then, a monomial function is in the equation:

$$f(x) = cx_1^{a_1} x_2^{a_2} ....x_n^{a_n} \quad (1)$$

Where, c>0. A sum of one or more monomial functions is called a posynomial function that is:

$$f(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} ....x_n^{a_{nk}} \quad (2)$$

Where, $c_k>0$. A Geometric Program (GP) is an optimization problem of the equation minimize f_0 (x) subject to:

Corresponding Author: E.U. Ubeku, Department of Electrical/Electronic Engineering, University of Benin, Benin City, Nigeria

$$f_i(x) \leq 1, \quad i=1,\ldots,m$$
$$g_i(x) = 1, \quad i=1,\ldots,p \tag{3}$$

Where:

$f_{0\text{-}m}$ = Posynomial functions

$g_{1\text{-}p}$ = Monomial functions

The geometric program of the Eq. 5 is the standard form which could be converted into the convex form that can be solved using interior point algorithms by changing variables y = log (x) or $x = e^y$ and replacing $f_i \leq 1$ with log $(f_i) \leq 0$ and log $(g_i) = 0$. The transformed geometric program in convex form is written as (Ribes-Mallada *et al*., 2011; Sahidul ANSI, 1986); minimize log (e^x) subject to:

$$\log\left(f_i\left(e^y\right)\right) \leq 0 \quad i = 1,\ldots,m$$
$$\log\left(g_i\left(e^y\right)\right) = 0 \quad i = 1,\ldots,p \tag{4}$$

Problem formulation: On the basis of the expressions used in geometric programming, researchers are formulating the problem in line with the applicable conditions-monomials and posynomials. Since, the problem in mind is cost minimization one of the cost component is the cost of conductors. To obtain this, researchers need the weight of the conductor W_c which can be written as:

$$W_c = \pi \frac{d^2}{4} w_c L_t \tag{5}$$

Where:

d = Ddiameter of grid conductor cross section (m)

w_c = Specific weight of conductor (kg m^{-3})

L_t = Total length of grid conductor (m)

This must be multiplied by a per weight cost factor k_1 to arrive at the cost to include in the objective function. The other costs are that of evacuation of the soil and joint welding. The expression researchers used for the total volume dugout V_d of a grid system is:

$$V_d = h^2 L_t \tag{6}$$

Where, h depth of burial of conductors (m). If researchers multiply Eq. 6 by the cost per volume k_2, then the cost of evacuation is obtained. The total volume of weld material V_w to be used in welding the joints could be expressed as:

$$V_w = \pi \frac{d^3}{4}(N+1)^2 \tag{7}$$

Where, N grid configuration. The cost per joint weld is k_3 which multiplies the total volume of weld material to arrive at the cost of welding. The objective function is the sum of the costs of conductor, evacuation and welding, respectively. That is:

$$C = k_1 \pi \frac{d^2}{4} w_c L_t + k_2 h^2 L_t + k_3 \pi \frac{d^3}{4}(N+1)^2 \tag{8}$$

Equation 8 satisfies the sum of monomials functions and thus met the one of the conditions for geometric programming.

Equality constraints are those which must be satisfied exactly in the cost minimization. The most important constraint is the Ground Potential Rise (GPR) which is the product of the grid resistance R_g and fault current I_f. For the GPR to meet the safety criterion, it must be less than or equal to the permissible touch voltage V_{touch}. That is:

$$I_G R_g \leq V_{touch} \tag{9}$$

Where, I_G maximum grid current (A). But, R_g is expressed as (ANSI/IEEE, 1986):

$$R_g = \rho_{soil}\left[\frac{1}{4}\sqrt{\frac{\pi}{A}} + \frac{1}{L_t}\right] \tag{10}$$

Where:

ρ_{soil} = Soil resistivity (Ω-m)

A = Total area enclosed by the earthing grid system (m^2)

$$I_G = 0.6 \frac{3V_{Line}}{\sqrt{3}.\left(2z_1 + z_0\right)} \tag{11}$$

V_{touch} from Lee and Shen (2009) and ANSI/IEEE (1986) are expressed as:

$$V_{touch} = \frac{\left(1000 + 1.5C_s\left(h_s, K\right)\rho_s\right)0.166}{\sqrt{t_S}} \tag{12}$$

for 50 kg body weight or;

$$V_{touch} = \frac{\left(1000 + 1.5C_s\left(h_s, K\right)\rho_s\right)0.157}{\sqrt{t_s}} \tag{13}$$

for 70 kg body weight;

$$C_s = 1 - \frac{1.02\left(1 - \frac{\rho_{soil}}{\rho_s}\right)}{2h_s + 1.02} \quad (14)$$

Where:

C_s = Surface area derating factor
ρ_s = Surface layer resistivity (Ω-m)
h_s = Surface layer thickness (m)
t_s = Duration of fault current (sec)

Using the permissible touch voltage for a 70 kg body weight and substituting Eq. 10-12 into Eq. 9 researchers have:

$$0.6\frac{3V_{Line}}{\sqrt{3}.(2Z_1 + Z_0)}\rho_{soil}\left[\frac{1}{4}\sqrt{\frac{\pi}{A}} + \frac{1}{L_t}\right] \le \left(1000 + 1.5C_s(h_s, K)\rho_s\right)\frac{0.157}{\sqrt{t_s}} \quad (15)$$

Let, $\sqrt{A} = DN$ then:

$$0.6\frac{3V_{Line}}{\sqrt{3}.(2z_1 + z_0)}\rho_{soil}\left[\frac{1}{4}\frac{\sqrt{\pi}}{DN} + \frac{1}{L_t}\right] \le \left(1000 + 1.5C_s(h_s, K)\rho_s\right)\frac{0.157}{\sqrt{t_s}} \quad (16)$$

Where, D is the grid's parallel conductor spacing. Equation 16 can be expressed as the generalized posynomials inequality as:

$$\frac{0.6 \times 3V_{Line}\sqrt{t_s} \times \rho_{soil}}{0.157\sqrt{3}.(2z_1 + z_0)\left(1000 + 1.5C_s(h_s, K)\rho_s\right)}\left[\frac{1}{4}\frac{\sqrt{\pi}}{DN} + \frac{1}{L_t}\right] \le 1 \quad (17)$$

The next equality constraint is the diameter d of the conductor cross A_{mm^2} section which is expressed, as a function of the fault current I_f is written as (Lee and Shen, 2009; ANSI/IEEE, 1986):

$$2 \times 10^{-3}\sqrt{\frac{A_{mm^2}}{\pi}} \le d_{min} \quad (18)$$

$$A_{mm^2} = \frac{3V_{Line}}{\sqrt{3}.(2z_1 + z_0)} \times \sqrt{\frac{t_c \alpha_r \rho_r}{TCAP \times 10^4 \times \ln\left(1 + \frac{K_0 + T_m}{K_0 + T_a}\right)}} \quad (19)$$

The monomial form of Eq. 19 is:

$$\frac{2 \times 10^{-3}}{d_{min}\sqrt{\frac{A_{mm^2}}{\pi}}} \le 1 \quad (20)$$

Where:

T_m = Maximum allowable temperature (°C)
T_a = Ambient temperature (°C)
K_0 = $1/\alpha_0$
α_0 = Thermal coefficient of resistivity at reference temperature T_r
T_r = Reference temperature for material constants in °C
t_c = Fault current duration (sec)
d_{min} = Minimum diameter of grid conductor cross section (m)
a_r = Thermal coefficient of resistivity (1/°C)
ρ_r = Resistivity of grid conductor at reference temperature (μΩ-m)
TCAP = Termal capacity per unit volume (J/(cm^2.°C))

Other constraints are (ANSI/IEEE, 1986):

$$N \le 25 \quad (21)$$

$$h = 0.25m \quad (22)$$

$$\frac{4d}{h} \le 1 \quad (23)$$

Having formulated the cost minimization problem, the geometric programme looks like this, minimize:

$$C = k_1\pi\frac{d^2}{4}w_c L_t + k_2 h^2 L_t + k_3\pi\frac{d^3}{4}(N+1)^2$$

Subject to:

$$\frac{0.6 \times 3V_{Line}\sqrt{t_s} \times \rho_{soil}}{0.157\sqrt{3}.(2z_1 + z_0)\left(1000 + 1.5C_s(h_s, k)\rho_s\right)}\left[\frac{1}{4}\frac{\sqrt{\pi}}{DN} + \frac{1}{L_t}\right] \le 1, \frac{2 \times 10^{-3}}{d}\sqrt{\frac{A_{mm^2}}{\pi}} \le 1, N \le 25, h \le 0.25m, \frac{4d}{h} \le 1 \quad (24)$$

After obtaining the optimal values for the number of mesh formation, the total length of grid conductor, the diameter of cross section of conductor and the depth of burial of conductor, the mesh voltage V_{mesh} and step voltage V_{step} can be obtained using (Lee and Shen, 2009; ANSI/IEEE, 1986).

$$V_{mesh} = \frac{\rho_{soil}.I_G.K_i.K_m}{L_t} \quad (25)$$

$$V_{mesh} = \frac{\rho_{soil}.I_G.K_i.K_s}{L_t} \quad (26)$$

$$K_i = 0.656 + 0.172N \quad (27)$$

$$K_s = \frac{1}{\pi}\left[\frac{1}{2h} + \frac{1}{D+h} + \frac{1}{D}\left(1 - 0.5^{N-2}\right)\right] \quad (28)$$

$$K_m = \frac{1}{2\pi}\left[\begin{array}{c}\ln\left(\frac{D^2}{16hd} + \frac{(D+2h)^2}{8Dd} - \frac{h}{4d}\right) + \\ \frac{k_{ii}}{k_h}\ln\frac{8}{\pi(2N-1)}\end{array}\right] \quad (29)$$

$$K_{ii} = \frac{1}{(2N)^{2/N}} \quad (30)$$

$$K_h = \sqrt{1 + \frac{h}{h_0}} \quad (31)$$

$$h_0 = 1m \quad (32)$$

A geometric programming tool box gplab written in Matlab environment was used in solving the formulated cost minimization problem.

RESULTS AND DISCUSSION

The data of the substation 115/13 kV obtained from ANSI/IEEE Std. 80-1986 is a case study of 70×70 m square grid and is shown in Table 1. It is used in this study, to validate to accuracy of geometric programming in earthing grid design.

In addition to the data in Table 1, the following were also taken note of: TCAP = 3.846, K_0 = 245, α_r = 0.00378, ρ_r = 5.862, T_m = 700°C, T_a = 40°C, K_1 = 1000 Naira kg^{-1}; K_2 = 50 Naira m^{-3} K_3 = 25 Naira/joint.

These data and Eq. 22 were inputted into ggplab and the resulting the optimal values in this case are as presented in Table 2. This was realized with only 42 iterations in just fraction of seconds.

Table 1: Data from ANSI/IEEE Std. 80-1986

Data	Values
Fault duration t_f	0.5 sec
Fault impedance, z_1	4.0+j 10.0 Ω
Fault impedance z_0	10.0+j 40.0 Ω
Current division factor, S_f	0.6
Soil resistivity, ρ_{soil}	400 Ω m
Crushed rock resistivity (wet), ρ_s	2500 Ω m
Thickness of crushed rock surfacing h_s	0.1 m
Line-line voltage at worst-fault location	115,000 V

Table 2: Optimal result of earthing grid

Variables	Results
L_t (m)	2100.00
N	14.00
h (m)	0.25
d (m)	0.01
R_g (Ω)	2.72
GPRV (V)	5170.70
V_{mesh} (V)	857.94
V_{step} (V)	527.01
Cost (Naira)	818.71

CONCLUSION

This study describes a reliable and efficient procedure for the design of an earthing grid via cost optimization using geometric programming. For the example used, the geometric programming enabled us to compute the essential parameters the total length of conductor, the grid configuration, the depth of burial and diameter of conductor cross section required to design an earthing grid system that meets the optimal safety criterion, as well as cost of installation.

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