

## Delta Operator Modelling and Control of Electromagnetic Levitation System

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**Abstract:** The dynamic model develop from physical relationship of an Electro Magnetic Levitation System (EMLS) is considered in this study to propose a unified framework for controller design. The EMLS is discretised using delta operator. The delta operator representation is implicitly discrete-time formulation which converges to its continuous-time counterparts at high sampling frequency. The EMLS is inherently unstable and strongly non-linear in nature which is linearised and different classical control scheme such as PD, Lead, Lag-lead, PID and combination of PI and Lead compensators are designed to meet the requirement of the overall controlled system stability and performance. The simulation results exhibit the usefulness of the proposed schemes.

**Key words:** Delta operator, unified approach, electromagnetic levitation system, lead, lag-lead, PID, PI plus lead control, India

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### INTRODUCTION

Magnetic levitation systems are devices that levitate ferromagnetic material with the aid of electromagnetism (Wong, 1986). A great interest is focussed in this field by many of the researchers since it eliminates energy losses due to friction. It has a large number of applications, such as high speed magnetic suspension trains (Vinokurov, 1997; Groning *et al.*, 1998), magnetic suspension shaft for its use as non-frictional rotating machinery (Suzuki *et al.*, 2000) and conveyor vehicle (Goethem and Henneberger, 2002). The low-cost magnetic levitation project kit is typical example of the academic study of magnetic levitation system (Lilienkamp and Lundberg, 2004). An attraction type magnetic levitation system is inherently unstable and non-linear in nature.

A linearised model of small scale magnetic levitation system has been developed, implemented and tested successfully (Wong, 1986; Shiao, 2001; Hurley *et al.*, 2004; Li, 2005; Banerjee *et al.*, 2008a). The modelling, analysis and control of discrete time systems are traditionally being carried out using shift operator in time domain while z-transformation is used in the complex domain. However, these discrete time systems cannot support very high sampling frequency and thus lose their stability with the decrease of sampling time. This

problem can be avoided if the discrete time systems are modelled using delta operator (Middleton and Goodwin, 1986, 1990, 1992). The delta operator formulation of the discrete-time systems provides a unified framework to obtain results similar to continuous time results at very large sampling frequency and further improves the numerical robustness as compared to shift operator description of the discrete time systems. This study deals with the design of classical controller in discrete delta domain using root locus technique for small scale single axis attraction type electromagnetic levitation system.

### MATERIALS AND METHODS

**Delta operator:** Middleton and Goodwin (1986, 1990, 1992) popularized the delta operator. The discrete-time system modeling using shift operator and/or z-transformation has several limitations while Nyquist frequency is the minimum requirement for discretization of the continuous-time signals and systems to retain the continuous time bandwidth in the corresponding discrete domain, there is no maximum limit of sampling frequency for discretization which largely depends on the intuitions of the designer, in addition the resultant discrete time system lost its corresponding continuous time insights in such formulation. The choice of the maximum sampling

frequency further depends on the numerical stability of the resultant discretization and the technological limit of the A/D converters available. Therefore, all practical purpose, it is not possible to develop a continuous time signal/system at fast sampling limit. On the contrary, delta operator modelling of a true discrete time model of a discrete time system converges to its continuous time system at high sampling frequency. Therefore, delta operator modelling of discrete time system highlights similarities rather than differences between discrete time and continuous time systems formulation. It is possible to use small sampling periods or high sampling frequency without incurring numerical difficulties such as a high sensitivity to round off errors in coefficient representation. The time domain delta operator is represented by:

$$\delta = \frac{q-1}{\Delta} \quad (1)$$

The small symbol  $\delta$  is called delta operator and  $z$  is the sampling period and  $q$  is the forward shift operator. Similarly, in complex domain the transform relation:

$$\gamma = \frac{z-1}{\Delta} \quad (2)$$

Where:

$\gamma$  = The complex domain transfer variable

$z$  = The complex domain z-transform variable

Since Eq. 1 and 2 are linear transform, therefore, any linear signal and system modelled with  $q$  and  $z$  will continue to obey all linear system properties in the corresponding  $\delta$  operator and  $\gamma$  domain modelling.

Therefore, all the linear system properties and parameterization of the complex  $z$  domain can be transformed to the  $\gamma$  domain with equal flexibility as offered by the  $z$ -domain. Figure 1 shows the  $\gamma$ -plane. The complex  $\gamma$  domain variable is the counterpart of the time domain  $\delta$  operator.

## RESULTS AND DISCUSSION

**Description and controller design of EMLS:** Electromagnetic levitation system is designed with a single E-core type electromagnet (Banerjee *et al.*, 2008a) The functional block diagram is shown in Fig. 2. There are 2 control loops in this system. The inner loop is for current control loop and outer loop is for position control loop.

The electromagnet act as an actuator and the levitate object is ferromagnetic material. These controllers are designed in sequence.

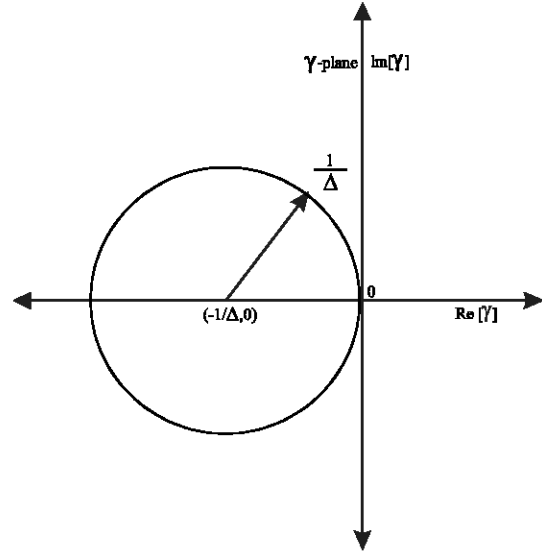


Fig. 1:  $\gamma$ -plane

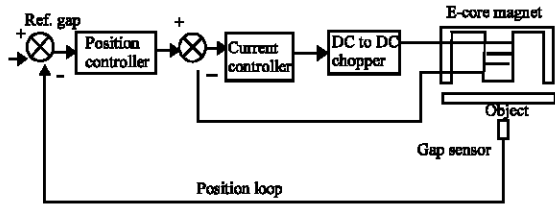


Fig. 2: Block diagram of the system

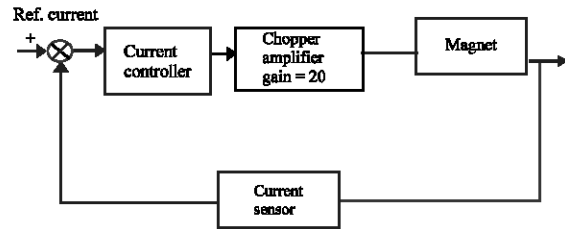


Fig. 3: Block diagram of current loop

**PI-type current controller:** To levitate the ferromagnetic object at the desired position by controlling the attraction force of the electromagnet need to control of the magnet coil current.

The coil current of the magnet is controlled by the single switch base DC-DC chopper circuit through the PWM control logic.

In current loop LA-55P used as current sensor. The linear factor of the current sensor circuit is set to be 1 volt/amp.

Figure 3 shows the block diagram of the current control loop. The transfer function of the current loop (Banerjee *et al.*, 2008b) is:

$$T_c(s) = \frac{1}{L} \frac{1}{s + \frac{R}{L}} \quad (3)$$

Here at 10 mm operating air gap  $R = 2.4$  ohm,  $L = 0.06065$  H and sampling interval. The domain transformation of Eq. 3 is:

$$T_c(\gamma) = \frac{15.03}{1 + 36.08\gamma} \quad (4)$$

The PI controller transfer function is given as:

$$G(\gamma)_{\pi 1} = K(1 + \frac{K_i}{\gamma})$$

After tuning the constant  $K$  and  $K_i$  properly, the specifications are met. Figure 4 and 5 shows the root locus plot and step response in the discrete delta domain. We get the performance from Table 1. The current dynamics are faster than the dynamics of the position. So the PWM-based current driver can be approximated simply by a constant gain as 1.

**Position control loop:** Hall-effect sensor is employed to sense the variation of the levitate object position around the equilibrium point. The position sensor gain is 1000 volts per meter.

Figure 6 shows the functional block diagram of the position control loop. Taking controlled current source as the excitation of the magnet coil transfer function of the levitate system may be written in the following form (Banerjee *et al.*, 2008b):

$$T_p(s) = -\frac{(K_i/m)}{(s^2 - K_z/m)} \quad (5)$$

Taking 10 mm operating air gap,  $K_i = 0.5358$  N/A,  $K_z = 72.86$  N/m,  $m = 58$  g and  $\Delta = 0.0005$ . The  $\delta$  transform of the Eq. 5 is:

$$T_p(\gamma) = \frac{0.002309\gamma + 9.237}{\gamma^2 - 0.628\gamma - 1256} \quad (6)$$

Here, we analyse the performance of the various classical controller like PD, LEAD, LAG-LEAD, PID and LEAD compensator with PI.

**PD controller:** The addition of a zero to the open loop transfer function has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response. The transfer of PD

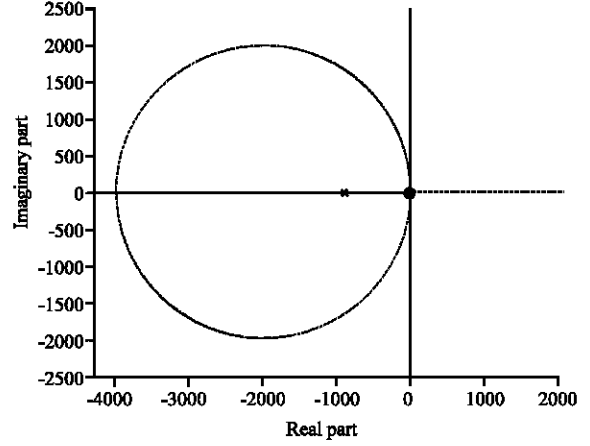


Fig. 4: Root locus plot of the current loop at 10 mm operating air-gap

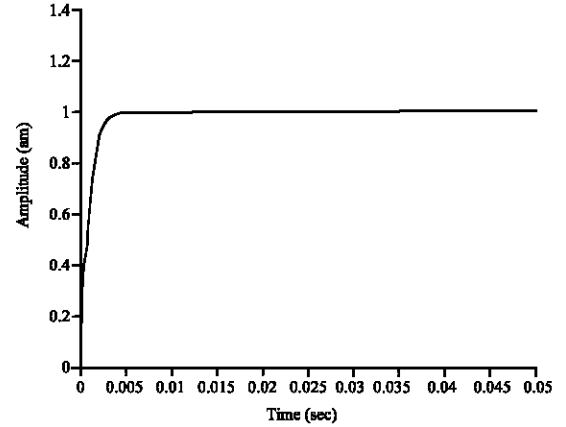


Fig. 5: Step response of the current loop with PI controller at 10 mm operating air-gap

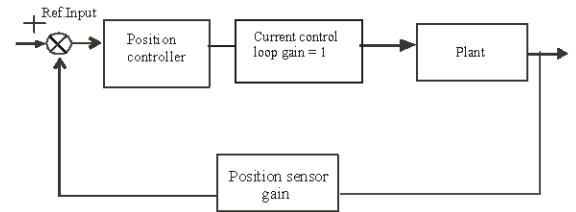


Fig. 6: Block diagram of the position control loop

Table.1: Performance of the current controller

Transfer function of PI controller	Settling time ( $T_{ss}$ )	Steady state error ( $E_{ss}$ )
$\frac{3(1+37/\gamma)}{\gamma}$	0.005	0

controller is taken as following form  $G(\gamma) = K(\gamma + K_d)$ . Figure 7 and 8 shows the root locus plot and step response of the position loop at 10 mm operating air gap.

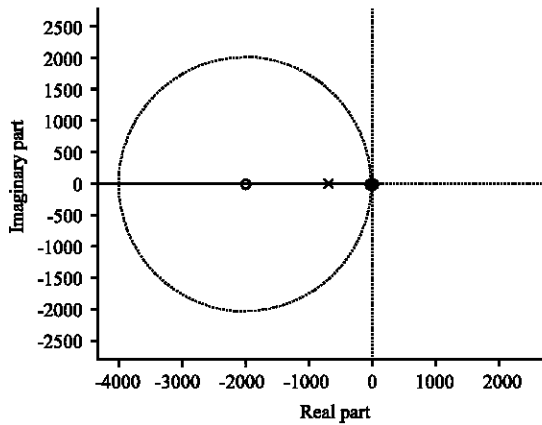


Fig.. 7: Root locus plot with PD controller at 10 mm operating air-gap

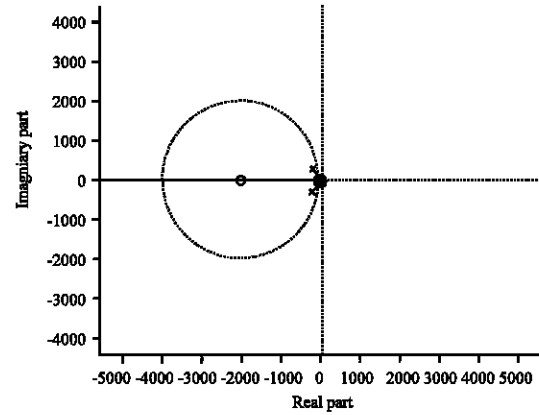


Fig. 9: Root locus plot with LEAD controller at 10 mm operating air-gap

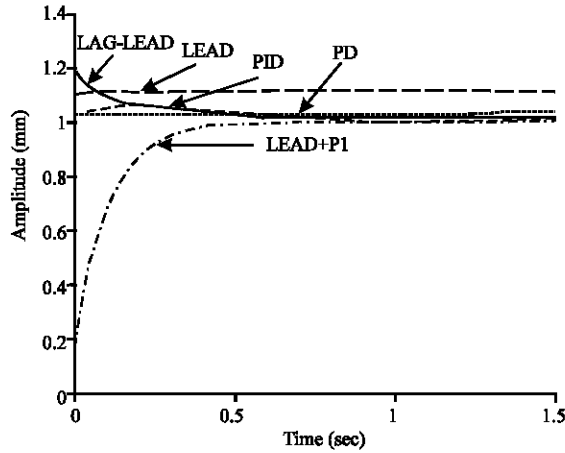


Fig. 8: Step response with various controllers at 10 mm operating air-gap

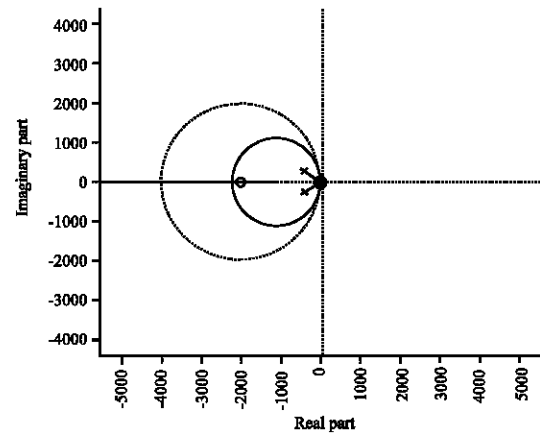


Fig.10: Root locus plot with LAG-LEAD controller at 10 mm operating air-gap

**LEAD controller:** To reduce the effects of high frequency noise, a pole is to be placed far away in along the negative real axis from this zero and effectively produces phase lead controller whose transfer function is taken as following form:

$$G_{lead} = \frac{K(\gamma + K_a)}{(\gamma + K_d)}$$

Figure 8 and 9 shows the root locus plot and step response at 10 mm operating air gap with LEAD controller of the position loop.

**LAG-LEAD controller:** For LAG-LEAD compensator there is combined effect for the LEAD and LAG compensator which improves the transient response,

steady state error. The transfer function is taken as following form :

$$G_{lead-lag}(\gamma) = \frac{K(\gamma + K_a)(\gamma + K_b)}{(\gamma + K_c)(\gamma + K_d)}$$

Figure 10 and 11 shows the root locus plot and step response at 10 mm operating air gap with LAG-LEAD controller of the position loop.

**PID controller:** The steady-state error can be completely removed by PID controller. As the system is inherently unstable, small change of proportional gain abruptly changes transient response so it is very typical to use the PID controller. The transfer function of the PID controller is taken as the following form  $G_{pid}(\gamma) = K(1 + K_p/\gamma + \gamma K_d)$ .

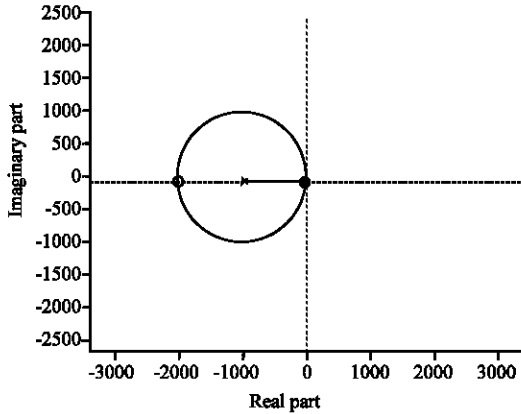


Fig 11: Root locus plot with PID controller at 10 mm operating air-gap

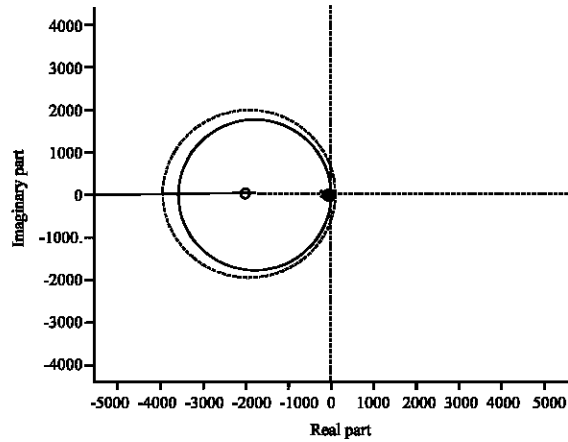


Fig. 13: Root locus plot with LAG + PI controller at 10 mm operating air-gap

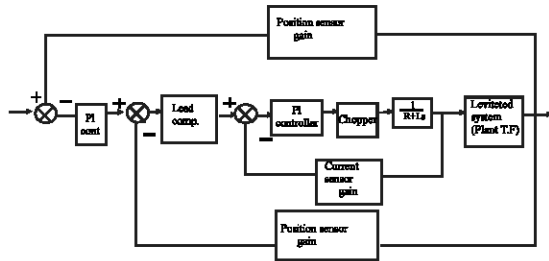


Fig. 12: The modified block diagram of the system

Table.2: Performance of the various position controllers

Type of controller	Transfer function	Max.(%) overshoot	Settling time( $T_{ss}$ )	Steady state error( $E_{ss}$ )
PD	$0.12(\gamma+36)$	0.006	0.034	
LEAD	$\frac{15.5(\gamma+32)}{(\gamma+370)}$	20	0.124	0.113
LAG-LEAD	$\frac{40(\gamma+15)(\gamma+69)}{(\gamma+0.5)(\gamma+840)}$	25.1	0.35	0.003
PID	$7(1+0.03\gamma+\frac{10}{\gamma})$	2.1	0.26	0
LEAD with PI	$0.09(1+\frac{53}{\gamma})$	0	1.474	0

Figure 8 and 11 shows the root locus plot and step response at 10 mm operating air gap with PID controller of the position loop.

**LEAD compensator with PI:** From Table 2, it is clear that with cascade LEAD compensated control scheme the overall closed loop of the system is stable but it has some steady state error.

This problem could be eliminated by introducing a PI controller. This PI controller cascaded with the overall closed loop system. Figure 12 shows the modified block

diagram of the system. The transfer function of the PI controller is taken as the following form:

$$G_{pi}(\gamma) = K(1 + K_i/\gamma)$$

Figure 8 and 13 shows the root locous plot and step response at 10 mm operating air gap with Lead and PI controller of the position loop.

## CONCLUSION

The classical root locus technique is used in the complex delta domain for design of different types of controllers for the EMLS. This delta domain formulation is able to avoid the restriction of the Nyquist frequency criterion for conventional discretization technique in complex z-domain. The performance analysis of these control schemes was shown through its step response characteristics. Therefore, delta operator based controller design schemes presented in this study is a viable alternative to the conventional discrete-time design in which shift operator is used, further, the continuous-time representation of the system can be retrieved at high sampling speed.

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