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Steady-State Performance of a Directly Connected PV Array/Six-Step VSI/Induction Motor System

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Abstract: In this study, the analysis of a PV array connected directly without an intermediate converter to a voltage-source inverter/induction motor system driving a centrifugal pump is presented. This method is based on that the VSI/induction motor system, operating at a constant air gap flux can be connected directly to the PV array by controlling the insolation level to which the PV array is subjected.

Key words: PV array, voltage source inverter, induction motor, pump, electrical energy, Egypt

INTRODUCTION

The PV array as a source of electrical energy is quiet, clean and reliable. It does not cause pollution and can be installed easily. Its maintenance cost is low but its capital cost of installation is relatively high and its efficiency of power conversion is low (Markvart, 1995).

The use of squirrel-cage induction motors in drive systems has many advantages such as its ruggedness and it relatively inexpensive. Inverter-driven induction motors are used in many industrial applications over a wide range and their operation is economical and the motor speed in these systems can be controlled in applications that require variable speed (Meiopoulos *et al*., 2002). There are a lot of investigations that had dealt with the voltage-source inverter/induction motor system when fed from a PV array (Bhat *et al*., 1987; Yao *et al*., 1994; Eskander and Zaki, 1997; Muljadi, 1997; Domijan and Buchh, 1998; Hamed, 2001; Mimouni *et al*., 2004; Betka and Moussi, 2004; Daud and Mahmoud, 2005; Akbaba, 2007).

In this study, a PV array directly connected to a VSI/induction motor system driving a centrifugal pump, Fig. 1 is investigated.

System modeling: The PV array consists of a number of modules and each module consists of a number of photovoltaic (solar) cells (Markvart, 1995). Depending on the load requirements, PV modules are connected in series to form a string and a number of parallel strings constitute a PV array.

The PV cell of a module is a nonlinear semiconductor power source (Appelbaum, 1986) that converts sunlight directly into electricity. The electrical equivalent circuit of a PV cell is shown in Fig. 2 (Markvart, 1995). The expression relating the output voltage, U and output current, I of the PV cell is shown by Markvart (1995):

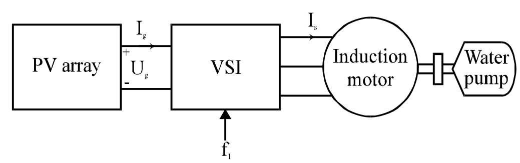

Fig. 1: PV array/VSI/induction motor system

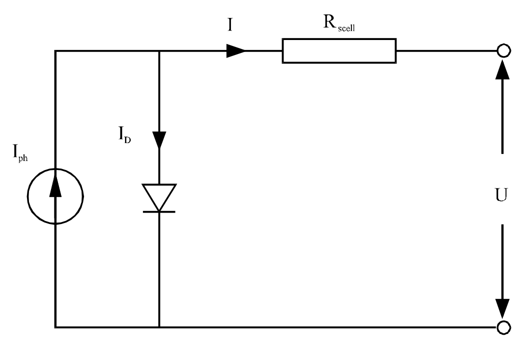

Fig. 2: PV cell equivalent circuit

$$I = I_{ph} - I_0 \cdot (e^{A \cdot (U + I \cdot R_{scell})} - 1) \quad (1)$$

Where:

$A = \frac{q}{AkT_{cell}}$, I_{ph} = The cell photocurrent
I_0 = The cell reverse saturation current
R_{scell} = The cell series resistance
q = The electron charge
A = An ideality factor
k = Boltzmann's constant
T_{cell} = The absolute cell temperature

Equation 1 can be rewritten in the following form (Appelbaum, 1986):

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$$U = -I.R_{scell} + \frac{1}{\Lambda}.\ln(1 + \frac{I_{ph} - I}{I_o}) \qquad (2)$$

When the PV array is formed from a number of parallel strings, N_p, with each string consisting of a number of series connected modules, N_m and each module consists of number of series connected cells, N_s, the PV array current, I_g can be related to the cell current by:

$$I_g = I.N_p \qquad (3)$$

and the PV array voltage can be related to the cell voltage by:

$$U_g = U.N_{sg} \qquad (4)$$

Where N_{sg} is the number of series cells in a string and is given by:

$$N_{sg} = N_m.N_s$$

To obtain an expression relating the PV array output voltage to its output current, Eq. 3 and 4 are substituted into Eq. 1 to get:

$$I_g = I_{phg} - I_{og}.(e^{\Lambda_g.(U_g + I_g.R_{sg})} - 1) \qquad (5)$$

Where $I_{phg} = I_{ph}.N_p$, $I_{og} = I_o.N_p$, $R_{sg} = R_{scell}.N_{sg}/N_p$, $\Lambda_g = \Lambda/N_{sg}$, I_{phg} is the PV array photocurrent, I_{og} is the PV array reverse saturation current, R_{sg} is the PV array series resistance and Λ_g is the factor of the PV array.

In Eq 5, the insolation level is not necessarily the value corresponding to Standard Test Conditions (STC). However, to apply this equation at any insolation level, the per unit insolation level, G should be included into the equation. Thus:

$$I_g = G.I_{phg} - I_{og}.(e^{\Lambda_g(U_g + I_g.R_{sg})} - 1) \qquad (6)$$

At STC, G = 1.0 pu. Thus, according to Hamed (2001):

$$U_g = -I_g.R_{sg} + \frac{1}{\Lambda_g}.\ln(1 + \frac{G.I_{phg} - I_g}{I_{og}}) \qquad (7)$$

Equation 7 is a non-linear relationship between the current and voltage of the PV array. If the VSI used is a six-step inverter and only its fundamental-frequency voltage component, U_{s1} is taken into consideration with the effect of voltage harmonics neglected then the rms

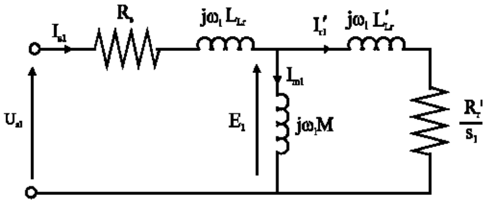

Fig. 3: Per phase equivalent circuit of the induction motor

value of the fundamental-voltage component is given by (Mohan *et al*., 2003):

$$U_{s1} = \frac{\sqrt{2}U_d}{\pi} \qquad (8)$$

The induction motor is modeled in terms of its per phase equivalent circuit which is shown in Fig. 3 (Dewan *et al*., 1984). The centrifugal pump of the system is modeled in terms of its power-speed equation which is given by Hamed (2001) and Dewan *et al*. (1984):

$$P_L = K_w\omega_m^3 \qquad (9)$$

Where:

K_w = The pump constant
ω_m = Speed in rad sec^{-1}

MATERIALS AND METHODS

The purpose of the analysis is to get the performance of the system under consideration (Fig. 1), at a given inverter frequency and certain operating condition of the mechanical load which is the pump when the inverter is fed from a PV array with the suitable insolation level of the PV array determined first.

In order to analyze the system shown in Fig. 1, the PV array should be sized first to meet the load demand. The sizing of the PV array will be discussed. The parameters for the chosen PV module are obtained based on a method described by Abdel-Halim (2004).

It is assumed that the motor is operated at a constant air-gap flux then the ratio E/ω must be held at any operating condition, constant at a value equal to the value at rated conditions i.e., E_r/ω_r which can be obtained from:

$$\frac{E_r}{\omega_r} = \frac{U_r}{\omega_r}\left[\frac{Z_m.Z'_r/(Z_m + Z'_r)}{Z_m.Z'_r/(Z_m + Z'_r) + Z_s}\right] \qquad (10)$$

Where:

E_1 = The fundamental back emf of the motor
ω_1 = The fundamental angular frequency
U_r = The rated phase voltage of the motor
ω_r = The rated angular frequency
Z_s = $R_s + j\omega_r L_{LS}$
Z_m = $j\omega_r M$
Z_r' = $R_r'/s_r + j\omega_r L_{Lr}'$

s_r = ω_r - p.ω_{mr}/ω_r
p = The motor pole pairs
ω_{mr} = The rated motor speed in rad sec^{-1}

The ratio E_r/ω_r will be denoted by the symbol K. Therefore, the ratio E_1/ω_1 at any operating condition is equal to K. For a certain inverter fundamental frequency and assuming that the motor is driving a centrifugal pump, the equation of the motor speed, ω_m is obtained to be as follows: When the friction and windage losses of the motor are neglected, the output power of the motor will be equal to the developed mechanical power and neglecting the harmonics produced by the VSI, the output power of the induction motor will be given by Dewan *et al.* (1984):

$$P_{out} = T_L\omega_m = P_{mech} = \frac{3(1-s_1)}{s_1}R'_r\left|\overline{I'_{r1}}\right|^2 \quad (11)$$

Where;

$$\left|\overline{I'_{r1}}\right| = \frac{E_1}{\sqrt{(R'_r/s_1)^2 + (\omega_1 L'_{lr})^2}} \quad (12)$$

Substituting Eq. 12 into 11 with $E_1 = K.\omega_1$, $T_L = K_w\omega_m^2$, when the motor is driving a centrifugal pump and gives Eq. 13:

$$s_1 = \frac{\omega_1 - p\omega_m}{\omega_1}$$

$$k_1\omega_m^4 + k_2\omega_m^3 + k_3\omega_m^2 + k_4\omega_m + k_5 = 0 \quad (13)$$

where, $k_1 = K_w(pL'_{Lr})^2$, $k_2 = -2pK_w\omega_1(L'_{Lr})^2$, $k_3 = K_w[(R'_r)^2 + (\omega_1 L'_{Lr})^2]$, $k_4 = 3p^2R'_rK^2$ and $k_5 = -3pR'_r\omega_1K^2$. Numerical solution of Eq. 13 gives the steady-state motor speed ω_m for a given fundamental frequency ω_1. The stator current is obtained from:

$$\overline{I}_{s1} = \overline{I}_{m1} + \overline{I}'_{r1} \quad (14)$$

Where:

$$\overline{I}_{m1} = \frac{\overline{E}_1}{j\omega_1 M},$$

$$\overline{I}'_{r1} = \frac{\overline{E}_1}{R'_r/s_1 + j\omega_1 L'_{lr}},$$

$$\overline{E}_1 = K.\omega_1$$

and

$$s_1 = \frac{\omega_1 - p.\omega_m}{\omega_1}$$

The stator voltage is obtained from:

$$\overline{U}_{s1} = \overline{E}_1 + \overline{I}_{s1}(R_s + j\omega_1 L_{Ls}) \quad (15)$$

The motor input power at certain frequency and speed can be obtained from:

$$P_{in} = 3.\left|\overline{U}_{s1}\right|.\left|\overline{I}_{s1}\right|\cos\varphi_{s1} \quad (16)$$

where, φ_{s1} is the phase angle between the stator voltage $\overline{U}_{s1}$ and current $\overline{I}_{s1}$. The DC inverter input voltage, U_d is obtained from Eq. 8 as:

$$U_d = \frac{\pi\left|\overline{U}_{s1}\right|}{\sqrt{2}} \quad (17)$$

The inverter input current, I_d can be obtained from:

$$I_d = \frac{P_{in}}{U_d} \quad (18)$$

Consequently, the expression of I_d is obtained using Eq. 16 and 18 as:

$$I_d = \frac{3\sqrt{2}}{\pi}I_{s1}\cos(\varphi_{s1}) \quad (19)$$

The insolation level should be adjusted such that the values of the output power, current and voltage of the PV array equal to the required inverter input power, current and voltage. The required insolation level for certain operating conditions can be determined as follows. The PV array output power, P_g which is the same power as the input power to the inverter is:

$$P_g = I_g U_g \quad (20)$$

Substituting Eq. 7 into Eq. 20 gives:

$$P_g = -I_g^2.R_{sg} + \frac{I_g}{\Lambda_g}.\ln(1+\frac{G.I_{phg} - I_g}{I_{og}}) \quad (21)$$

Since the PV array is directly connected to the six-step VSI/induction motor system thus, the inverter input current, I_d is equal to the array output current, I_g and the inverter input power, P_{in} is equal to the array output power P_g. Therefore, from Eq. 21 we get:

$$P_{in} = -I_d^2.R_{sg} + \frac{I_d}{\Lambda_g}.\ln(1+\frac{G.I_{phg} - I_d}{I_{og}}) \quad (22)$$

where the resistance of the array, R_{sg} is related to the series resistance of the module by:

$$R_{sg} = R_{s\,mod}.\frac{N_m}{N_p}$$

and

$$I_{phg} = I_{ph\,mod}.N_p$$

$$I_{og} = I_{o\,mod}.N_p$$

$$\Lambda_g = \frac{\Lambda_{mod}}{N_m}, R_{s\,mod}$$

is the PV module series resistance, $I_{ph\,mod}$ is the PV module photocurrent and $I_{o\,mod}$ is the PV module reverse saturation current. Equation 22 is a transcendental equation in one unknown which is the insolation level, G. Therefore, in order to obtain the value of G for certain required operating conditions, Eq. 22 is solved numerically.

Required PV array size: In order to determine the size of the required PV array the value of the Peak Sun Hours (PSH) for the site in which the system is installed has to be determined first (Markvart, 1995). For example the value of the PSH obtained for Cairo, Egypt is 5.57 h day^{-1}.

Then, the required load energy, E_L for a typical day in kWh is determined from:

$$E_L = P_{inrat}H \quad (23)$$

Where, P_{inrat} is the rated input power to the motor driving the pump in kW and H is the time of operation of the system in hours per day.

The obtained load energy required per day is corrected to take into consideration the system losses using:

$$E_{Lt} = E_L(1+K_a) \quad (24)$$

Where K_a is an allowance factor and is taken to be 0.4. The obtained energy required from the PV array and the PSH are used to obtain the total power required from the PV array as:

$$P_{gr} = \frac{E_{Lt}}{PSH} \quad (25)$$

The required PV array output voltage, U_{gr} can be obtained neglecting inverter harmonics from (Hamed, 2001; Dewan *et al*., 1984):

$$U_{gr} = \frac{\pi U_r}{\sqrt{2}} \quad (26)$$

Where U_r is the rated motor rms phase voltage. The current required from the PV array is obtained from:

$$I_{gr} = \frac{P_{gr}}{U_{gr}} \quad (27)$$

For a PV module whose maximum power point current is I_{MPPmod} and maximum power point voltage is U_{MPPmod}, the total number of series connected modules per string, N_m can be obtained from:

$$N_m = \frac{U_{gr}}{U_{MPP\,mod}} \quad (28)$$

The total number of parallel connected strings, N_p can also be obtained from:

$$N_p = \frac{I_{gr}}{I_{MPP\,mod}} \quad (29)$$

RESULTS AND DISCUSSION

The method of analysis described in this study was used to obtain results for the performance of the system under consideration using MATLAB programming language. The parameters of the PV modules used in the PV array and the parameters of the induction motor and the centrifugal pump are given in Appendix 1. Figure 4 shows a three-dimensional plot relating the motor speed, the operating frequency and the corresponding required insolation level of the PV array for 4, 6 and 8 operating hours per day considered in sizing the array. From Fig. 4, it is noticed that for certain number of operating hours per day the required insolation level increases as the frequency and motor speed increase. This is because increasing the motor speed results in increasing the output power of the motor.

The increase in the motor output power requires an increase in the motor input power which is the array output power.

The increase in the array output power requires an increase in the insolation level of the PV array. Also from this, it is noticed that as the required operating hours increase at a given frequency and motor speed, the insolation level decreases. This is because increasing the operating hours results in an increase in the energy required from the PV array to be sized which in turn increases the number of series connected modules and parallel connected strings which in turn increases the

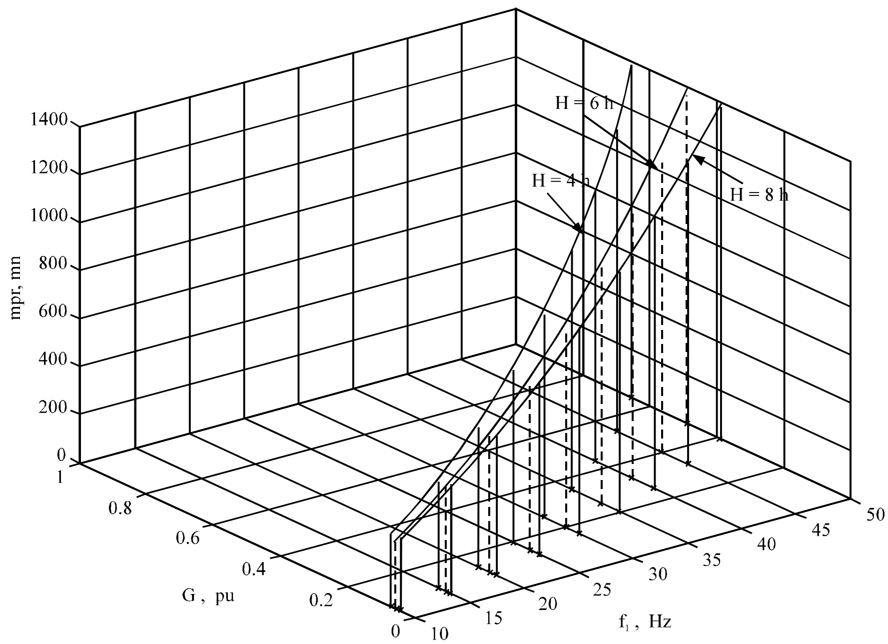

Fig. 4: Speed-frequency-insolation level characteristics

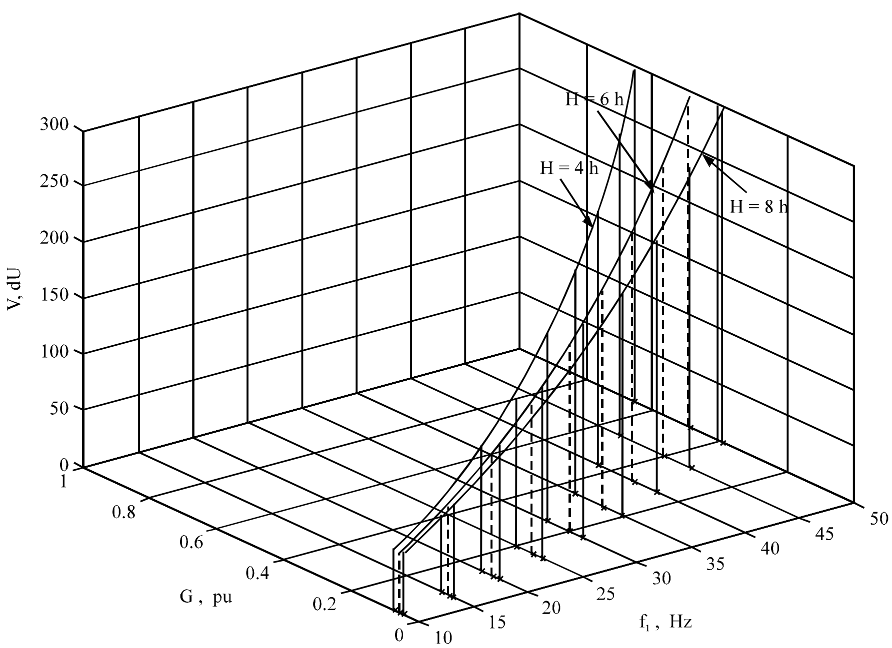

Fig. 5: Inverter input voltage-frequency-insolation level characteristics

output power generated from the array at a given insolation level compared to that sized at lower value of operating hours. Thus, in order to keep the array output power the same at a given frequency and motor speed when the operating hours increases, the insolation level must be decreased.

Figure 5 shows the relationship between the inverter input voltage which is equal to the array output voltage, the operating frequency and the insolation level for the same operating hours used in Fig. 4. From Fig. 5, it is noticed that as the operating frequency increases the inverter input voltage increases and also the required

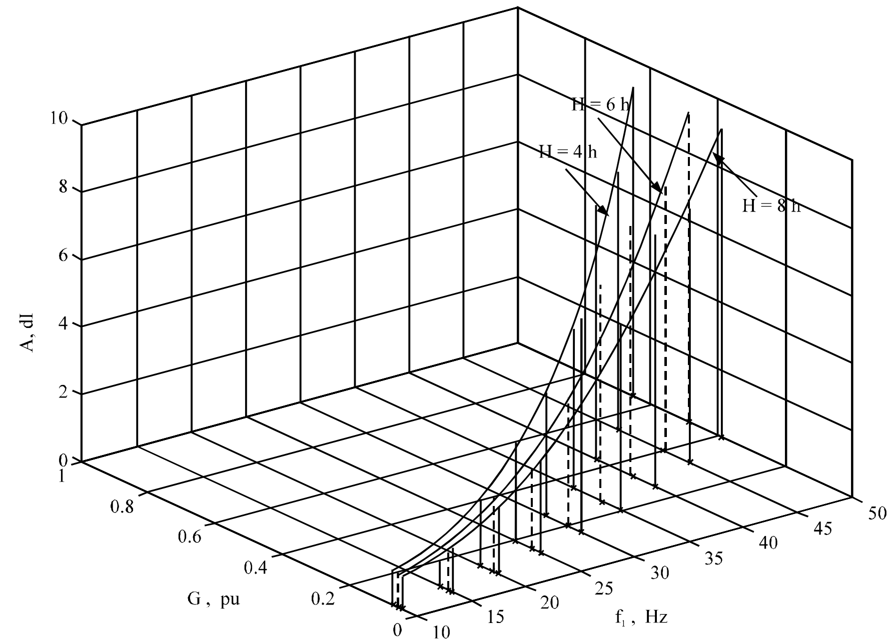

Fig. 6: Inverter input current-frequency-insolation level characteristics

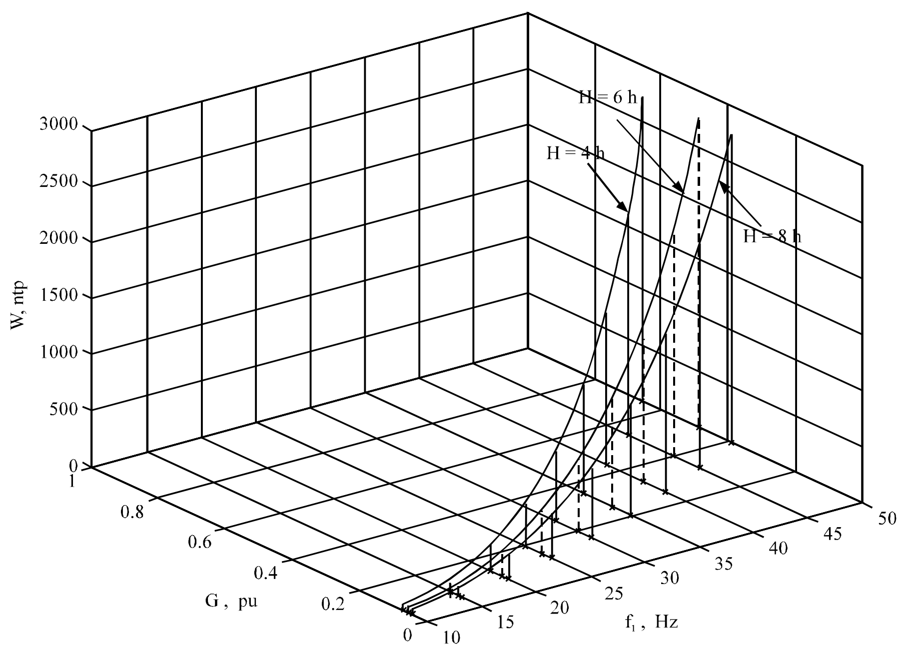

Fig. 7: Inverter input power-frequency-insolation level characteristics

insolation level increases. This is because increasing the operating frequency results in increasing in the insolation level as was shown in Fig. 4. Also from Fig. 4, it is noticed that at a given frequency and array output voltage as the operating hours increases the required insolation level must decrease as was mentioned before. Figure 6 shows the relationship between the inverter input current which is equal to the array output current, the operating frequency and the insolation level for the same numbers of hours considered before . From Fig. 6, it is noticed that as the inverter frequency increases the inverter input current increases and the required insolation level increases. This is because increasing the inverter frequency results in an increase in the insolation level (Fig. 4) which in turn increases the array photocurrent which results in an increase in the array output current which is equal to the inverter input current. Also from this, it is noticed that at a given frequency and array output current as the number of operating hours per day increases the required insolation level must decrease as mentioned before for Fig. 4.

Figure 7 shows the relationship between the inverter input power which is equal to the array output power, the operating frequency and the insolation level for the same numbers of operating hours considered. From Fig. 7, it is noticed that as the inverter frequency increases the inverter input power increases and the required insolation level increases.

This is because increasing the inverter frequency results in increasing the inverter input voltage (Fig. 5) and current (Fig. 6), hence the inverter input power must increase.

Also from Fig. 6, it is noticed that at certain operating frequency and array output power, as the number of operating hours per day increases the required insolation level must decrease, as was mentioned earlier.

CONCLUSION

The steady-state performance of a system composed of a PV array, six-step VSI an induction motor and a centrifugal pump was obtained. From studying the performance of this system, it can be concluded that the PV array can be connected directly without an intermediate converter to the six-step VSI/induction motor system which is controlled such that it operates at a constant air-gap flux while its speed changes by controlling the insolation level of the PV array.

APPENDIX 1

Photovoltaic module data: The following data are at standard test conditions for a SP75 PV module.

Maximum power (P_{MPPmod})	75 W
Maximum power point voltage (U_{MPPmod})	17 V
Maximum power point current (I_{MPPmod})	4.4 A
Open circuit voltage (U_{ocmod})	21.7 V
Short circuit current (I_{scmod})	4.8 A

The calculated parameters are as follows: The PV module series résistance, $R_{smod} = 0.546013\ \Omega$. The PV cell or module reverse saturation current, $I_{omod} = 3.07978 \times 10^{-10}$ A. The PV cell or module photocurrent, $I_{phmod} = 4.8$ A.

Induction motor and pump parameters: The three-phase induction motor is a delta-connected, 2.25 kW, 230 V, 50 Hz, 9 A $\eta = 28.9\%$, 1380 rpm, 4-pole, J = 0.0195 kg m^{-2} motor with the following equivalent circuit parameters: $R_s = R_r' = 3.24\ \Omega$, $L_{ls} = L_{lr}' = 0.03$ H and M = 0.33 H. Centrifugal pump constant (Hamed, 2001):

$$K_W = 7.4552\times10^{-4}\ \text{W/(rad/sec)}^3.$$

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