Linear Programming Decomposition of Real Power Dispatch in Power System Control

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Abstract: This study presents, a framework for decomposing strongly interacting complex electric power systems to controllable sub-units using the development of linear programming. The proposed algorithm relies on a characteristic subsystem description, which reduces the complex power plant to a set of scalar representations and provides the user with the same control flexibility as that associated with single-input single-output system configuration.

Key words: Real power dispatch, strongly interacting power systems, linear programming decomposition, complex large-scale plants

INTRODUCTION

The essence of process control in large-scale electric power plants (Obinabo, 2008; Wang *et al.*, 1993) is to maintain system frequency and voltage levels, while guaranteeing security of supply at minimum cost. The interconnection has evolved over several decades to meet the needs of an ever-growing demand for electricity. The basic topology of modern electric power plants has been configured into, large-scale units, which are often remote from load centres, utilities supplying their customers without depending much on the neighbouring utilities and utilities interconnecting for reliability reasons to help each other during major equipment failures. Consequently, the electric power grid (Ilic, 2007) has several voltage levels, converted from one to the other step-up and step-down transformers. This has led to an extra high voltage meshed transmission backbone network and distribution (local) lower voltage networks closer to the power consumers.

It can be seen from the foregoing that the interconnection of power systems in order to provide security of supply, economic operation and to reduce capital costs of system expansion has introduced more complex control problem, for example, frequency does not suffice any longer as the only index of imbalance between area load and generation. To coordinate these activities, centralized control schemes have been proposed (Liu *et al.*, 2007), which ensure that global information of the entire system is available. However, centralized controllers are very difficult to design and implement in complex large-scale plants due to technical and economic reasons. Furthermore, the coupled scheduling and planning formulation (Contreras and Wu, 1999; Armbruster and Gosnell, 2005) poses a complex operations and planning as a single optimization problem evolving at the same time. The centralized controller designs are also, dependent upon the system structure and cannot handle the structural changes. To overcome these problems, the distributed nature of the plant is taken into consideration when specifying criteria for optimal performance. This is done by splitting, the plant into suitable areas and controlling each area locally. However, because interaction exists between the areas, it cannot be certain that the overall system control is optimal. To overcome this, it may be necessary to retain some unsolved local variables in the sub-problems and if a central computer is made to exercise control over them, it can modify the local controls in order to optimize the overall system.

In this study, the problem of real power dispatch in electric power systems is formulated as a linear program by splitting the plant into a number of the constituent elements and recombining their solutions to obtain an overall solution of the original problem. The study sets the problem in a suitable form for decomposed solution from which a general formulation of optimization of dynamic systems by linear programming is obtained.

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THE CONCEPT OF LINEAR PROGRAMMING

The problems of design and control of large-scale electric power systems can be reduced to one of determining optimum cost or operating conditions (De Farias and Van Roy, 2003; Denardo, 1970) for minimum cost with product cost depending on a large number of interrelated control parameters. Mathematically, the approach (Fletcher, 1987) would be to minimize the integral of the squared difference between a specified function and an approximation to it generated as a function of the control parameters.

Process optimization problems by linear programming (Hordijk and Kallenberg, 1979; Manne, 1960) are formulated for either maximization or minimization and are expressed mathematically as:

$$\phi = \sum_{i=1}^{n} \alpha_{oi} x_i$$

subject to a number of linear constraints,

$$\sum_{i=1}^{n} \alpha_{oi} x_i \leq \alpha_{jo}, \ j = 1, 2, ..., n$$

and non-negative restrictions $x_i \geq 0$, $j = 1, 2, ..., n$. The constraints are transformed into a system of Eq.

$$\sum_{i=1}^{n} \alpha_{ji} x_i + x_{n+j} = \alpha_{jo}, \ j = 1, 2, ..., n \qquad (1)$$

by introducing the slack variable, x_{n+j}, which must also, satisfy the additional non-negativity restrictions $x_{n+j} \geq 0$, $j = 1, 2, ..., n$. If only two variables characterize the problem, we will, without loss of generality, be considering a system split into 2 subsystems (Fig. 1 and 2),

Where:
m   = Vector of manipulated variables.
m_i = Vector of manipulated variables for subsystem i.
y   = Vector of output variables.
y_i = Vector of output variables for subsystem i.
x_i = Vector of interaction variables from subsystem 1-2.
x_2 = Vector of interaction variables from subsystem 2-1.

The interactions between the 2 subsystems are unbalance, that is,
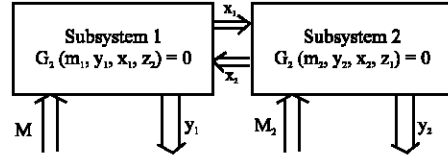
$$x_1 \neq z_1 \qquad (2)$$

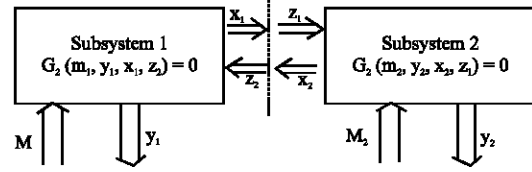Fig. 1: Decomposition of the original problem into 2 subsystems

Fig. 2: Interaction decoupling: Min $P(m, y, x) = P_1(m_1, y_1, x_1) + P_2(m_2, y_2, x_2)$, subject to the conditions indicated for 1 and 2

$$z_2 \neq x_2 \qquad (3)$$

$$\text{Min } P(m, y, x) = P_1(m_1, y_1, x_1) + P_2(m_2, y_2, x_2) \qquad (4)$$

The second level problem must therefore attempt to force the interactions into balance, that is,

$$x_1 = z_1 \qquad (5)$$

$$z_2 = x_2 \qquad (6)$$

at the same time as optimizing the objective functions. This may be done by the inclusion of a penalty term

$$P(m, y, x, z, \lambda) = P_1(m_1, y_1, x_1) + P_2(m_2, y_2, x_2) + \lambda^T (x - z) \qquad (7)$$

where, λ is a vector of weighting parameters (positive or negative as needed), which penalize any unbalance. Expanding the penalty term

$$\lambda^T (x - z) = \lambda_1^T (x_1 - z_1) + \lambda_2^T (x_2 - z_2) \qquad (8)$$

we arrive at the coordination strategy in which the goals of the subsystems have been modified to achieve coordination. Solution would proceed by iteration between the 2 levels. The overall objective function is additively separable,

$$P(m, y, x) = P_1(m_1 y_1, x_1) + P_2(m_2, y_2, x_2) \quad (9)$$

Now for the time being fixed, the interactions at any arbitrary value $\underline{z}$ is:

$$\text{Constraint } \underline{x} = \underline{z} \quad (10)$$

We now split the problem into first and second level problems, such that the first level chooses m and y to minimize P with the given interaction z and the second (higher) level chooses a new z to minimize H. The co-ordination proceeds by modifying the subsystem process models. The minimization proceeds sequentially, with the first and second level problems being solved alternatively. Generally, the optimization problem would be to maximize the objective function,

$$\phi = \underline{a}_R^T \underline{x} \quad (11)$$

subject to

$$A\underline{x} = \underline{a}_c \quad (12)$$

$$\underline{x} \geq 0$$

where,

$$\underline{x}^T = (x_1, x_2, ..., x_{n+m})$$

and

$$\underline{a}_R^T = (a_{o1}, a_{o2}, ..., a_{on}, a_{o,n+1}, ..., a_{o,n+m})$$

and

$$a_{o,n+1} = a_{o,n+2} = ... = a_{o,n+m} = 0$$

A is an $m \times (m+n)$ matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} & 1 & 0 & ... & 0 \\ a_{21} & a_{22} & ... & a_{2n} & 0 & 1 & ... & 0 \\ . & & & & & & & \\ . & & & & & & & \\ a_{m1} & a_{m2} & ... & a_{mn} & 0 & 0 & ... & 1 \end{bmatrix} \quad (13)$$

Thus, the constraint system of Eq. 12 consists of m equations in (m+n) unknown

$$x_1, x_2, ..., x_n \quad n = M + N \quad (14)$$

If A has rank m, a set of n variables can always be chosen from among the (m+n) variables $x_1, x_2, ..., x_{m+n}$, such that the system

$$A\underline{x} = \underline{a}_c \quad (15)$$

is uniquely solvable in terms of the remaining m variables. If the problem requires that the problem is to maximize the function

$$L = \sum_{i=1}^{n} c_i^T x_i \quad (16)$$

subject to

$$\sum_{i=1}^{n} A_i x_i = b \quad (17)$$

$$B_i x_i = b_i, \ i = 1, ..., n \quad (18)$$

$$x_i \geq 0 \quad (19)$$

where, A is an $m \times n$ matrix, B_i is an $m_i \times n_i$ matrix. C_i and x_i are n_i vectors, b is an m vector. Also, b_i is an m_i vector. We define a bounded convex domain S_i.

$$S_i = \{ x_i / x_i \geq 0; \ B_i x_i = b_i \} \quad (20)$$

Let, the set of all corner points of the domain S_i be denoted by

$$W_i = \{ x_{i1}, x_{i2}, ..., x_{ik_i} \} \quad (21)$$

If we introduce new variables z_{ik} we can represent any point x_i in S_i by a convex combination of the corner points x_{ik}, i.e.,

$$x_i = \sum_{k=1}^{k_i} z_{ik} x_{ik} \quad (22)$$

$$\sum_{k=1}^{k_i} z_{ik} = 1, \ \text{for all i.} \quad (23)$$

$$z_{ik} \geq 0, \ \text{for all k.} \quad (24)$$

This is analogous to attaching weights z_i to the corners of the hyper-surface and the finding its centre of gravity. We now have a way of representing all the feasible points in the sub problem in terms of some new variables z_{ik}. The rest of the linear, program is now transformed into an equivalent form by introducing some new control problems

$$P_{ik} = A_i x_{ik}, \ k = 1, ..., k \quad (25)$$
$$\text{and } i = 1, ..., n$$

$$c_{ik} = c_i^T x_{ik} \qquad (26)$$

Through these quantities, P_{ik} and c_{ik} the effect of the new variables z_{ik} on the original problem's cost function and overall constraints is clearly obvious. We now have a linear program, which is a linear transformation of the original problem

$$\text{Min} \sum_{i=1}^{n} \sum_{k=1}^{k_i} c_{ik} z_{ik} \qquad (27)$$

Subject to

$$\sum_{i=1}^{n} \sum_{k=1}^{k_i} P_{ik} z_{ik} = b \qquad (28)$$

$$\sum_{k=1}^{k_i} z_{ik} = 1, \text{ for all } i. \qquad (29)$$

$$z_{ik} \geq 0, \text{ for i and k} \qquad (30)$$

Theorem: Let the quantities z_{ik} be the solution to Eq. (27-30).
Then the vectors

$$x_i = \sum_{k=1}^{k_i} x_{ik} z_{ik}, \quad i = 1, ..., n$$

form the solution of Eq. (16-19)

Proof: Let problem Eq. (16-19) be denoted by I and Eq. (27-30) by II. Then we need to show only that the minima of the objective functions c_I and c_{II} are the same and that each feasible point of I corresponds uniquely to a feasible point of II.

Let $\dot{x}_i$ be the solution of I, then by Eq. (17 and 18) we have $\dot{x}_i \in S_i$. Hence, $\dot{x}_i$ can be represented by:

$$\sum_{k=1}^{k_i} z_{ik} x_{ik}, \text{with} \sum_{k=1}^{k_i} z_{ik} = 1, z_{ik} \geq 0 \qquad (31)$$

and Eq. (17) changes to

$$\sum_{i=1}^{n} A_i x_i = b = \sum_{i=1}^{n} A_i \sum_{k=1}^{k_i} z_{ik} x_{ik} = \sum_{i=1}^{n} \sum_{k=1}^{k_i} z_{ik} P_{ik} \quad (32)$$

Thus, the constraint of II are satisfied and we have

$$c_I = \sum_{i=1}^{n} c_i^T x_i = \sum_{i=1}^{n} c_i^T \sum_{k=1}^{k_i} z_{ik} x_{ik} = \sum_{i=1}^{n} \sum_{k=1}^{k_i} c_{ik} z_{ik} \geq c_{II}$$

$$(33)$$

Conversely, it is easy to show that the optimal point of II corresponds to a feasible point of I and that therefore,

$$c_{II} \geq c_I \qquad (34)$$

DECOMPOSITION OF THE REAL POWER DISPATCH

A linearized state space representation of a multivariable power system with voltage and frequency dependent loads is considered in this study. The system was assumed to consist of n generating nodes and m load nodes connected by transmission lines and transformers. The linearization was achieved by assuming negligible transformer voltages and stator resistances. Saturation was ignored and machine damping, including the effect of damper windings was taken into account by introducing a damping constant in the swing equation. A system of equations was consequently obtained, which can be written as (Brucoli *et al.*, 1985; Obinabo, 2008):

$$\dot{x} = Ax + Br \qquad (35)$$

where, x is a composite state vector of dimension $3n \times m$ and r a 2n- dimensional composite input vector. A and B are constant matrices of appropriate dimensions. The presence of frequency-dependent loads implies a system description, which in addition to the 3n state variables characterizing the n generating units, includes m state variables relative to m loads.

The control problem under consideration is to set the demanded active power dispatch of each generator in a system, such that the production cost is minimized and at the same time security of supply and operational limits are adequately satisfied. The overall problem is therefore, a very large scale constraints static optimization. The dynamics of the system are assumed to be slow so that they can be accommodated by repetitive intervention by the control computers. The problem is linearized as a reasonable approximation and to keep it manageable with the available time. Firstly, the production cost is approximated by a linear function to be minimized as follows:

$$\min \underline{c}^T \underline{x} \qquad (36)$$

The elements of $\underline{x}$ are the power of the generator elements of $\underline{c}$ are their incremental cost in f per WM. This minimization must be carried out subject to operational constraints, which divide naturally into individual constants, group constraints and overall constraints. The output of each generator must be within upper and lower limits.

$$\underline{X}_{min} \le \underline{x} \le \underline{X}_{max} \qquad (37)$$

The sum of the outputs of generators in one station or group must also be within limits, often derived from boiler or network limitation:

$$S_{min} \le \sum_{i=1}^{n_i} x_i \le S_{max} \qquad (38)$$

The total generation, must equal the total P_o

$$\sum_{i=1}^{N} x_1 = P_o \qquad (39)$$

It is also, possible to represent spare capacity and network flow limitations as linear constraints.

The spare capacity is be represented

$$S_i \le K_{2i}$$

$$S_i \le K_{1i} + h_{1i}\, x_i$$

$$S_i \le K_{3i} - h_{2i}\, x_i$$

$$\sum_{i=1}^{n} S_1 \ge S_o \qquad (40)$$

and the network constraints.

$$\sum_{\text{certain i}} x_i \ge G_1 \ (\text{import constraint}) \qquad (41)$$

$$\sum_{\text{certain i}} x_i \le G_2 \ (\text{export constraint}) \qquad (42)$$

The vectors $\underline{c}^T\, \underline{x}$ are the generator costs. The vectors $\underline{A}_i$ are the overall constraints (overall generation requirement, import-export restriction and total spare). The matrices $\underline{B}_i$ are the sub-system and individual constraints (station and generator effective maxima and minima).

CONCLUSION

The results established in this study are, compared with those obtained for cascade and parallel decomposition of large-scale composite systems from the associated transfer functions to equivalent state equations (Obinabo, 2008). First, our result is seen strictly as a system theoretic approach of decomposing linear multimachine interconnections into equivalent

controllable sub-units using the development of linear programming. In parallel decomposition, a state set is decomposed into direct summand machines (Hartmantis and Stearns, 1966; Kalman *et al.*, 1969), each summand operating independently of the others so that the next state of the machine depends only on the input and its own current state. The current state in this application is restructured by some functions of the current states of the component machines. The basic feature here is to decompose a linear system Σ into simpler linear systems by finding sub modules Γ and $\wedge$ of the state module X such that $\Gamma \cap \wedge = 0$. Subsequently, X is embedded in $X/\Gamma \oplus X/\wedge$ and a new system Σ' is constructed with this state set, the input/output properties of Σ and Σ' being identical. The problem is thus, to find Γ and $\wedge$ that lead to simplification in the description of Σ'. For real power dispatch in electric power systems, this means decomposing the state module into a direct sum of, say, R(z) module. On the other hand, cascade decomposition emphasizes simulation of the original machine by simpler machines that do not necessarily operate independently. In particular, the component machines are conceived as an arrangement in chain where the next state of a given machine depends not only on the input and its own current state, but also on the current state of the preceding machine in the chain. Intuitively therefore, it should seem reasonable the linear programming decomposition of the interconnected composite system considered in this study can be achieved under much simpler and straightforward conditions than those for parallel and serial decomposition since, the feature of absolute independence is slackened.

We have shown that the problem of controlling interconnected multi-machine power systems can be set in suitable form for decomposed solution and formulated as a linear program. This was accomplished by setting the desired real power of each generator in the system such that the production cost was minimized, while satisfying security of supply and operational limits. The study has given the fundamental theoretical insight into the operation of a large class of dynamic systems of the type considered by Obinabo (2008). More importantly, the method can be used as a starting point for investigating systems with more detailed structures and can be applied to situations, where the network constraints are finite.

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