

A Novel Generalized Predictive Controller for the Speed Control of PMBLDC Motor

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Abstract: Usually speed and position of a permanent magnet brushless direct current motor (PMBLDCM) rotor is controlled in a conventional cascade structure. The inner current control loop runs at a larger bandwidth than the outer speed control loop to achieve an effective cascade control. This study proposes a multirate based General Predictive Control (GPC) law performed for the conventional cascaded PI-PI scheme. Both speed and torque tracking objectives are achieved in matched and mismatched parameter case. The measured and unmeasured disturbances can be effectively rejected and the motor can run at desired speed at constant load. In addition, non-minimum phase characteristics and system constraints can effectively be handled by proposed GPC algorithm. The simulation results are provided to show that the proposed GPC strategy results in better performance than that of the conventional well tuned cascade PI-PI control strategy. The proposed controller has also been implemented to control the speed of a PMBLDCM and the results show an improvement over conventional scheme.

Key words: PMBLDC motor, cascade control, GPC and cost function

INTRODUCTION

In recent years, PMBLDC machines have gained widespread use in electric drives. These machines are ideal for use in clean, explosive environments such as aeronautics, robotics, electric vehicles, food and chemical industries and dynamic actuation. For using these machines in high-performance drives requires advance and robust control methods (Rubaii and Yalamanchili, 1992). Conventional control techniques require accurate mathematical models describing the dynamics of the system under study. These techniques result in tracking error when the load varies fast and overshoot during transients. In lieu of provisions for robust control design, they also lack consistent performance when changes occur in the system. If advance control strategies are used instead, the system will perform more accurately or robustly. It is, therefore, desired to develop a controller that has the ability to adapt even structure online, according to the environment in which it works to yield satisfactory control performance. An interesting alternative that could be investigated is the use of GPC in the PMBLDCM drive.

The conventional approach to the design of speed and current controllers, cascade structure is used

(Ling *et al.*, 2004). The conventional schemes lead to over sampling to the outer loops due to the multirate sampling. This will increase the computational complexity, sometimes incompatible with chosen sampling frequency; hence it requires a new prediction horizon.

Over the last decade, GPC has received increasing attention in many control applications (Clarke 1994). For example, GPC has been used in an operating theater to control the on-line administration of muscle relaxant drugs (Linkens and Mahfouf, 1994). In, Kassapakis and Warwick (1994), it is used in the autopilot control of a roll movement in a jet fighter aircraft. In the process industries, it has been used in steel casting (Jolly and Bentsman, 1993), glass processing (Kay-Soon *et al.*, 1998), oil refineries, etc. All these applications have demonstrated that GPC has good performance, efficiency and robustness against unmodeled disturbances as compared to some conventional control methodology. A cascade predictive structure with an adaptive predictive controller for inner loop and conventional PID controller for outer loop was implemented in a distributed collector solar field. To control an open-loop unstable Continuous Stirred Tank Reactor System, Nagraath *et al.* (1997) developed a state estimation-based model predictive control approach which employs a single MPC strategy

that incorporates both loops' measurements and manipulates the system input. The predictive cascade controllers for the control of position and speed of an induction motor have been reported by Silva *et al.* (1997) and Dumur *et al.* (1996). In this research, an alternative design approach to conventional cascade speed control scheme is proposed, in which both the current and speed control loops are configured with new General Predictive Control (GPC) algorithm. The system parameters are modified based on predictive cost functions. In addition, GPC handles non minimum phase characteristics and system constraints on speed and current loops. Through the prediction system output via a receding horizon method over several sampling intervals, an optimal control at every instant can be achieved. The performance of the drive is examined by subjecting the motor to the inertia loads and the time-varying loads. The proposed approach has been implemented in MATLAB software tool and the results are presented. The dynamic performance and robustness of the control schemes are also discussed.

PMBLDCM DRIVE SYSTEM

A control system for the speed control of a PMBLDC motor drive system is presented in Fig. 1. The cascade control scheme is the common standard for the control of electric drive systems. Such a controller is practically complex and requires lot of manipulations and retuning methods.

The speed controller $G_s(s)$ computes an output signal that is the torque needed to accelerate the motor to the desired speed. The desired current $I_{ref}(s)$ that the motor needs to produce the torque is calculated from a mathematical model of the motor. The inner loop controls the current that is needed to produce the torque.

The output of the controller $G_i(s)$ is used as set point to the power converter which produces the necessary input voltage to the motor. The transfer function from the rotor current set point $I_{ref}(s)$ to the rotor current is the closed loop transfer function of the inner loop and is given by:

$$G_{icl}(s) = \frac{I_a(s)}{I_{ref}(s)} = \frac{G_i(s)G_r(s)G_{p1}(s)}{1 + G_i(s)G_r(s)G_{p1}(s)} \quad (1)$$

where, $G_i(s)$ is transfer function model of the power converter, given by:

$$G_r(s) = \frac{K_r}{1 + sT_r}$$

and $G_{p1}(s)$ is the electrical part of the PMBLDCM model.

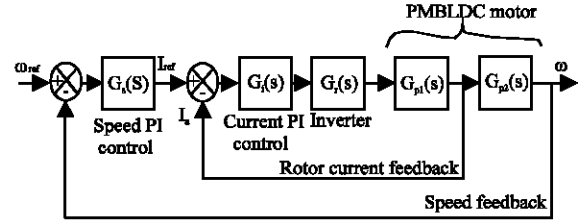


Fig. 1: Block diagram of cascade PI speed control for PMBLDCM system

If the gain of the speed controller is large, then the inner closed loop controller will approach to unity and will also be quite insensitive to variations in the power converter and/or motor transfer functions. Non-linear behaviour of the motor and converter can often be modeled by transfer functions with variable coefficients. From the output of the speed controller, there are 3 quite simple systems in series, the gain K_T , the current control loop $G_{ia}(s)$ ($G_{ia}(s)=1$) and the mechanical part of the motor model $G_{p2}(s)$. Thus, the cascade structure eliminates many of the inherent complexities in the power converter and motor dynamics. However, windup problem in cascade control systems needs special attention. To avoid wind up in the speed controller, saturation of secondary controller should be known. In some process control systems the inner controller is set to manual mode when the secondary controller saturates. For computer control, it is important to consider the execution order of both inner current controller and outer speed controller. The sampling time for the current control loop may be significantly longer. If the speed controller is executed first, then setpoint to the current controller is updated. Otherwise, the secondary controller may be fed with an unnecessary old setpoint value. The output from the speed controller is directed to the current controller which is its setpoint value. The conventional cascade PI controller has been tuned to behave well, as if there were no time delay. The difficulty with system delays creates stability problems. Hence, the problem of controlling systems with time delay was used (Maaziz *et al.*, 1999).

The main objective of this study is to have fast response, less overshoot and accurate tracking with the desired setpoint, however, these characteristics need a carefully designed and practical control strategy, preserving enough robustness to cope with uncertainty in the plant. To achieve the above requirements, this study applies a double GPC to the PMBLDCM system in an existing cascade control structure as shown in Fig. 2. Because GPC control is an online optimization approach to satisfy multiple, changing performance criteria, under existing PMBLDC motor control hardware scheme.

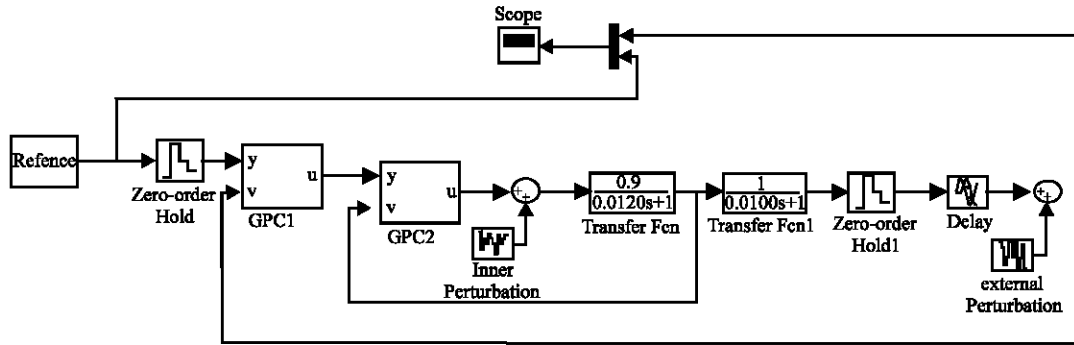


Fig. 2: Block diagram of cascade double GPC for PMBLDCM system

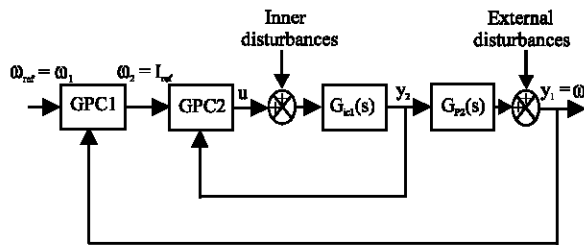


Fig. 3: Simulation diagram for Cascade double GPC

CASCADED DOUBLE GPC

Basics of GPC: The interest in developing a multirate cascade control system using GPC is the possibility to control the speed and current together. To realize this, two GPC control algorithm is computed as shown in the Fig. 3:

Where,

- u : The resulting control signal applied to the system.
- ω_r : The speed setpoint ($\omega_1 = \omega_r$).
- ω_2 : The inner signal coming from the minimization of GPC1.

The speed and current loops of the cascaded structure imply the definition of two GPC algorithms and consequently the minimization of cost function. It is also necessary to define the numerical models of GPC to the individual loops.

NUMERICAL MODEL OF GPC

A cascaded predictive strategy first requires the definition of the numerical model for both speed and current loops of the PMBLDCM system. Often, the time varying system dynamics can be described by a controlled autoregressive and integrated moving average

(CARIMA) model for a general r inputs and n outputs system with n_d measurable input disturbances can be expressed as

$$A(q^{-1})y(t) = B(q^{-1})u(t-d) + \frac{C(q^{-1})\epsilon(t)}{\Delta} + D(q^{-1})u_d(t-d) \quad (2)$$

where, A , B , C and D are matrices of polynomial in the delay operator q^{-1} with dimension $n \times n$, $n \times r$, $n \times n$ and $n \times n_d$, respectively.

The input is delayed by a time (d), Δ is the differencing operator $1 - q^{-1}$. In most of cases $C(q^{-1}) = 1$ (Shinskey, 1981):

$$\begin{aligned} y(t) &= [y_1(t) y_2(t) \dots y_n(t)]^T \\ u(t) &= [u_1(t) u_2(t) \dots u_r(t)]^T \\ u_d(t) &= [u_{d1}(t) u_{d2}(t) \dots u_{nd}(t)]^T \\ \epsilon(t) &= [\epsilon_1(t) \epsilon_2(t) \dots \epsilon_n(t)]^T \end{aligned}$$

Controller formulation: The main objective of the GPC is to detect the changes in manipulated variable with a system delay time of one sampling instant is

$$\Delta u(t) = u(t) - u(t-d) \quad (3)$$

This would make the output best match to a target value $y_r(t+N)$ in the presence of disturbances and system constraints. In long range predictive control, a predicted projection of outputs $y_r(t)$ over p -future time intervals ($t+N_1$) to ($t+N_2$) is matched to the setpoint trajectory ω_r by prescribing the sequence of m - future moves:

$$\Delta u(t), \dots, \Delta u(t+m-1)$$

Where,

- N_1 : The minimum costing horizon
 N_2 : The maximum costing horizon

Inner GPC2 model:

$$A_2(q^{-1})y_2(t) = B_2(q^{-1})u(t-d) + \frac{\varepsilon_2(t)}{\Delta} \quad (4)$$

Outer GPC1 model:

$$A_1(q^{-1})y_1(t) = B_1(q^{-1})y_2(t) + \frac{\varepsilon_1(t)}{\Delta} \quad (5)$$

where, A_1 , A_2 , B_1 and B_2 are polynomial in the backward shift operator q^{-1}

$$\begin{aligned} A_1(q^{-1}) &= 1 + a_{11}q^{-1} + a_{12}q^{-2} + \dots + a_{1n}q^{-n_{a1}} \\ A_2(q^{-1}) &= 1 + a_{21}q^{-1} + a_{22}q^{-2} + \dots + a_{2n}q^{-n_{a2}} \\ B_1(q^{-1}) &= b_{01} + b_{11}q^{-1} + b_{12}q^{-2} + \dots + b_{1m}q^{-m_{b1}} \\ B_2(q^{-1}) &= b_{02} + b_{21}q^{-1} + b_{22}q^{-2} + \dots + b_{2m}q^{-m_{b2}} \end{aligned}$$

$\varepsilon_1(t)$ and $\varepsilon_2(t)$ are uncorrelated random sequence with zero mean for outer and inner GPC models. $\Delta(q^{-1})$ ensures an integral control law.

Since, speed control PMBLDCM is a SISO system, the predicted outputs $\hat{y}(t + N^1)$ and $\hat{y}(t + N^2)$ can be obtained by recursively iterating Eq. 2.

The predicted output can be expressed in vector notation as follows:

$$\hat{Y} = GU + P \quad (6)$$

Where,

$$\begin{aligned} \hat{Y} &= [y(t + N_1) \dots y(t + N_2)]^T \\ \Delta u &= [\Delta u(t) \dots y(t + N_u - 1)]^T \\ P &= [p(t + N_1) \dots p(t + N_2)]^T \end{aligned}$$

G is the step response matrix

$$G = \begin{bmatrix} g_{N_1} & g_{N_1} - 1 & 0 & 0 \\ g_{N_1} + 1 & g_{N_1} & \dots & 0 \\ g_{N_2} - 1 & g_{N_2} - 2 & \dots & g_{N_2} - N_u \\ g_{N_2} & g_{N_2} - 1 & \dots & g_{N_2 - N_u + 1} \end{bmatrix}$$

P is called free response as it is affected by past control action only. $P(t + j)$ can be easily calculated for all j values by iterating PMBLDCM model and a future control equals the previous control variable $u(t-1)$.

Considering multistage cost function (Silva *et al.*, 1997)

$$J_{GPC} = \sum_{j=N_1}^{N_2} [y(t+j) - w(t+j)]^2 + \sum_{j=N}^{N_u} \lambda \Delta u^2(t+j-1) \quad (7)$$

Where,

$w(t+j)$: A future reference trajectory, which is a pre-specified set point $y_r(t)$

λ : A weighting upon future control increments.

Thus the cost function can be written as:

$$J_{GPC} = (GU + P - W)^T (GU + P - W) + \lambda U^T U \quad (8)$$

The solution minimizing J_{GPC} gives an optimal value, which is suggested for control increment sequence. The Optimum value for the prediction sequence will be:

$$U_{opt} = (G^T G + \lambda I_N)^{-1} G^T (w - p) \quad (9)$$

From the above Eq. 7-9, the cost function for the inner GPC2 model and the external GPC1 models are derived as:

$$J_{GPC2}(N_{12}, N_{22}, N_{u2}) = \sum_{j=N_{12}}^{N_{22}} [\hat{y}_2(t+j) - w(t+j)]^2 + \sum_{j=1}^{N_{u2}} \lambda_2(j) [\Delta u_2(t+j-1)]^2 \quad (10)$$

$$J_{GPC1}(N_{11}, N_{21}, N_{u1}) = \sum_{j=N_{11}}^{N_{21}} [\hat{y}_1(t+j) - w_1(t+j)]^2 + \sum_{j=1}^{N_{u1}} \lambda_1(j) [\Delta u_1(t+j-1)]^2 \quad (11)$$

As a standard generalized predictive control (Hamed *et al.*, 1990; Clarke and Mohtadi, 1989), Eq. 10 and 11 can be written as

$$\hat{Y}_1 = G_1 U_1 + P_1 \quad (12a)$$

$$\hat{Y}_2 = G_2 U_2 + P_2 \quad (12b)$$

It is noted from Fig. 2 that the external GPC1 control variable ω_2 acts as set point to the inner GPC2 and the external loop ω_1 should track setpoint ω . G_1 and G_2 are the step response matrices for the external loop and inner loop system, respectively.

Substituting Eq. 12 in 10 and 11 we get,

$$J_{GPC1} = (G_1 U_1 + P_1 - w_1)^T (G_1 U_1 + P_1 - w_1) + \lambda_1 U_1^T U_1 \quad (13)$$

$$J_{GPC2} = (G_2 U_2 + P_2 - w_2)^T (G_2 U_2 + P_2 - w_2) + \lambda_2 U_2^T U_2 \quad (14)$$

Through computing $\partial J_{GPC1} / \partial U_1 = 0$ and $\partial J_{GPC2} / \partial U_2 = 0$, the optimal control variables for the optimal speed control system are obtained as follows:

$$U_{opt1} = (G_1^T G_1 + \lambda_1 I_{N11})^{-1} G_1^T (w_1 - p_1) \quad (15)$$

$$U_{opt2} = (G_2^T G_2 + \lambda_2 I_{N22})^{-1} G_2^T (w_2 - p_2) \quad (16)$$

The proposed cascade double GPC algorithm is implemented as follows

Step 1: Set sampling time for the external and inner loops as T_1 and T_2 .

Step 2: Set maximum, minimum predictive horizon and control horizon for the 2 loops.

Step 3: Estimate the CARIMA model to yield G_1 , G_2 and P_1 , P_2 .

Step 4: Compute matrix G_1 , G_2 and $(G_1^T G_1 + \lambda_1 I_{N11})$, $(G_2^T G_2 + \lambda_2 I_{N22})^{-1}$.

Step 5: Determine the variables U_{opt1} and U_{opt2} based on Eq. 15 and 16.

Step 6: Set the increment as $k = k + 1$, go back to Step 3.

RESULTS AND DISCUSSION

A generalized predictive control algorithm has been presented and its application to a PMBLDCM has been investigated. The plant is characterized by fast-dynamic nonminimum phase behaviour with nonlinearities.

Plant parameters are:

- Number of turns per phase = 100.
- Resistance per phase = 1.4Ω .
- Self inductance per phase = 2.44 mH.
- Mutual inductance per phase = 1.5 mH.
- Maximum value of flux density = 60 wb/m².
 - Rotor length = 0.03 m.
 - Rotor radius = 0.02 m.
 - Value of viscous friction = 0.002.
- Moment of Inertia = 0.0002 Kg-m².
 - Total number of phases = 4.

The time constant of the current loop is $T_c = 20$ ms, while that of the outer loop is almost $T_s = 230$ ms. In order to show the interaction effects, multiple rates sample time should be considered. Here $T_1 = 8$ ms and $T_2 = 2$ ms. The GPC parameters have been chosen to design controllers as follows (Table 1).

Very often N_{u1} and N_{u2} is chosen so that $N_{u1} < N_{21}$ and here we stressed the fact $N_{u1} = 1$ and $N_{u2} = 1$.

Simulation results: Figure 4-10 show different responses of cascade double GPC based on different types of set points. Let the set point equal to the step sequence (Fig. 4 and 5) In order to examine the track performance along with operating condition change, a square wave set point is introduced as shown in Fig. 6. From the Fig. 7 and 8, the set-point performance is extremely good for both positive and negative ramps. The Fig. 4-8 show that cascade GPC can stabilize system output around desired trajectories with minor oscillation.

Table 1: GPC parameters

N_{11}	N_{21}	N_{u1}	N_{12}	N_{22}	N_{u2}	λ_1	λ_2
1	12	1	2	20	1	0.000002	0.0000004

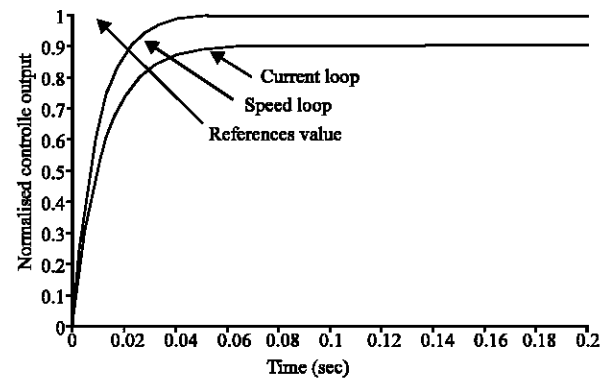


Fig. 4: The control variables of current and speed loops for the given step reference

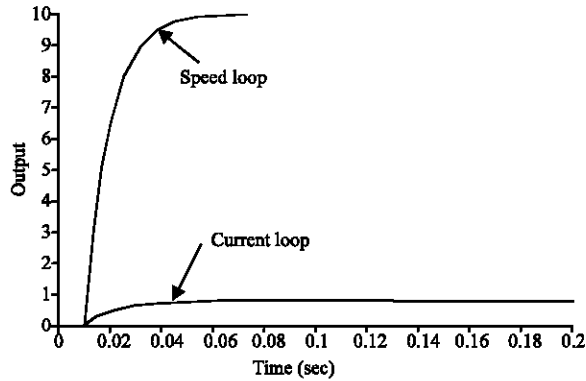


Fig. 5: Output responses to speed and current loops

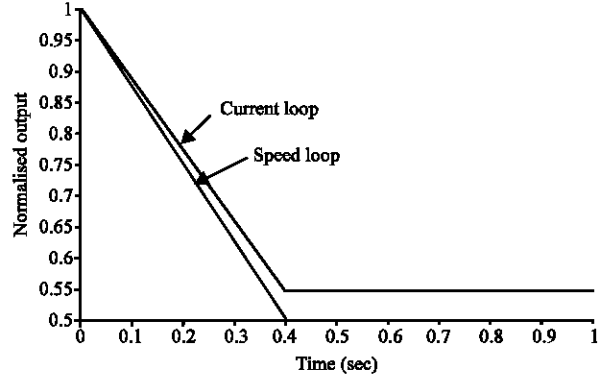


Fig. 8: The output of current and speed loops with ramp decreasing reference

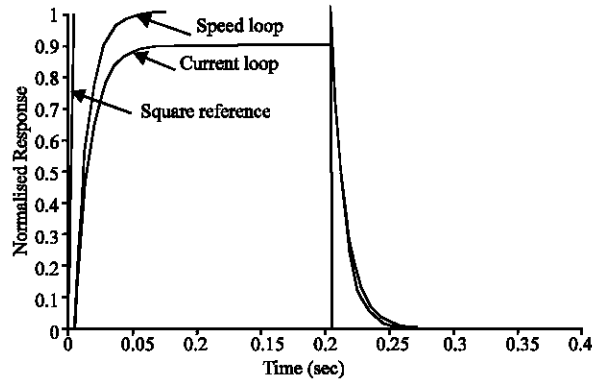


Fig. 6: Output of speed and current loops for the given square reference

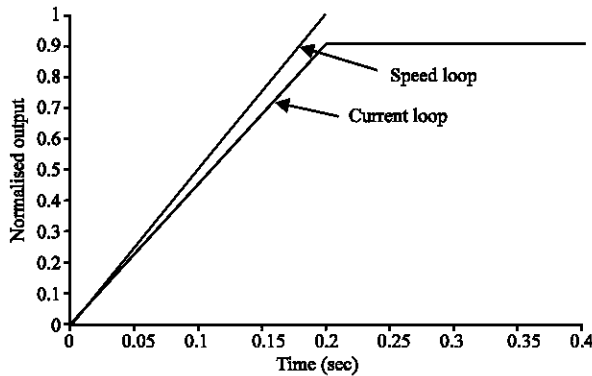


Fig. 7: The output of current and speed loops with increasing ramp reference

Second, a type of white noise, commonly encountered in real time systems, is introduced in the inner loop. Figure 9 (a and b) shows simulation results with different kinds of set-point trajectory. Not only does the outer controller tackle model uncertainty problems,

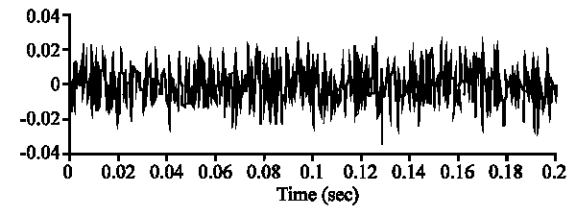


Fig. 9a: The disturbance of inner loop

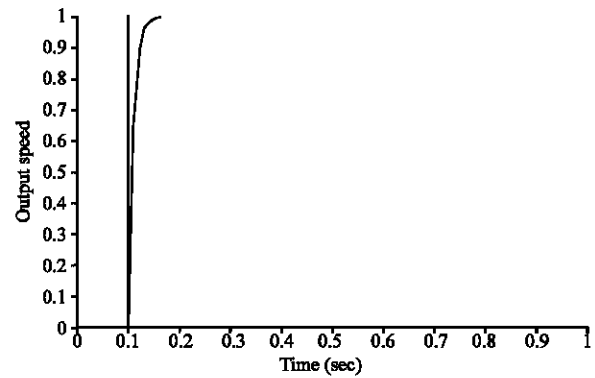


Fig. 9b: The output of the speed loop for the inner loop disturbance

but the inner one rejects disturbance. Finally, cascade PI and cascade GPC are compared at the same operating conditions (Fig. 10).

As shown in Fig. 10, the cascade GPC scheme exhibits a satisfactory performance that achieves a fast and non-oscillatory convergence of system output. However, cascade PI has more than 10% overshoots as set point steps from 0 to 1, which may cause the actuator to switch frequently. It is concluded that cascade GPC makes full use of advance knowledge of future requirements to achieve improved performance over the well tuned cascade PI controller.

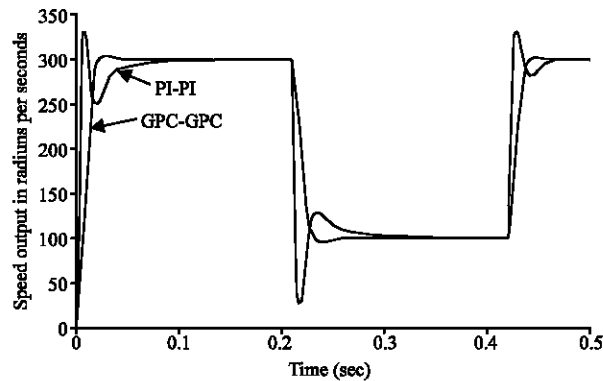


Fig. 10: Comparison of cascade PI algorithm with cascade double GPC algorithm with variable step

CONCLUSION

A cascade GPC for speed control of PMBLDCM is presented in this study. The inner loop has used an adaptive based model predictive controller, exploiting information conveyed by accessible disturbances, while outer loop used a GPC to restrain the error from nonlinear identification of the generalized system. Based on PMBLDCM models, simulation results showed that cascade GPC outperformed than the well tuned cascade PI controller. Experiment demonstrated that a satisfactory system output and smooth feasible control actions can be achieved. The novel control scheme which successfully replaced the well tuned cascade PI control algorithm usually adopted in many motor control drives.

REFERENCES

Clarke, D.W. and C. Mohtadi, 1989. Properties of generalized predictive control. *Automatica*, 25 (1): 859-875.

Clarke, D.W., 1994. Advances in model-based predictive control. *Advances in Model-Based Predictive Control*. Oxford, U.K: Oxford Science, pp: 3-21.

Dumur, D., P. Boucher and T. Kolb, 1996. Application of cascaded constrained receding horizon predictive control to an Induction machine. *Proc. IEEE. Int. Conf. Control Applications*, Dearbon MI, pp: 888-893.

Hamed M. Al-rahmani and Gene F. Franklin, 1990. A new optimal multirate control of linear periodic and time-invariant systems. *IEEE. Trans. Automatic Control* 35 (4): 406-415.

Jolly, T. and J. Bentsman, 1993. Generalized predictive control with dynamic filtering for process control applications. In: *Proc. Am. Control Conf.* San Francisco, CA, 2: 1741-1745.

Kassapakis, E.G. and K. Warwick, 1994. Predictive algorithm for autopilot design. In: *Advances in Model-Based Predictive Control*. Oxford, U.K.: Oxford Science, pp: 458-470.

Kay-Soon Low, Koon-Yong Chiun, and Keck-Voon Ling, 1998. Evaluating generalized predictive control for brushless dc drive. *IEEE. Trans. Power Elec.*, 13 (6): 1191-1198.

Ling, K.V., W.U. Bingfang, H.E. Minghua and Zhang Yu, 2004. A Model predictive controller for multirate cascade system. *Proc. Am. Control Conf. ACC USA*, pp: 1575-1579.

Linkens, D.A. and M. Mahfouf, 1994. Generalized Predictive Control (GPC) in clinical anaesthesia. In: *Advances in Model-Based Predictive Control*. Oxford, U.K.: Oxford Science, pp: 429-445.

Maaziz, M.K., P. Boucher and D. Dumur, 1999. A new RST Cascade predictive control scheme for Induction machine. *Proc. IEEE. Int. Conf. Control Applications*, Hawaii, USA, pp: 927-932.

Rubaii, A. and R. Yalamanchili, 1992. Dynamic study of an electronically brushless DC machine via computer simulation. *IEEE. Trans. Energy Conversion*, 7: 132-138.

Russo, L.P. and B.W. Bequette, 1997. State space versus input/output representation for cascade control of unstable systems. *Industrial and Eng. Chem. Res.*, 36 (6): 2271-2278.

Shinskey, F.G., 1981. *Controlling Multivariable Processes*. Instrument Society of America, Triangle Research Part, N.C.

Silva, R.N., L.M. Rato, J.M. Lemos and F. Coito, 1997. Cascade control of a distributed collector solar field. *J. Process Control*, 7 (2): 111-117.