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Sliding Mode Control of the Stator Currents of the Induction Motor

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Abstract: The sliding mode control had a large success these last years because it is largely applicable to non-linear systems, besides, it takes into account the parameter variation. In this study, this theory is mathematically developed and then applied to the regulation of the stator currents of the induction machine. Simulation results show a perfect regulation even in the presence of parametric variation which testifies the hardiness of the developed method.

Key words: Sliding mode control, non-linear systems, induction machine, robustness

INTRODUCTION

The modelisation of ac machines in order to control them is due mainly to the works of Kron (1942), based on an tensorielle approach that grave birth to the notion of the generalized machine. A particular case of this concept is the Hautier and Caron's (1993) model, making reference today for most the advanced synthesis control. Equations of Hautier and Caron's (1993) are henceforth famous for the development of the knowledge model translating the dynamic behavior of the electric and electromagnetic fashions of the induction machine (Kazmierkowsky and Kasprowicz, 1995). This model, describes by an non-linear algébro-differential system (Marino *et al*., 1993), admits several class of state representations. These class of models depend directly on the control objectives (torque, speed, position), the nature of the feeding source (voltage or current), the referential frame ((α, β) or (d, q)) and finally, of the choice of the state component vector (flux or current, stator or rotor). Whatever is the studied class, the application of theories of the linear control or non-linear has for main object the increase of the dynamic performances of the process outputs. One of the robust control that has been developed these last years is the sliding mode control, which is applicable to non-linear systems presenting parameters uncertainties. In this study, this control is retailed and then applied for the regulation of the stator currents of the induction machine.

MODELISATION OF THE INDUCTION MACHINE

In this study, we present briefly and no in exhaustive way, a mathematical model of the induction machine (Krause, 1993; Cgreuet and Clerk, 1996; Caron and Hautier, 1998), fluently used to synthesis the control laws, this model is defined in a referential frame (indication (d, q). This referential frame of the induction machine with the help of an adapted mathematical transformation. The dynamics of the component of the stator current and rotor flux defined in a referential rotating (d, q), are given by:

$$\frac{d_{iqs}}{d_t} = -\frac{R_t}{\sigma L_s} i_{ds} + \frac{L_m}{\sigma L_s L_r T_r}\varphi_{dr} + \frac{L_m}{\sigma L_s L_r}\omega\varphi_{qr} + \frac{1}{\sigma L_s} u_{ds} \quad (1)$$

$$\frac{d_{iqs}}{dt} = -\frac{R_t}{\sigma L_s} i_{qs} + \frac{L_m}{\sigma L_s L_r}\omega\varphi_{dr} - \frac{L_m}{\sigma L_s L_r T_r}\varphi_{qr} + \frac{1}{\sigma L_s} u_{qs} \quad (2)$$

$$\frac{d\varphi_{dr}}{dt} = -\frac{L_m}{T_r} i_{ds} - \frac{1}{T_r}\varphi_{dr} - \omega\varphi_{qr} \quad (3)$$

$$\frac{d\varphi_{qr}}{dt} = \frac{L_m}{T_r} i_{qs} + \omega\varphi_{dr} - \frac{1}{T_r}\varphi_{qr} \quad (4)$$

$$R_t = Rs + \frac{Lm^2}{Lr^2} Rr \qquad \sigma = 1 - \frac{Lm^2}{LsLr}, Tr = \frac{Lr}{Rr} \quad (5)$$

Where,

R_s, L_s : Resistance et inductance stator.
R_r, L_r : Resistance et inductance rotor.
L_m : Mutual inductance between stator and rotor.
R_t : Total resistance brought back to the rotor.
σ : Total leakage coefficient.
T_r : Rotor time constant.
ω : Rotor angular frequency.

The associated mechanical equations are given by:

$$C_{em} = p\frac{Lm}{Lr}(i_{qs}\varphi_{dr} - i_{ds}\varphi_{qr}) \quad (6)$$

$$\frac{d\Omega}{dt} + \frac{f}{J}\Omega = \frac{1}{J}(C_{em} - C_r) \quad (7)$$

C_{em} : Electromagnetic torque of the machine.
C_r : Load torque.
Ω : Mechanical speed of the rotor $\Omega = \omega / p$.
J : Moment of inertia.
f : Damping constant.
p : Number of motors's pole.

The outputs to be controlled are the stator currents i_{ds} and $_{IQs}$.

RECALLS OF THE SLIDING MODE CONTROL

Principle: Let a nonlinear system of n dimension described by the following equations:

$$\begin{cases} \dot{x}_i = x_{i+1}, i = 1, ..., (n-1) \\ . \\ . \\ \dot{x}_n = f(x,t) + g(x,t).u \\ y = x_1 \end{cases} \quad (8)$$

The sliding surface is given by:

$$S = \sum_{i=1}^{n} a_i.x_i \text{ avec } a_n = 1 \quad (9)$$

the $a_i(i = i,...,n)$ defined in Eq. (9) are chosen such that the polynomial $P(s) = a_0 + a_1.s + ... + a_{n-1}.s^{n-1} + s^n$ is an Hurwitz polynomial, the reaching mode condition is given by: (Utkin, 1993, 1997; Emelyanov, 1967):

$$S.\dot{S} \prec 0 \quad (10)$$

The condition Eq. (10) is verified by taking

$$\dot{S} = -M.\text{sign}(S) \quad (11)$$

with

$$\text{sign}(S) = \{1 \text{ si } S \succ 0,\ 0 \text{ si } S = 0,\ \text{-}1 \text{ si } S \prec 0\}$$

and M is a constant strictly positive called sliding gain. indeed

$$S.\dot{S} = S.(-M\text{sign}(S)) = -M.|S| \prec 0 \text{ si } S \neq 0$$

The derivative of Eq. (9) give:

$$\dot{S} = \dot{x}_n + \sum_{i=1}^{n-1} a_i.x_{i+1} \quad (12)$$

while equalizing (11) and (12), one will have:

$$\dot{x}_n = -\sum_{i=1}^{n-1} a_i.x_{i+1} - M.\text{sign}(S) \quad (13)$$

Equation (13) and (8) permit the determination of the control in sliding mode.

$$u = \frac{1}{g}.\left[-\sum_{i=1}^{n-1} a_i.x_{i+1} - M.\text{sign}(S) - f\right] \quad (14)$$

This control appears as the sum of a basis frequency signal U_{eq} and of a high frequency signal Δu assuring the sliding mode.

$$U_{eq} = \frac{1}{g}.\left[-\sum_{i=1}^{n-1} a_i.x_{i+1} - f\right] \quad (15a)$$

$$\Delta u = \frac{-M.\text{sign}(S)}{g} \quad (15b)$$

When the sliding surface is reached, ($u = U_{eq}$), the dynamics of the system Eq. (8) is governed by the following equation:

$$\begin{cases} \dot{x}_i = x_{i+1}, i = 1, \dots, (n-1) \\ \cdot \\ \cdot \\ \dot{x}_n = -\sum_{i=1}^{n-1} a_i . x_{i+1} \\ y = x_1 \end{cases} \quad (16)$$

and the dynamics of the system is independant of the functions f and g, in addition the order of the system in sliding mode is reduced by an unit (ordre = n-1). The control Eq. (14) present oscillations due to the existence of the term sign (S).

Calculus of the reachability time: The integration of the differential Eq. (11) between the initial instant t = 0 where S = S (0) and the reachability instant t_f where S (t_f), gives.

$$t_f = \frac{|S(0)|}{M} \quad (17)$$

Therefore, more M is big, more the attractivite time is small, more the response is fast.

APPLICATION TO THE INDUCTION MOTOR

The sliding surface is given by:

$$S_1 = i_{ds} - i_{dsref} \quad (18)$$

the surface S_1 is attractive if:

$$\frac{dS_1}{dt} = -M_1.\text{sign}(S_1) \quad (19)$$

the derivation of S_1 and the utilization of the relations Eq. (19) and (1), gives the control u_{ds}which is given by:

$$u_{ds} = R_t.i_{ds} - \frac{L_m}{L_r.T_r}.\varphi_{dr} - \frac{L_m}{L_r}.\omega.\varphi_{qr} - \sigma.L_s.\frac{d}{dt}(i_{dsref}) - \sigma.L_s.M_1.\text{sign}(S_1) \quad (20)$$

the surface $S_2$is given by:

$$S_2 = i_{qs} - i_{qsref} \quad (21)$$

the surface S_2 is attractive if:

$$\frac{dS_2}{dt} = -M_2.\text{sign}(S_2) \quad (22)$$

In the same way as previously, the control u_{qs} is given by:

$$u_{qs} = R_t i_{qs} - \frac{L_m}{L_r}\omega\varphi_{dr} - \frac{L_m}{L_r T_r}\varphi_{qr} + \sigma L_s \frac{d}{dt}(i_{qsref}) - \sigma L_s M_2 \text{sign}(S_2) \quad (23)$$

SIMULATIONS

The simulation has been made on a horizon of 20 sec by using the MATLAB software. N.B a value without unit refers to the international system. The real parameters of the machine are given by:

R_s = 52 mΩ, R_r = 70 mΩ, L_m = 31 mΩ
L_s = 31.7 mH, L_r = 32.3 mH, p = 2
J = 041. Kg m^{-2}, f = 0.5 N,
C_r = 0.1 Nm

but at the time t = 10s, the parameters of the machine change as follows: R_s and R_r increase by 10% to reach R_s= 52 mΩ, R_r= 77 mΩ and the load torque passes to C_r = 0.1 Nm.
The initial conditions are given by:

$i_{ds}(0) = 30$, $i_{qs}(0) = 30$, $\varphi_{dr}(0) = 0$, $\varphi_{qr}(0) = 0$, Ω (0) = 0.

The desired outputs are

$$i_{dsref} = 50 \sin(10\,t)$$
$$i_{qsref} = 50.\ \text{Sin}(10\,t - \pi/2)$$

The choose of M1 = M2 = 130 assures the attractivity of the surfaces S_1 and S_2even in the parametric variation

INTERPRETATION

Figure 1 and 2 show the perfect tracking of the stator current toward their references, we can also see it from the sliding surfaces which goes to zero after a lapse of instant (Fig. 3 and 4). The excitation voltages (Fig. 5 and 6) are quadratic and sinusoidal, but the module presents small oscillations around the middle value (Fig. 7) and it is justified by the existence of the function sign in the control law. Figure 8 is the sum of the product of

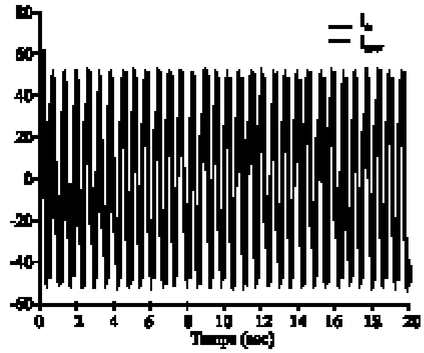

Fig. 1: Current i_{ds} (__), current i_{dsref} (...)

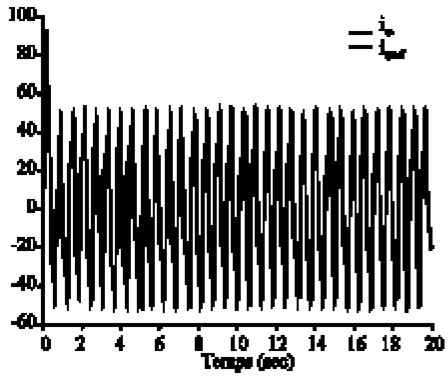

Fig. 2: Current i_{qs} (__), current i_{qsref} (...)

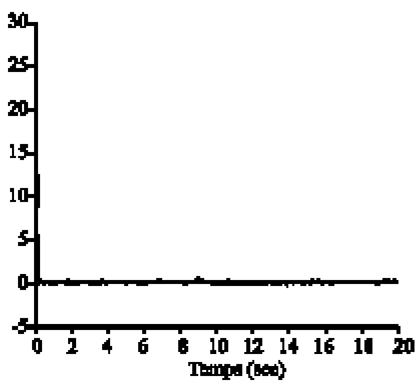

Fig. 3: Sliding surface S_1

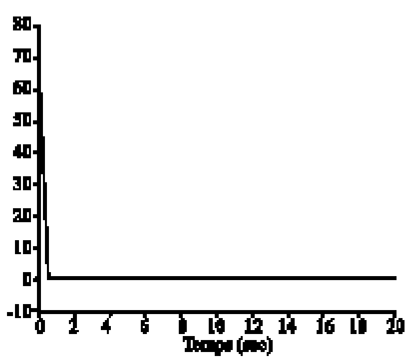

Fig. 4: Sliding surface S_2

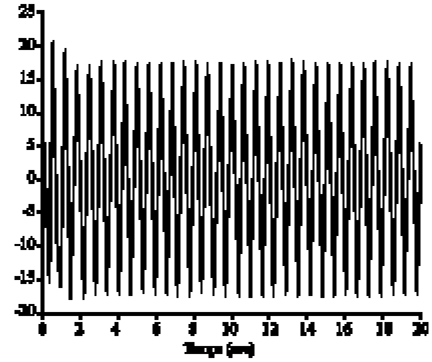

Fig. 5: Tension u_{ds}

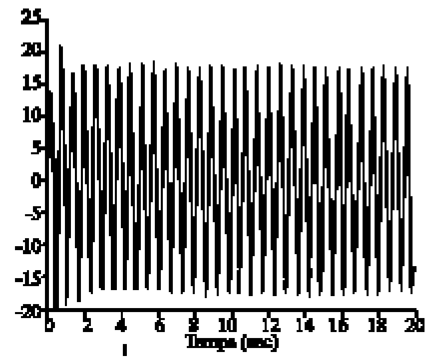

Fig. 6: Tension u_{qs}

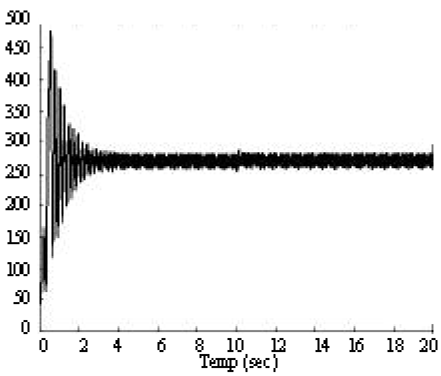

Fig. 7: $u_{ds} \times u_{ds} \times u_{qs} \times u_{qs}$

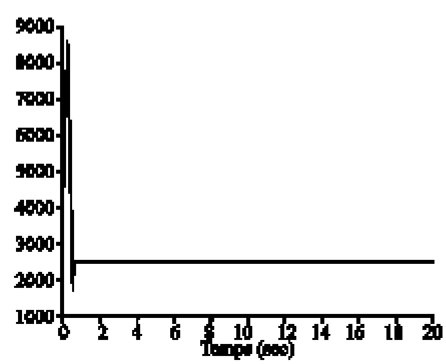

Fig. 8: $i_{ds} \times i_{ds} \times i_{qs} \times i_{qs}$

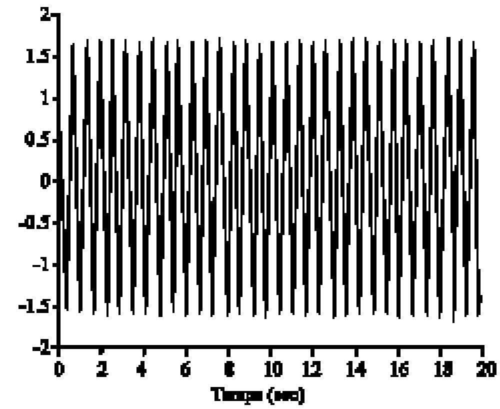

Fig. 9: Flux φ_{dr}

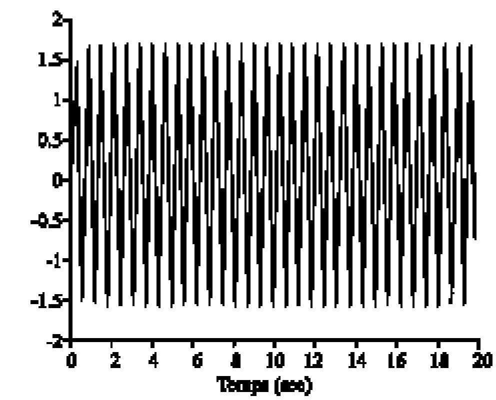

Fig. 10: Flux φ_{q}

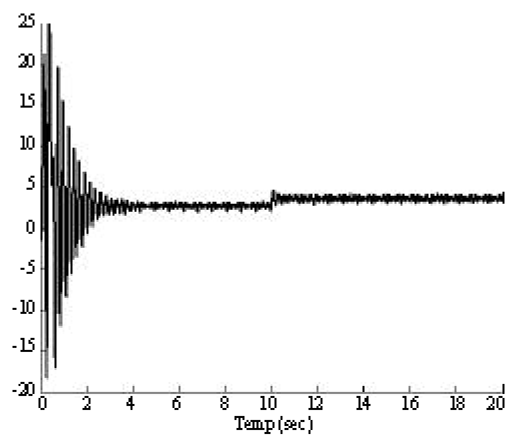

Fig. 11: Torque C_{em}

stator currents, it is a constant which justifies that the stator currents are sinusoidal and quadratic.

The rotor flux in the direction d and q are sinusoidal (Fig. 9 and 10) with the same period but differ of pi/2. The Fig. 11 represents the electromagnetic torque and the mechanical speed oscillates slightly around the value 5 (Fig. 12).

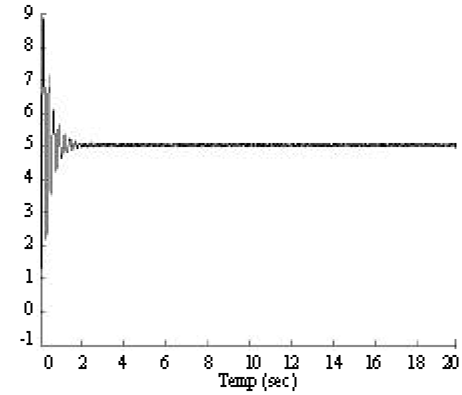

Fig. 12: Speed Ω

CONCLUSION

In this study, we have developed a method of control of non-linear systems by using the sliding mode theory. This method has been applied with success to the regulation of the stator currents of the induction motor in the presence of parametric uncertainties in the stator and rotor resistance and in the load torque. Results of the simulation testify the hardiness of the method. An extension of this work is considered in order to apply this method to others type of motors.

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