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Reliability Maximization of Power System Using Ant Colony Approach

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Abstract: This study describes and uses an ant colony meta-heuristic optimization method to solve the reliability optimization problem. This problem is known as total reliability maximization of parallel-series system configuration. Redundant elements are included to achieve a high desired level of reliability. System reliability is represented by a multi-state reliability function. The systems elements are characterized by their performance (capacity), reliability and cost. These elements are chosen among a list of products available on the market. The proposed meta-heuristic seeks to the best maximal reliability system configuration with limited system investment. To estimate the parallel-series system reliability, a fast method based on Universal Moment Generating Function (UMGF) is suggested. The ant colony approach is used as an optimization technique.

Key words: Ant colony, reliability optimization, multi-state systems, Universal Moment Generating Function (UMGF)

INTRODUCTION

One of the most important problems in many industrial applications is the reliability redundancy optimization problem. This latter is well known combinatorial optimization problem where the design goal is achieved by discrete choices made from elements available on the market. The natural objective function is to find the maximal reliability configuration of a parallel-series system under cost constraints. The system is considered to have a range of performance levels from perfect working to total failure. In this case the system is called a Multi-State System (MSS). Let consider a multi-state system containing n components C_i (i = 1, 2, …, n) in series arrangement. For each component C_i there are various technologies, which are proposed by the suppliers on the market. Elements are characterized by their cost, performance and reliability according to their technologies. For example, these elements can represent machines in a manufacturing system to accomplish a task on product in our case they represent the whole of electrical power system (generating units, transformers and electric carrying lines devices). Each component C_i contains a number of elements connected in parallel. Different technologies of elements may be chosen for any given system component. Each component can contain elements of different technologies as depicted in Fig. 1.

A limitation can be undesirable or even unacceptable, where only identical elements are used in parallel (i.e., homogeneous system) for two reasons. First, by allowing different technologies of the devices to be allocated in the same system, one can obtain a solution that provides the desired reliability level with a lower cost than in the solution with identical parallel devices. Second, in practice the designer often has to include additional devices in the existing system. It may be necessary, for example, to modernize a production power system according to a new demand levels from customers or according to new reliability requirements.

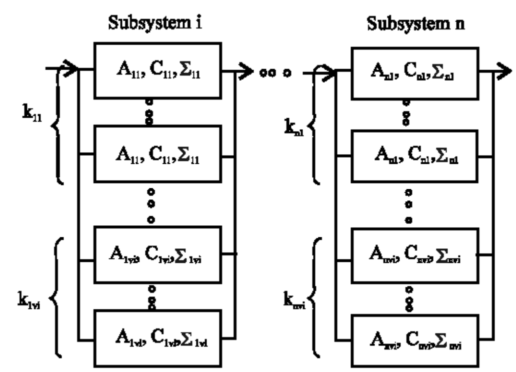

Fig. 1: Parallel-series power system topology

The vast majority of classical reliability analysis and optimization assume that components and system are in either of two states (i.e., complete working state and total failure state). However, in many real life situations we are actually able to distinguish among various levels of performance for both system and components. For such situation, the existing dichotomous model is a gross oversimplification and so models assuming multi-state (degradable) systems and components are preferable since they are closer to reliability. Recently much works treat the more sophisticated and more realistic models in which systems and components may assume many states ranging from perfect functioning to complete failure. In this case, it is important to develop MSS reliability theory. In this study, an MSS reliability theory will be used, where the binary state system theory is extending to the multi-state case. As is addresses in recent review of the literature for example by Ushakov (2002) and Levitin and Lisnianski (2001). The methods of MSS reliability assessment are based in this reearch on the Universal Moment Generating Function (UMGF) approach.

Ushakov (2002) gives a comparison between the classical and the UMGF approaches, highlights that the UMGF approach is fast enough to be used in the optimisation problems where the search space is sizeable.

The problem of total reliability maximization, subject to cost constraints, is well known as the Reliability Redundancy Optimization Problem (RROP). The RROP is studied in many different forms as summarized by Tillman *et al.* (1997) and more recently by Ku and Prasad (2000). The RROP for the multi-state reliability was introduced by Ushakov (1987). In Lisnianski *et al.* (1996) and Levitin *et al.* (1997), genetic algorithms were used to find the optimal or nearly optimal power system structure.

This research uses an ant colony optimization approach to solve the RROP for multi-state system. The idea of employing a colony of cooperating agents to solve combinatorial optimization problems was recently proposed by Dorigo and Gambardella (1997). The ant colony approach has been successfully applied to the classical travelling salesman problem (Dorigo and Gambardella, 1997) and to the quadratic assignment problem (Maniezzo and Colorni, 2001). Ant colony shows very good results in each applied area. It has been recently adapted for the reliability design of binary state systems (Liang and Smith, 2001). The ant colony has also been adapted with success to other combinatorial optimization problems such as the vehicle routing problem (Bullnheimer, 1997). The ant colony method has not yet been used for the reliability redundancy optimization of multi-state systems.

Approach and outlines: The problem formulated in this paper lead to a complicated combinatorial optimization problem. The total number of different solution to be examined is very large, even for rather small problems. An exhaustive examination of all possible solutions is not feasible given reasonable time limitations. Because of this, the Ant Colony Optimization (or simply ACO) approach is adapted to find optimal or nearly optimal solutions to be obtained in a short time. The newer developed meta-heuristic method has the advantage to solve the RROP for MSS without the limitation on the diversity of technologies of elements. Ant colony optimization is inspired by the behavior of real ant colonies that exhibit the highly structured behavior. Ants lay down in some quantity an aromatic substance, known as pheromone, in their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone in laid down by others ants, therefore the best path has more intensive pheromone and higher probability to be chosen.

During the optimization process, artificial ants will have to evaluate the reliability of a given selected structure of the parallel-series system. To do this, a fast procedure of reliability estimation is developed. This procedure is based on a modern mathematical technique: the z-transform or UMGF which was introduced by Ushakov (1986). It was proven to be very effective for high dimension combinatorial problems: see e.g. Ushakov (2001), Levitin (2001). The universal moment generating function is an extension of the ordinary moment Generating Function (UGF) (Ross, 1993). The method developed in this study allows the reliability function of reparable parallel-series MSS to be obtained using a straightforward numerical procedure.

FORMULATION OF THE RELIABILITY OPTIMIZATION PROBLEM RROP

Parallel-series system with different redundant elements: Let consider a parallel-series system containing n subcomponents C_i (i = 1, 2, …, n) in series as represented in Fig. 1. Every component C_i contains a number of different elements connected in parallel. For each component i, there are a number of element technologies available in the market. For any given system component, different technologies and number of elements may be chosen. For each subcomponent i, elements are characterized according to their technology v by their Cost (C_{iv}), reliability (A_{iv}) and performance (Σ_{iv}). The structure of system component i can be defined by the numbers of parallel elements (of each technology) k_{iv} for $1 \le v \le V_i$, where V_i is a number of technologies available for element of type i. Figure 2 illustrates these notations for a given component i. The entire system structure is defined by the vectors $k_i = \{k_{ivi}\}$ ($1 \le i \le n$, $1 \le v \le V_i$). For a given set of vectors $k_1, k_2, \ldots, k_n$ the total cost of the system can be calculated as:

$$C = \sum_{i=1}^{n} \sum_{v=1}^{V_i} k_{iv} \ C_{iv} \tag{1}$$

Reliability of reparable multi-state systems: The parallel-series system is composed of a number of failure prone

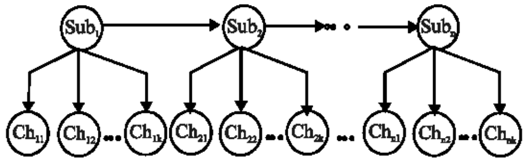

Fig. 2: Definition of parallel-series system with tree subsystems into a graph

elements, such that the failure of some elements leads only to a degradation of the system performance. This system is considered to have a range of performance levels from perfect working to complete failure. In fact, the system failure can lead to decreased capability to accomplish a given task, but not to complete failure. An important MSS measure is related to the ability of the system to satisfy a given demand.

In electric power systems, reliability is considered as a measure of the ability of the system to meet the load Demand (D), i.e., to provide an adequate supply of electrical energy (Σ). This definition of the reliability index is widely used in power systems (Ross, 1993; Murchland, 1995; Levitin *et al.*, 1997a, b, 1998). The Loss of Load Probability index (LOLP) is usually used to estimate the reliability index (Billinton and Allan, 1990). This index is the overall probability that the load demand will not be met. Thus, we can write $R = \text{Probab}(\Sigma \geq D)$ or $R = 1\text{-LOLP}$ with $\text{LOLP} = \text{Probab}(\Sigma < D)$. This reliability index depends on consumer demand D.

For reparable MSS, a multi-state steady-state reliability E is used as $\text{Probab}(\Sigma \geq D)$ after enough time has passed for this probability to become constant (Levitin *et al.*, 1998). In the steady-state the distribution of states probabilities is given by Eq. 2, while the multi-state stationary reliability is formulated by Eq. 3:

$$P_j = \lim_{t \to \infty} [\text{probab}(\textstyle\sum(t) = \sum_j)] \tag{2}$$

$$E = \sum_{\Sigma_j \geq D} P_j \tag{3}$$

If the operation period T is divided into M intervals (with durations $T_1, T_2, \ldots, T_M$) and each interval has a required demand level ($D_1, D_2, \ldots, D_M$, respectively), then the generalized MSS reliability index A is:

$$A = \frac{1}{\sum_{j=1}^{M} T_j} \sum_{j=1}^{M} \text{Probab}(\Sigma \geq D_j)\, T_j \tag{4}$$

We denote by D and T the vectors $\{D_j\}$ and $\{T_j\}$ ($1 \leq j \leq M$), respectively. As the reliability A is a function of $k_1, k_2, \ldots, k_n$, D and T, it will be written $A(k_1, k_2, \ldots, k_n, D, T)$. In the case of a power system, the vectors D and T define the cumulative load curve (consumer demand). In reality the load curves varies randomly; an approximation is used from random curve to discrete curve (Wood and Ringlee, 1970). In general, this curve is known for every power system.

Optimal design problem formulation

Dual problem: The multi-state system Reliability Redundancy Optimization Problem (RROP) of electrical power system can be formulated as follows: find the maximal reliability system configuration $k_1, k_2, \ldots, k_n$, such that the corresponding to the cost less than or equal the specified cost C_0. That is,

$$\text{Minimize } A(K_1, K_2, \ldots, K_n, D\ t) = \underbrace{\prod_{j=1}^{J_m} u_j(z) \sum_{i=1}^{n} P_i z^{ai}}_{\max_{i=1}^{n}} \tag{5}$$

$$\text{Subject to } C = \sum_{i=1}^{n} \sum_{v=1}^{V_i} k_{iv}\, C_{iv} \leq C_o \tag{6}$$

MULTI-STATE SYSTEM RELIABILITY ESTIMATION METHOD

The procedure used in this study is based on the universal z-transform, which is a modern mathematical technique introduced by Ushakov (1986). This method, convenient for numerical implementation, is proved to be very effective for high dimension combinatorial problems. In the literature, the universal z-transform is also called Universal Moment Generating Function (UMGF) or simply u-function or u-transform. In this study, we mainly use the acronym UMGF. The UMGF extends the widely known ordinary moment generating function (Ross, 1993).

Definition and properties: The UMGF of a discrete random variable Σ is defined as a polynomial:

$$u(z) = \sum_{j=1}^{J} P_j z^{\Sigma j} \tag{7}$$

Where the variable Σ has J possible values and P_j is the probability that Σ is equal to Σ_j.

The probabilistic characteristics of the random variable Σ can be found using the function u(z). In particular, if the discrete random variable Σ is the MSS stationary output performance, the reliability E is given by the probability Probab(Σ ≥ D) which can be defined as follows:

$$\text{Probab}(\Sigma \geq D) = \Psi\left(u(z)z^{-D}\right) \tag{8}$$

Where Ψ is a distributive operator defined by expressions (9) and (10):

$$\Psi(Pz^{\sigma-D}) = \begin{cases} P, & \text{if } \sigma \geq D \\ 0, & \text{if } \sigma < D \end{cases} \tag{9}$$

$$\Psi\left(\sum_{j=1}^{J} P_j z^{\Sigma_j - D}\right) = \sum_{j=1}^{J} \Psi\left(P_j z^{\Sigma_j - D}\right) \tag{10}$$

It can be easily shown that Eq. 7-10 meet condition Probab

$$(\Sigma \geq D) = \sum_{\Sigma_j \geq D} P_j.$$

By using the operator Ψ, the coefficients of polynomial u(z) are summed for every term with $\Sigma_j \geq D$ and the probability that Σ is not less than some arbitrary value D is systematically obtained.

Consider single elements with total failures and each element i has nominal performance Σ_i and reliability A_i. Then, Probab($\Sigma = \Sigma_i$) = A_i and Probab(Σ = 0) = 1-A_i. The UMGF of such an element has only two terms and can be defined as:

$$\begin{aligned} u_i &= (1 - A_i)z^0 + A_i z^{\Sigma_i} \\ &= (1 - A_i) + A_i z^{\Sigma_i} \end{aligned} \tag{11}$$

To evaluate the MSS reliability of a series-parallel system, two basic composition operators are introduced. These operators determine the polynomial u(z) for a group of elements.

Composition operators

Properties of the operators: The essential property of the UMGF is that it allows the total UMGF for a system of elements connected in parallel or in series to be obtained using simple algebraic operations on the individual UMGF of elements. These operations may be defined according to the physical nature of the elements and their interactions. The only limitation on such an arbitrary operation is that its operator ϕ should satisfy the following Ushakov's (1986) conditions:

$$\phi(p_1 z^{g1}, p_2 z^{g2}) = p_1 p_2 z^{\phi(g_1, g_2)}$$
$$\phi(g) = g,$$
$$\phi(g_1, \ldots, g_n) = \phi(\phi(g_1, \ldots, g_k), \phi(g_{k+1}, \ldots, g_n)).$$
$$\phi(g_1, \ldots, g_k, g_{k+1}, \ldots, g_n) = \phi(g_1, \ldots, g_{k+1}, g_k, \ldots, g_n) \text{ for any k.}$$

Parallel components: Let consider a system component m containing J_m elements connected in parallel. As the performance measure is related to the system productivity, the total performance of the parallel system is the sum of performances of all its elements. In power systems engineering, the term capacity is usually used to indicate the quantitative performance measure of an element (Lisnianski *et al*., 1996). It may have different physical nature. Examples of elements capacities are: generating capacity for a generator, pipe capacity for a water circulator, carrying capacity for an electric transmission line, etc. The capacity of an element can be measured as a percentage of nominal total system capacity. In a manufacturing system, elements are machines. Therefore, the total performance of the parallel machine is the sum of performances (Dallery and Gershwin, 1992).

The u-function of MSS component m containing J_m parallel elements can be calculated by using the Γ operator:

$$u_p(z) = \Gamma(u_1(z),\ u_2(z),\ \ldots,\ u_n(z)),$$

$$\text{where } \Gamma(g_1,\ g_2,\ \ldots,\ g_n) = \sum_{i=1}^{n} g_i$$

Therefore, for a pair of elements connected in parallel:

$$\Gamma(u_1(z),\ u_2(z)) =$$
$$\Gamma(\sum_{i=1}^{n} P_i z^{a_i},\ \sum_{j=1}^{m} Q_j z^{b_j}) = \sum_{i=1}^{n}\sum_{j=1}^{m} P_i Q_j z^{a_i + b_j}.$$

Parameters a_i and b_j are physically interpreted as the respective performances of the two elements. n and m are numbers of possible performance levels for these elements. P_i and Q_j are steady-state probabilities of possible performance levels for elements. One can see that the Γ operator is simply a product of the individual u-functions. Thus, the component UMGF is:

$$u_p(z) = \prod_{j=1}^{J_m} u_j(z)$$

Given the individual UMGF of elements defined in Eq. 11, we have:

$$u_p(z) = \prod_{j=1}^{J_m} (1 - A_j + A_j z^{\Sigma_i})$$

Series components: When the elements are connected in series, the element with the least performance becomes the bottleneck of the system. This element therefore, defines the total system productivity. To calculate the u-function for system containing n elements connected in series, the operator η should be used:

$$u_s(z) = \eta(u_1(z),\ u_2(z),\ \ldots,\ u_m(z)), \text{where}$$
$$\eta(g_1,\ g_2,\ \ldots, g_m) = \min\{g_1,\ g_2,\ \ldots,\ g_m\}$$

so that

$$\eta(u_1(z),\ u_2(z)) =$$
$$\eta\left(\sum_{i=1}^{n} P_i z^{a_i},\ \sum_{j=1}^{m} Q_j z^{b_j}\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} P_i Q_j z^{\min\{a_i,\ b_j\}}$$

Applying composition operators Γ and η consecutively, one can obtain the UMGF of the entire series-parallel system. To do this we must first determine the individual UMGF of each element.

Elements with "partial" failures: This is the general case where failures may cause reduction of element performance and, therefore, different performance degradation levels should be considered. In this "partial" failure case, the UMGF of an element is

$$u^*_i(z) = \sum_{j=1}^{K} P_{ij} z^{\Sigma_{ij}}$$

with the following notations:

K = Number of possible states (or performance levels) of element i
j = Index for performance level (j = 1, 2, …, K)
Σ_{ij} = The performance of the element i in state j
P_{ij} = The steady-state probability of the state with the corresponding Σ_{ij}

The first state (j = 1) may be considered as total failure (Σ_{ij} = 0) and the K-th state as a nominal element performance level.

Using the Γ operator, we can obtain the UMGF of the i-th system component containing k_i parallel elements as:

$$u_i(z) = \left(u^*_i(z)\right)^{k_i}$$

The UMGF of the entire system containing n components connected in series is:

$$u_s(z) = \eta\left(\left(u^*_1(z)\right)^{k_1},\ \left(u^*_2(z)\right)^{k_2}, \ldots,\ \left(u^*_n(z)\right)^{k_n}\right)$$

Elements with total failures: Let consider the usual case where only total failures are considered (K = 2) and each element of type i and technology v_i has nominal performance Σ_{iv} and reliability A_{iv}. In this case, we have:

Probab($\Sigma = \Sigma_{iv}$) = A_{iv} and Probab(Σ = 0) = 1-A_{iv}. The UMGF of such an element has only two terms and can be defined as in Eq. 11 by

$$u^*_i(z) = (1 - A_{iv})z^0 + A_{iv} z^{\Sigma_{iv}}$$
$$= 1 - A_{iv} + A_{iv} z^{\Sigma_{iv}}$$

Using the **Γ** operator, we can obtain the UMGF of the i-th system component containing k_i parallel elements as:

$$u_i(z) = \left(u^*_i(z)\right)^{k_i} = (A_{iv}{}^{\Sigma iv} + (1 - A_{iv}))^{ki}$$

The UMGF of the entire system containing n components connected in series is:

$$u_s(z) = \eta\Big(\left((A_{1v} z^{\Sigma 1v}) + (1 - A_{1v})\right)^{k1},\ \left((A_{2v} z^{\Sigma 2v}) + (1 - A_{2v})\right)^{k2}, \ldots,\ \left((A_{nv} z^{\Sigma nv}) + (1 - A_{nv})\right)^{kn}\Big) \quad (12)$$

To evaluate the probability Probab(Σ ≥ D) for the entire system, the operator **Ψ** is applied to Eq. 12:

$$\text{Probab}(\Sigma \geq D) = \psi\left(u_s(z) z^{-D}\right) \quad (13)$$

The above procedure was implemented and tested on a PC computer and shown to be effective and fast. The UMGF method, convenient for numerical implementation, is efficient for the high dimension combinatorial problem formulated in this research. In our optimization technique to solve this problem, artificial ants will evaluate the reliability of given selected structures of the series-parallel system. To do this, the fast implemented procedure of reliability estimation will be used by the optimization program.

The next study presents the ant colony meta-heuristic optimization method to solve the redundancy optimization problem for multi-state systems.

THE ANT COLONY OPTIMIZATION APPROACH

The problem formulated in this study is a complicated combinatorial optimization problem. The total number of different solutions to be examined is very large, even for rather small problems. An exhaustive examination of the enormous number of possible solutions is not feasible given reasonable time limitations. Thus, because of the search space size of the RROP for MSS, a new meta-heuristic is developed in this study. This meta-heuristic consists in an adaptation of the ant colony optimization method.

The ACO principle: Recently, Dorigo and Gambardella (1997) introduced a new approach to optimization problems derived from the study of any colonies, called "Ant System". Their system inspired by the work of real ant colonies that exhibit the highly structured behavior. Ants lay down in some quantity an aromatic substance, known as pheromone, in their way to food. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone in laid down by others ants, therefore the best paths have more intensive pheromone and higher probability to be chosen. This simple behavior explains why ants are able to adjust to changes in the environment, such as new obstacles interrupting the currently shortest path.

Artificial ants used in ant system are agents with very simple basic capabilities mimic the behavior of real ants to some extent. This approach provides algorithms called ant algorithms. The Ant System approach associates pheromone trails to features of the solutions of a combinatorial problem, which can be seen as a kind of adaptive memory of the previous solutions. Solutions are iteratively constructed in a randomized heuristic fashion biased by the pheromone trails, left by the previous ants. The pheromone trails, τ_{ij}, are updated after the construction of a solution, enforcing that the best features will have a more intensive pheromone. An Ant algorithm presents the following characteristics. It is a natural algorithm since it is based on the behavior of ants in establishing paths from their colony to feeding sources and back. It is parallel and distributed since it concerns a population of agents moving simultaneously, independently and without supervisor. It is cooperative since each agent chooses a path on the basis of the information, pheromone trails, laid by the other agents with have previously selected the same path. It is versatile that can be applied to similar technologies the same problem. It is robust that it can be applied with minimal changes to other combinatorial optimization problems.

The solution of the Travelling Salesman Problem (TSP) was one of the first applications of ACO. Various extensions to the basic TSP algorithm were proposed, notably by Dorigo and Gamberdella (1997). The improvements include three main aspects: the state transition rule provides a direct way to balance between exploration of new edges and exploitation of a priori and accumulated knowledge about the problem, the global updating rule is applied only to edges which belong to the best ant tour and while ants construct solution, a local pheromone updating rule is applied. These extensions have been included in the algorithm proposed in this study.

ACO-based solution approach: In our reliability optimization problem, we have to select the best combination of parts to maximize the total reliability given a cost constraint. The parts can be chosen in any combination from the available components. Components are characterized by their reliability, capacity and cost. This problem can be represented by a graph (Fig. 2) in which the set of nodes comprises the set of subsystems and the set of available components (i.e., max (M_j), j = 1..n) with a set of connections partially connect the graph (i.e., each subsystem is connected only to its available components). An additional node (blank node) is connected to each subsystem.

In Fig. 2, a parallel -series system is illustrated where the first and the second subsystem are connected, espectively to their 3 and 2 available components. The nodes ch_{i3} and ch_{i4} represent the blank components of the two subsystems. At each step of the construction process, an ant uses problem-specific heuristic information, denoted by η_{ij} to choose the optimal number of components in each subsystem. An imaginary heuristic information is associated to each blank node. These new factors allow us to limit the search surfaces (i.e. tuning factors). An ant positioned on subsystem i chooses a component j by applying the rule given by:

$$j = \begin{cases} \arg \quad \max\limits_{m \in AC_i} ([\tau_{im}]^{\alpha} [\eta_{im} = \dfrac{A_{ij}}{1 + C_{ij}}]^{\beta}) & \text{if} \quad q \leq q_o \\ J & \text{if} \quad q > q_o \end{cases} \tag{14}$$

and J is chosen according to the probability:

$$p_{ij} = \begin{cases} \dfrac{\left[\tau_{ij}\right]^{\alpha}\left[\eta_{ij} = \dfrac{A_{ij}}{1+C_{ij}}\right]^{\beta}}{\sum_{m \in AC_i}\left[\tau_{im}\right]^{\alpha}\left[\eta_{im}\right]^{\beta}} & \text{if} \quad j \in AC_i \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

α : The relative importance of the trail.
β : The relative importance of the heuristic information η_{ij}.
AC_i : The set of Available Components choices for subsystem i.
q : Random number uniformly generated between 0 and 1.

The heuristic information used is : $\eta_{ij} = A/(1+c_{ij})$ where c_{ij} represents the associated cost of component j for subsystem i and A is the corresponding availability. A "tuning" factor $t_i = \eta_{ij} = A/(1+c_{i(Mi+1)})$ is associated to blank component (M_i+1) of subsystem i. The parameter q_o determines the relative importance of exploitation versus exploration: Every time an ant in subsystem i have to choose a component j, it samples a random number $0 \le q \le 1$. If $q \le q_o$ then the best edge, according to Eq. 14, is chosen (exploitation), otherwise an edge is chosen according to Eq. 15 (biased exploration).

The pheromone update consists of two phases: local and global updating. While building a solution of the problem, ants choose components and change the pheromone level on subsystem-component edges. This local trail update is introduced to avoid premature convergence and effects a temporary reduction in the quantity of pheromone for a given subsystem-component edge so as to discourage the next ant from choosing the same component during the same cycle. The local updating is given by:

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho\tau_o \quad (16)$$

Where ρ is a coefficient such that $(1-\rho)$ represents the evaporation of trail and τ_o is an initial value of trail intensity. It is initialized to the value $(n.TC_{nn})^{-1}$ with n is the size of the problem (i.e. number of subsystem and total number of available components) and TC_{nn} is the result of a solution obtained through some simple heuristic.

After all ants have constructed a complete system, the pheromone trail is then updated at the end of a cycle (i.e. global updating), but only for the best solution found. This choice, together with the use of the pseudo-random-proportional rule given by Eq. 14 and 15, is intended to make the search more directed: ants search in a neighbourhood of the best solution found up to the current iteration of the algorithm. The pheromone level is updated by applying the following global updating rule:

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho\Delta\tau_{ij} \quad (17)$$

$$\Delta\tau_{ij} = \begin{cases} \dfrac{1}{TC_{best}} & \text{if}\,(i,j) \in \text{best tour} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

The algorithm: An ant-cycle algorithm is stated as follows. At time zero an initialization phase takes place during wish NbAnt ants select components in each subsystem according to the Pseudo-random-proportional transition rule given by Eq. 14 and 15. When an ant selects a component, a local update is made to the trail for that subsystem-component edge according to Eq. 16. In this equation, ρ is a parameter that determines the rate of reduction of the pheromone level. The pheromone reduction is small but sufficient to lower the attractiveness of precedent subsystem-component edge. At the end of a cycle, for each ant k, the value of the system's reliability A_k and the total cost TC_k are computed. The best feasible solution found by ants (i.e., maximal reliability and assignments) is saved. The pheromone trail is then updated for the best solution obtained according to Eq. 17 and 18. This process is iterated until the tour counter reaches the maximum number of cycles NC_{max} or all ants make the same tour (stagnation behavior).

The followings are formal description of the algorithm.

1. Set NC: = 0 (NC: cycle counter)
   For every edge (i,j) set an initial value $\tau_{ij}(0) = \tau_o$
2. For k = 1 to NbAnt do
   For i = 1 to NbSubSystem do
   For j = 1 to MaxComponents do
   Choose a component, including blanks, according to Eq. 5 and 6 .
   Local update of pheromone trail for chosen subsystem- component edge (i,j) :

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho\tau_o$$

   End For
   End For
3. Calculate A_k (system reliability for each ant)
   Calculate the total cost for each ant TC_k

Update the best found feasible solution

4. Global update of pheromone trail:
   For each edge (i,j) ∈ best feasible solution, update the pheromone trail according to:

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho\Delta\tau_{ij}$$

$$\Delta\tau_{ij} = \begin{cases} \dfrac{1}{A/1+C_{ij\ best}} & \text{if}\,(i,j) \in \text{best tour} \\ 0 & \text{otherwise} \end{cases}$$

End For

5. cycle = cycle +1
6. if ($NC < NC_{max}$) and (not stagnation behavior)
   Then
      Go to step 2
   Else
      Print the best feasible solution and components selection.
      Stop.

ILLUSTRATIVE EXAMPLE

In order to illustrate the proposed ant colony algorithm, a numerical example is solved by use of the data given in Table 1-3. Each element of the sub-system is considered as a unit with total failures.

The maximum numbers of components put in parallel are set to (7, 5, 4, 9, 4). The number of ants used to find the best solution is 50. The simulation results depend greatly on the values of the coefficients α and β. Different t_i values (tuning factors associated to blank components) were tested and shown to influence greatly the algorithm. The best found values of t_i are (t_1 = -0.13, t_2 = -0.04, t_3 = 2.3, t_4 = -0.35, t_5 = 0.35). Several simulations are made for α = 5 , β = 5 , τ_0 = 0.0070 and ρ = 0.0070 and the best solution is obtained in 2500 cycles. Table 3 presents the obtained maximal reliability configuration.

Description of the system to be optimized: The electrical power station system which supplies the consumers is designed with five basic sub-systems (stations) as depicted in Fig. 3. The Fig. 1 showed the detailed process of the electrical power station system distribution. The process of electrical power system distribution follows as: The electrical power is generated from the station units (Subsystem 1). Then transformed for High Tension (HT) by the HT transformers (Subsystem 2) and carried by the HT lines (Subsystem 3). A second transformation in HT/MT transformers (Subsystem 4) which supplies the MT load by the MT lines (Subsystem 5). Each element of the system is considered as unit with total failures.

To provide a desired cost corresponding to maximal reliability topology, the system should be expended by the choice among several products available on the market. The characteristics for each type of component are presented in Table 1. This latter shown for each component (Comp i) reliability A, nominal capacity Σ and cost unit C. With out loss of generality both the equipment capacity and the demand levels can be measured as a percentage of the maximum boiler capacity (Demand) at each stage. Interval duration of load can be measured as a fraction (percentages %) of the total operation time.

Table 1: Data examples

Component #	Technology #	Reliability A	Cost C	Capacity Σ
Power units	1	0.798	0.590	100
1	2	0.977	0.535	100
	3	0.982	0.470	85
	4	0.978	0.420	85
	5	0.983	0.400	48
	6	0.990	0.180	31
	7	0.980	0.220	26
HT transformer	1	0.695	0.205	100
2	2	0.896	0.189	92
	3	0.797	0.091	53
	4	0.997	0.056	28
	5	0.898	0.042	21
HT lines	1	0.871	7.525	100
3	2	0.973	4.720	60
	3	0.971	3.590	40
	4	0.676	2.420	20
HT/MT				
transformers	1	0.977	0.180	100
4	2	0.678	0.160	100
	3	0.978	0.150	91
	4	0.983	0.121	72
	5	0.881	0.102	72
	6	0.971	0.096	72
	7	0.783	0.071	55
	8	0.982	0.049	25
	9	0.877	0.044	25
MT Lines	1	0.984	0.986	100
5	2	0.883	0.825	100
	3	0.987	0.490	60
	4	0.981	0.475	51

Table 2: Parameters of the cumulative demand curve

Demand level (%)	100	80	50	20
Duration (h)	4203	788	1228	2536
Probability	0.479	0.089	0.140	0.289

Table 3: Optimal solution obtained by ant colony algorithm

C_0	Subsystem s of configuration	Best reliable configuration	Reliability A	Cost C MI $
17.00	Subsystem 1	4(6)-3(7)		
	Subsystem 2	1(3)-2(4)-2(5)		
	Subsystem 3	4(4)	0.87	13.980
	Subsystem 4	1(4)-2(5)-1(6)-2(7)-2(8)-1(9)		
	Subsystem 5	2(3)-2(4)		

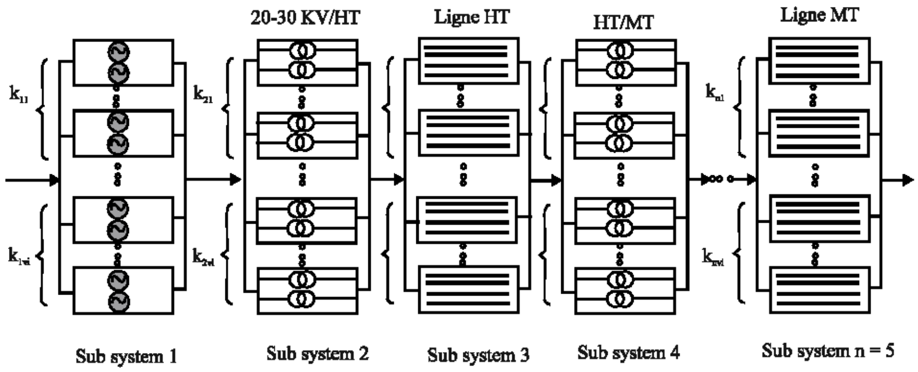

Fig. 3: Parallel-series power system

RESULTS AND DISCUSSION

Our natural objective function is to define the maximal reliability design which provides the requested level of real budget. The whole of the results obtained by the proposed ant algorithm for given values of C_0 is illustrated in Table 3. This latter also shows the computed budget C, the maximal reliability A of the best power system. In this experiment the values parameters of the ant algorithm are the set to the following values: The best found values of t_i are (t_1 = -0.13, t_2 = -0.04, t_3 = 2.3, t_4 = -0.35, t_5 = 0.35). Several simulations are made for $\alpha = 5$, $\beta = 5$, $\tau_0 = 0.0070$ and $\rho = 0.0070$ and the best solution is obtained in 2500 cycles. The choice of these values affects strongly the solution. These values were obtained by a preliminary optimisation phase. The ant algorithm is tested well for quite a range of these values. In the ant algorithm 50 ants are used in each iteration. The stopping criterion is when the number of iterations reaches 2500 cycles. The space search visited by the 30 ants is composed of 25000 solutions (50*50 cycles) and the huge space size of an exhaustive search (combinatorial algorithm) is not realistic. Indeed, a large comparison between the ant algorithm and an exhaustive one, clarify the goodness of the proposed ant algorithm meta- heuristic with respect to the calculating time.

CONCLUSION

In this study, we solve the reliability optimization problem which is a very interesting problem often reencountered in energy industry or manufacturing industry. It is formulated as redundancy optimization problem. The resolution of this problem uses a developing ant colony method. This new algorithm for choosing the best component to maximize the reliability design of parallel-series power system subject to the cost level constraints is proposed. This algorithm seeks and selects components technologies among a list of available products according to their reliability, nominal capacity (performance) and cost. Also defines the number and the kind of parallel-series power component to put in each subsystem when consumers' demand changes. The proposed method allows a practical way to solve wide instances of reliability optimization problem of multi-state systems without limitation on the diversity of components technologies put in parallel-series. A combination is used in this algorithm based on the universal moment generating function and an ACO algorithm.

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