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Modeling and Analysis of Sliding Mode Controller for Buck-Boost Converter

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Abstract: Non-linear control of power converters is an active area of research in the fields of power electronics. The variable structure nature of DC-DC converters lends itself to Sliding Mode Control (SMC). The state space averaged model of a DC-DC buck boost converter has been derived and sliding mode control techniques are applied to the system to obtain the desired output. The presence of a RHP zero in the control-to-output transfer function resulted in undershoot in the output response. To tide over the disadvantages of the conventional control techniques, the non-linear Sliding Mode Control (SMC) scheme has been adopted. The simulations are carried out using SIMULINK software.

Key words: Sliding mode control, buck-boost converter, analysis, DC-DC converters

INTRODUCTION

Recent applications in the design of power supply employs buck-boost converters because the required output is inverted directly from the input voltage and the output voltage can be either higher or lower than the input voltage. The buck-boost power converters are widely used in applications like automotive, marine, etc., have to be controlled by appropriate means in order to supply the needs of the loads (Matsui *et al*., 2002) conventional control techniques such as P, PI and PID controllers have the common characteristic that the control algorithms are described by equations (Nagrath and Gopal, 2001; Slotine and Li, 1991). A mathematical description of the controlled system is essential, however, such techniques leave much to be desired in terms of high grade of performance required from converters in several applications.

Sliding Mode Control (SMC) is a technique that is suitable for the variable structure nature of the DC-DC Converter (Venkataraman *et al*., 1985). A variable structure system is one that has a defined number of sub structures, based on the condition of some non-linear elements. Sliding mode control is a control technique wherein the state trajectory is directed towards an invariant surface on the phase plane to achieve desired system behaviour (Mattavelli *et al*., 1993).

The objective of this research, is to implement a sliding control scheme for a buck-boost DC-DC converter using MATLAB and analyse its performance under various conditions.

CONVERTER MODEL AND DERIVATION OF STATE SPACE MODELS

State space models are very useful for dynamic modeling of power converters (Mohan *et al*., 1995; Skvarenina, 2002). They also provide a basis for applying various linear control techniques. A buck-boost converter possesses two different circuit configurations, corresponding to the ON state and the OFF state. The behaviour of the buck-boost converter circuit in these states can be depicted by state space models.

The general form of the state space model is given by

$$\dot{X} = AX + Bu$$
$$Y = CX + Du$$

Where X is the state vector, u is the input or control vector, A, B, C and D are the state matrix, the input matrix, the output matrix and the direct transmission matrix, respectively.The switching function $\delta(t)$ is used to describe the switch states. $\delta(t) = 1$ corresponds to the ON state circuit and $\delta(t) = 0$ corresponds to the OFF state.

Thus, over a time period T, divided into two switching intervals, the state space models for each switching interval can be given as

$$\dot{X} = A_1X + B_1u$$
$$Y = C_1X + D_1u \quad \text{for } 0 \le t \le \delta_1 T, \ \delta(t) = 1$$

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$$\dot{X} = A_2X + B_2u$$
$$Y = C_2X + D_2u \text{ for } \delta_1T \le t \le T,\ \delta(t) = 0$$

The differential equations governing the dynamics of the state vector $X = [i_L, v_0]^T$ are obtained as:
ON state equation:
$0 \le t \le \delta_1T$, $\delta(t) = 1$, S is ON, D is OFF;

$$L_i\frac{di_L}{dt} = V_{DC};\ C_0\frac{dv_0}{dt} = -\frac{v_0}{R_0}$$

OFF state equation:
$\delta_1T \le t \le T$, $\delta(t) = 0$, S is OFF, D is ON

$$L_i\frac{di_L}{dt} = -v_0;\ C_0\frac{dv_0}{dt} = i_L - \frac{v_0}{R_0}$$

From these differential equations, the A_1, B_1, C_1 and D_1 matrices can be obtained for the ON state and the A_2, B_2, C_2 and D_2 matrices can be obtained for the OFF state. The state space averaged model can be obtained from these.

$$\begin{bmatrix}\dot{\bar{i}}_L\\ \dot{\bar{v}}_o\end{bmatrix} = \begin{bmatrix}0 & -(1-\delta_1)/L_i\\ (1-\delta_1)/C_0 & -1/R_0C_0\end{bmatrix}\begin{bmatrix}\bar{i}_L\\ \bar{v}_o\end{bmatrix} + \begin{bmatrix}\frac{\delta_1}{L_i}\\ 0\end{bmatrix}\begin{bmatrix}\bar{V}_{DC}\end{bmatrix}$$

$$\begin{bmatrix}\bar{v}_0\\ \bar{i}_L\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\bar{i}_L\\ \bar{v}_o\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}\begin{bmatrix}\bar{V}_{DC}\end{bmatrix}$$

Small signal transfer function model: State space averaging provides an elegant solution for the application of widely known linear control techniques to most power converters (Rashid, 2001). Since the converter outputs must be regulated actuating on the duty cycle and the converter inputs usually present perturbations due to load and supply variations, state variables are decomposed to small AC perturbations and DC steady state quantities.

The transfer functions are derived from these state space averaged models.
Line to Output Transfer Functions:

$$\frac{\bar{i}_L(s)}{\bar{v}_0(s)} = \frac{\Delta_1(1+sC_0R_0)}{s^2L_iC_0R_0 + sL_i + R_0(1-\Delta_1)^2}$$

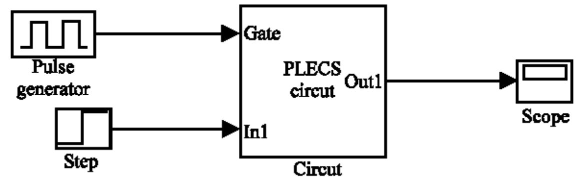

Fig. 1: Open loop model of buck-boost converter

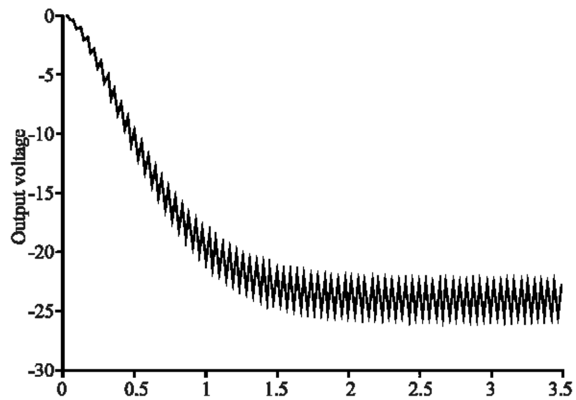

Fig. 2: Open loop characteristics

Small signal duty cycle to output transfer functions:

$$\frac{\bar{v}_0(s)}{\bar{\delta}(s)} = \frac{V_{DC}(R_0 - sL_i\Delta_1/(1-\Delta_1)^2)}{s^2L_iC_0R_0 + sL_i + R_0(1-\Delta_1)^2} \text{ Time}$$

The input to output steady state relations are derived and found to be:

$$\frac{I_L}{V_{DC}} = \frac{\Delta_1}{R_0(1-\Delta_1)^2}$$
$$\frac{V_0}{V_{DC}} = \frac{\Delta_1}{1-\Delta_1}$$

These relations are the steady-state transfer relationships of the buck-boost converter.

Open loop performance: The buck-boost converter model is designed using MATLAB as shown in Fig. 1 with the values of f_s = 20 kHz, V_{DC} = 28 V, V_0 = 24 V,
The output of the simulation is shown in Fig. 2.

As seen from Fig. 2, the required characteristics cannot be obtained with open loop since there is no feedback to control the duty ratio. Hence wide fluctuations are found in the output and the system becomes unstable. To get the desired characteristics, closed loop control is chosen.

SLIDING MODE CONTROL OF DC-DC CONVERTERS

Switched mode DC-DC converters are non-linear and time varying systems and so it is not easy to design linear controllers directly for these systems. The state space averaging can be used to design linear controllers, which operate in a narrow linear range. SMC is a logical control choice, which provides a more consistent way of handling the control problems in power converters. The converter switches are driven as a function of the instantaneous values of the state variables. The main feature of sliding mode control is its ability to result in very robust control systems (Rashid, 2001).

Basic principles: A variable structure system is one that is made up of a number of 'sub topologies', each of which is defined by the condition of non-linear elements, i.e., a system where the control law is deliberately changed during the process according to predefined rules that depend on the state of the system. The overall dynamics of the system is quite different from that of the individual sub topologies.

Consider the following Single Input Single Output (SISO) dynamic system:

$$x^{(n)} = f(x, t) + b(x, t)u(t) + d(t),$$
$$y(t) = x(t)$$

Where $x(t) = [x(t), x^{(1)}(t), x^{(2)}(t), \ldots x^{(n-1)}(t)]$ is the state vector ($x^{(i)}(t)$ is the i^{th} time derivative of the state variable x, f(x, t) is the system matrix, y(t) is the output vector, u(t) is the control input, d(t) is the disturbance input and n is the order of the system.

It is desired to track a state trajectory $x_d(t) = [x_d(t), x^{(1)}{}_d(t), x^{(2)}{}_d(t), \ldots x^{(n-1)}{}_d(t)]$, in the presence of modeling uncertainties and disturbances. Let $e(t) = x_d(t) - x(t)$ be the tracking error vector. Defining a scalar equation:

$$S(e, t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} e = 0,$$

Where λ is a strictly positive constant.

S is a surface on the phase plane of the system. This surface is called as the sliding surface. The motion of the system trajectories on this surface, i.e., the dynamics described by S(e, t) = 0, is known as the sliding mode and the line S is known as the sliding line or the switching line. This equation simply denotes that S is the weighted sum of the state error and its derivatives.

Once in the sliding regime, the resultant system performance is completely different from that dictated by any of the substructures of the VSS and is completely controlled by the control law. For the system to operate in the sliding mode, the following conditions must hold:

SLIDING MODE CONTROL OFBUCK-BOOST DC - DC CONVERTERS

All existing controllers for power converters are in fact, variable structure controllers, in the sense that the control action changes rapidly from one to another of, usually, two possible values of the switching signal $\delta(t)$, thereby cyclically changing the converter topology. This results in a ripple as the state trajectories move back and forth a certain average surface in the state space. This inherent ripple and the variable structure nature of the power converters lend themselves to the application of sliding mode control, with which superior performance can be achieved.

In this approach, the converter topologies, as non-linear time-variant systems, are controlled to switch from one dynamics to another when just needed. If this switching occurs at a very high frequency (ideally at infinite frequency), the state dynamics can be forced to slide along a certain prescribed state space trajectory. The converter is then said to be in sliding mode, the allowed deviations from the trajectory (the ripple) imposing the practical switching frequency.

This study deals with the derivation of the control law for the buck-boost dc-dc converter. Consider the basic circuit of a buck boost converter shown in Fig. 3.

Let us assume the output voltage v_o to be the controlled variable. The state space averaged model of this topology, derived already, is reproduced below:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_o \end{bmatrix} = \begin{bmatrix} 0 & -(1-\delta(t))/L_i \\ 1-\delta(t)/C_o & -1/R_oC_o \end{bmatrix} \begin{bmatrix} i_L \\ v_o \end{bmatrix} + \begin{bmatrix} \delta(t)/L_i \\ 0 \end{bmatrix} [V_{DC}]$$

To convert this into the phase variable canonical form, let us choose the errors in the output voltage and its derivative as the phase variables:

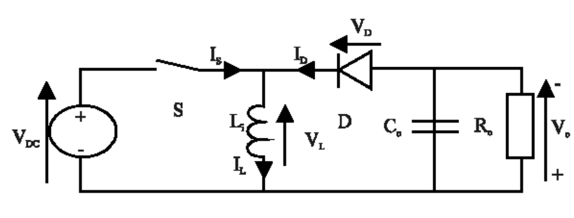

Fig. 3: A buck boost dc-dc converter

$$x_1 = V_{or} - V_o$$
$$x_2 = V'_{or} - V'_o = x_1'$$

Where V_{or} is the reference output voltage and V_o is the controlled output voltage. Choose the sliding surface S as:

$$S(x, t) = g_1 x_1 + g_2 x_2$$
$$g_1(V_{or} - V_o) + g_2(V'_{or} - V'_o) = 0$$

Assuming $\delta(t) \approx Vo / (Vo + V_{DC})$ is constant and substituting from the above equations, we get:

$$C_o \frac{g_1}{g_2}\left(\frac{v_{o_r} + V_{DC}}{V_{DC}}\right)\left[(v_{o_r} - v_o) + \frac{g_2}{g_1}\frac{v_o}{R_o C_o}\right] - i_L = 0$$

If S(x, t) = 0, then S'(x, t) = 0. Therefore,

$$C_o \frac{g_1}{g_2}\left(\frac{v_{o_r} + V_{DC}}{V_{DC}}\right)\left[-\frac{dv_o}{dt} + \frac{g_2}{g_1}\frac{1}{R_o C_o}\frac{dv_o}{dt}\right] - \frac{di_L}{dt} = 0$$

According to the existence condition,
If $S(x, t) > 0$, then $S'(x, t) < 0$, then
$di_L / dt > 0$ and $\delta(t) = 1$
If $S(x, t) < 0$, then $S'(x, t) > 0$, then
$di_L / dt > 0 => \delta(t) = 0$
Therefore, the control law is given as:

$$\delta(t) = \begin{cases} 1, S(x,t) > +\varepsilon \\ 0, S(x,t) < -\varepsilon \end{cases}$$

Dynamics in the sliding regime: In this study, it will be proved that the dynamics in the sliding regime is governed only by the error coefficients g_1 and g_2. The system is given in the state variable notation as:

$$\dot{X} = AX + BU$$

Let $G = [g_1\ g_2]$, $X = [x_1\ x_2]^T$ and hence the sliding surface is

$$S = GX = g_1 x_1 + g_2 x_2$$

The dynamics in the sliding regime is governed by $S'(x, t) = 0 => GX = 0$
Upon simplification,

$$u_{eq} = -(GB)^{-1}GAX$$

$$\dot{X} = (I - B(GB)^{-1}G)AX = A_{eq}X$$

$$A_{eq} = I - B(GB)^{-1}G$$

Where u_{eq} is the equivalent control input to the system and A_{eq} is the system matrix in the sliding regime. It can be easily shown that, in the phase variable representation, for the system in sliding mode,

$$A_{eq} = \begin{bmatrix} 0 & 1 \\ 0 & -g_1/g_2 \end{bmatrix}$$

This shows that the dynamics in the sliding regime is completely governed only by the ratio g_1/g_2. Note that the above derivation is based on the assumption that $(GB)^{-1}$ exists.

For positive values of both g_1 and g_2, the system is marginally stable and the ratio determines the pole location and hence the dynamics. As the model is a state space averaged model, $\lambda_m.T << 1$, where $\lambda_m = g_1/g_2$ is the maximum eigen value of the system (related to the maximum frequency of the system) and T is the time period of the switching signal.

RESULTS

The derived control law is simulated by using SIMULINK software. The converter circuit is simulated using PLECS software. The block diagrams used for simulation are shown in Fig. 4 and 5.

In the simulation, $C_0 . g_1/g_2 = 4$ and $\varepsilon = 0.3$, $v_{or} = 32V$ (boost mode) and 24V (buck mode) and $V_{DC} = 28$ V. The circuit parameters used in linear control

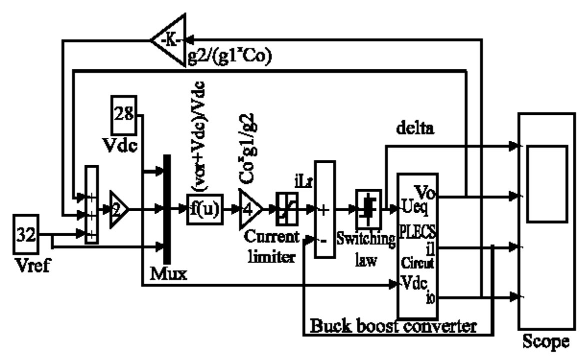

Fig. 4: Block diagram of sliding mode control of buck-boost converter

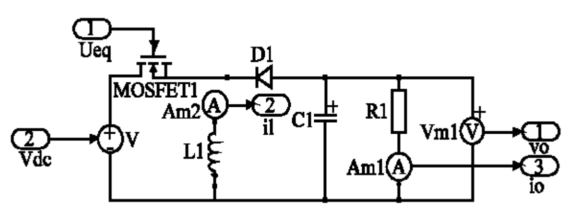

Fig. 5: Buck boost converter

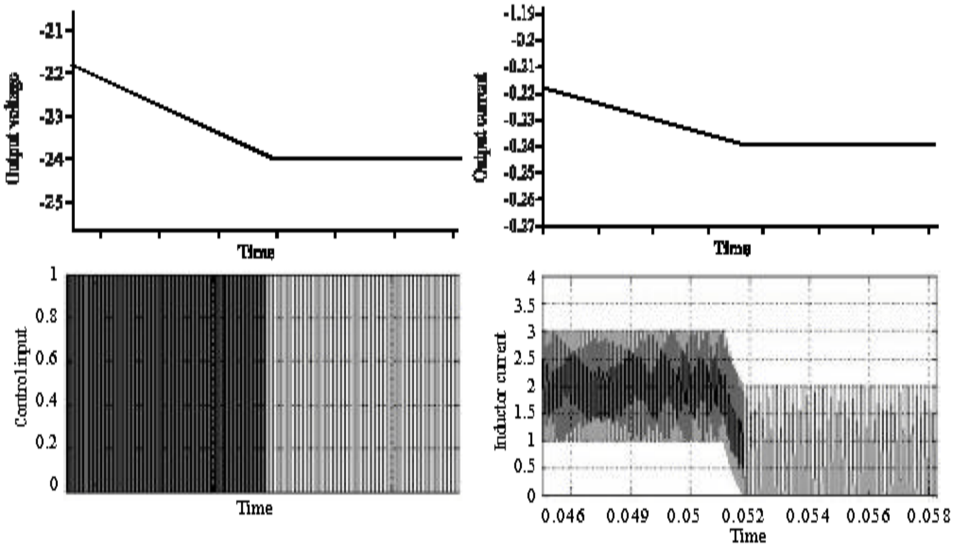

Fig. 6: Buck mode output (v_{or} = 24V)

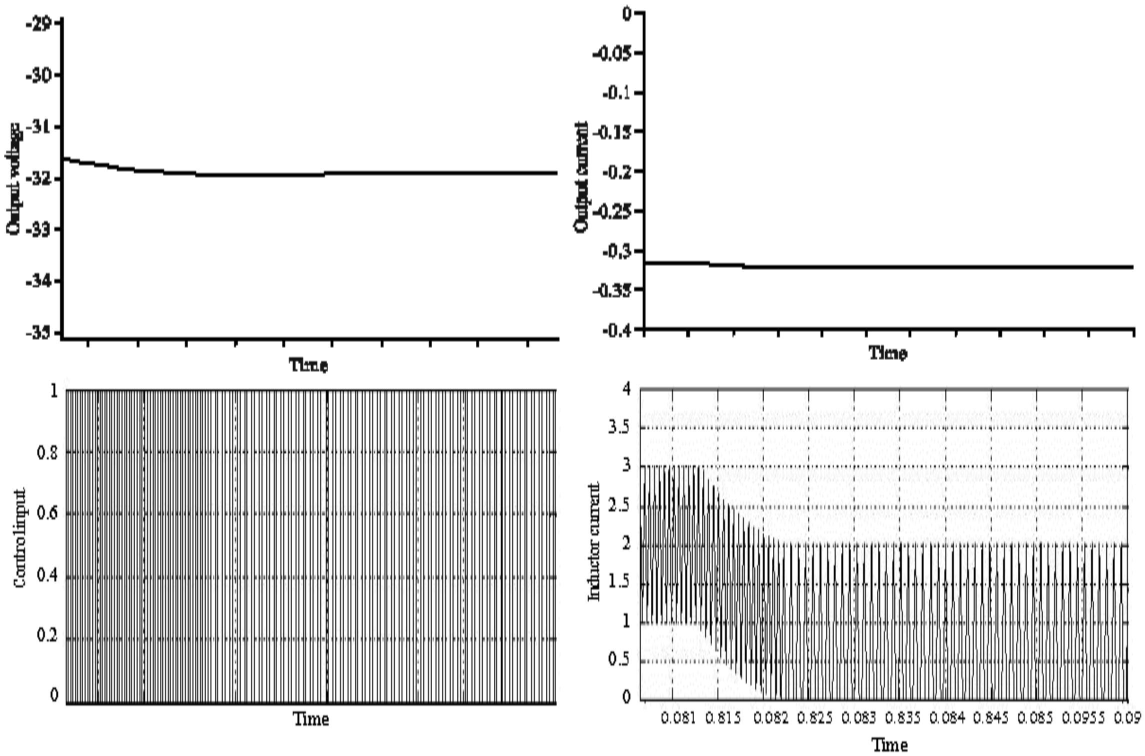

Fig. 7: Boost mode output (v_{or} =32V)

simulation are retained. The simulated output is as shown in Fig. 6 and 7.

Compared to the linear controller, it can be easily seen that sliding mode control gives a superior performance. Also undershoot (due to the presence of the RHP zero in the control to output transfer function) is not present in the response (Fig. 8) for reasons advanced in the earlier study.

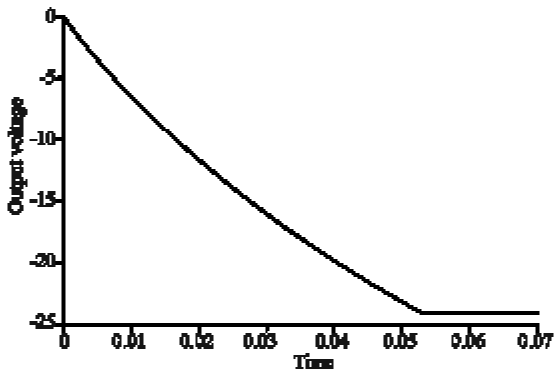

Fig. 8: Output response with SMC

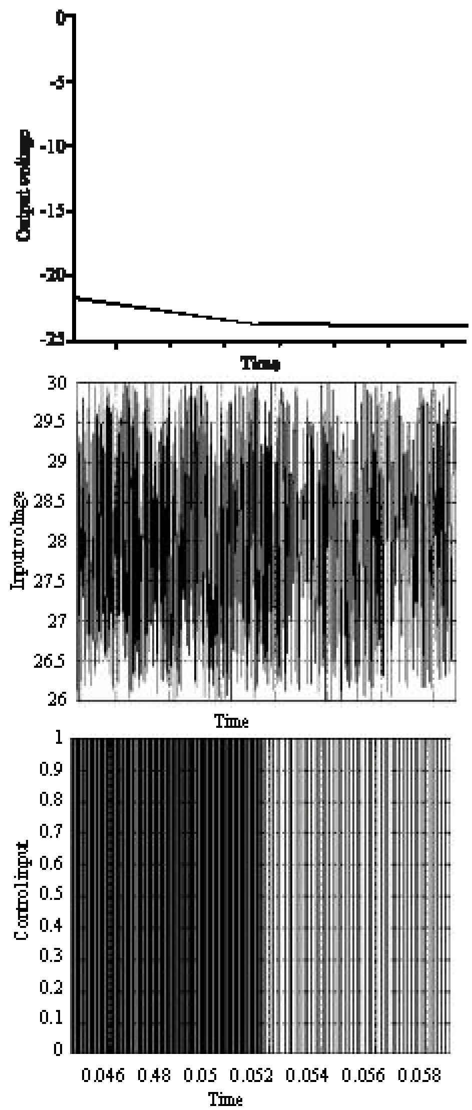

Fig. 9: Output response with input disturbance

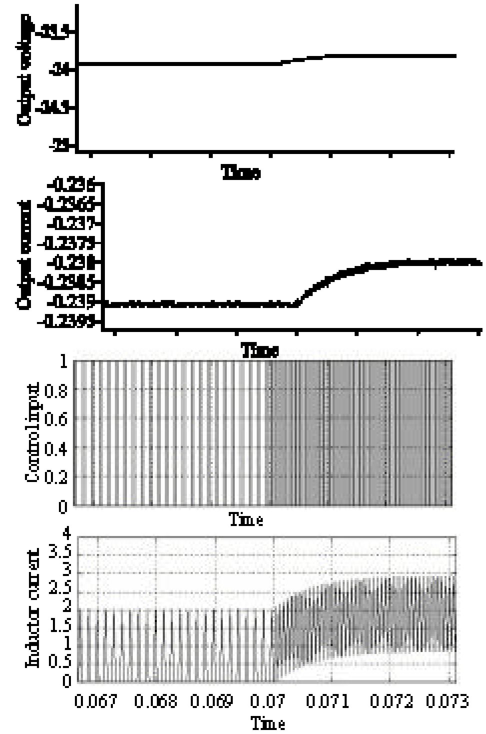

Fig. 10: Output response with load disturbances

To prove the robust nature of the sliding mode controller, input disturbance (noise super imposition) and load disturbance (switching on a 2500 Ω load parallel to existing load at t = 0.07s) were simulated and the results obtained are as shown in Fig. 9 and 10. It is seen that the output response is insensitive to these variations, a manifestation of the controller's robustness.

CONCLUSION

The performance of the sliding mode controller for buck-boost converter are analysed under normal and disturbance conditions. However, under input and output disturbance conditions there are noise present in the response characteristics. Such noise can be eliminated by proper operating conditions and by use of fuzzy or neural models. However, this research gives better performance than conventional controllers.

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