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Optimum Full Load Losses of a Transformer by Graphical Method

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Abstract: The estimation of losses are essential for obtaining efficiency. The estimation of optimum losses are much important for obtaining maximum efficiency. The conventional design procedure is a trial and error method. Further it is uncertain to obtain optimum values. Here in this study, the design equations are suitably modified such that the objective function and constraint equations are expressed with the help of only 2 variables. Graphical method is explained where the optimum values for both the variables are obtained simultaneously. The values are unique in nature. A single phase transformer is considered and the optimum values for full load losses are obtained and analyzed.

Key words: Efficiency, losses, optimum, transformer, unique, design, estimation

INTRODUCTION

The transformer is a static device consisting of magnetic circuit and electric circuit. To accommodate it, space is needed which leads to physical dimensions. Frank and Durwood (1977) has explained in their study about designing transformers with given structure for maximum output VA. Further he said that a number of computer-aided design procedures for power transformers have appeared in the literature, while in some design techniques the word optimum has been used to describe the resulting design, each is in effect based on a heuristic technique often involving a modified process of direct enumeration. The principal design objectives in these proposed techniques are usually minimum physical sizes or mass, maximum efficiency and cost. Unfortunately, a number of assumptions are often made which are not only contradictory, but tend to artificially constrain the resulting design away from the optimum the technique was intended to find.

Since a transformer consists basically a structure housing the circuits, some designers took structural parameters as fixed and allowing magnetic and electric parameters to vary or vice versa,. Accordingly they have implied that a transformer design cannot be unique for a given set of ratings and specifications. The lack of a unique design however is not an inherent property of the transformer design. It can be shown that uniqueness or non-uniqueness is solely dependent on how the design procedure is organized. For example, if the design procedure is based on the solution to a well posed mathematical optimization problem, then the uniqueness of the resulting design can be guaranteed relative to the design objectives.

The variables or parameters associated with the transformers may be grouped into two categories, namely free parameters or variables and fixed parameters. Optimization is done as per the choice of the designer. However, for the transformer, it is worth to bring that either fix electrical and magnetic parameters and vary the geometry/structural parameters or vice versa to minimize an overall objective such as weight, volume, losses or cost.

The second type of approach, used in this study, is preferred for at least three reasons. First, transformer cores are generally available only in discrete sizes. Second, even if a wider choice of core geometries were available, one would ultimately be faced with the problem of choosing the best values for certain electrical and magnetic parameters in order to complete the design. Third, the designer's ability to assess in the fixed parameters is greatly facilitated.

The problem taken for design is to obtain minimum full load losses and at the same time to get rated VA as output of the given transformer with given ratings and specifications.

Before proceeding with the solution to this problem, it may be worth listing a few assumptions imposed during this study. These are as follows

- Full load Iron and copper losses (P_i and P_c) are modeled by the below mentioned Eq. 1 and 2, respectively.

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$$P_i = P_h + P_e \quad (1)$$

Where P_h is hysteresis loss, W and P_e is eddy current loss, W
$P_h = \eta f B_m^2 V$ and
$P_e = K_e f B_m^2 V$
η = Steinmetz hysteresis coefficient (191 J m^{-3} for silicon steel (Theraja and Theraja, 2002)).
B_m = Maximum flux density in the core, Wb m^{-2}
f = Frequency, Hz
V = Volume of iron, m^3
$K_e = \pi^2 t^2/6\rho$
t = Thickness of lamination, m
ρ = Resistivity of silicon steel, Ω-m.

$$P_c = I_p^2 R_p + I_s^2 R_s \quad (2)$$

Where I_p and I_s are primary and secondary winding currents, A respectively and R_p and R_s are primary and secondary winding resistances Ω, respectively.

- Exciting current can be neglected; i.e., primary and secondary mmfs are equal.
- Primary and secondary current densities are identical.

These assumptions lead to simple mathematical results. Since the intent in this paper is to demonstrate an improved design strategy for transformers with given structure, it is important to keep the mathematics as simple as possible without adversely compromising the practicalities of the design problem.

Accordingly, in this study, the design equations are suitably modified such that the objective function and constraint equation are formed only with two variables. However, length of flux path is treated as a parameter for obtaining optimum values. Graphical procedure which is an analytical method is explained and optimum values for the variables are obtained simultaneously. The values thus obtained are unique in nature.

DERIVATION FOR MINIMUM FULL LOAD LOSSES OR MAXIMUM EFFICIENCY

At any fraction x of full load, the total full load losses P_T, are equal to $Pi+x^2Pc$ (Sawhney, 2001). Even though copper loss is variable, it is constant at one specified load, say full load. It means that full load is fixed. If Q is the kVA output at full load, the output at any fraction x of full load is xQ.

Therefore, efficiency at output xQ, η_x is

$$\eta_x = xQ/(xQ + P_i + x^2 P_c)$$

This efficiency is maximum when $d\eta_x/dx = 0$, Thus we get the condition $P_i = x^2 P_c$ which indicates that the maximum efficiency is obtained when the variable losses (copper loss) are equal to constant losses (iron loss). However, the full load losses at x = 1 is $P_i = P_c$. Therefore, the total full load loss, $P_T = P_i + P_c$, which is also constant.

The efficiency at full load and at UPF, η_{fl}, is given by

$$\eta_{fl} = \{Q/(Q+P_T)\} \times 100\%$$

Formation of objective function and constraint equation:

As stated earlier, the objective is to obtain minimum total full load losses

$$P_T = P_i + P_c \quad (3)$$

Hence the above Eq. 3 will be our objective function. It may be noted that there are only two variables.

The constraint is to get rated kVA output of the transformer. Considering a single phase core type distribution transformer with ratings and other structural specifications are available, the output can be written as

$$Q = 2.22 f B_m \delta\, K_w A_w A_i \times 10^{-3} \text{ kVA} \quad (4)$$

In the above Eq. 4, excepting magnetic parameter B_m and electric parameter δ, which are variable parameters, the other quantities are known. The task now is to modify the above equation so that P_i and P_c appear as variables in place of B_m and δ.

We know

$$\begin{aligned} P_i &= P_h + P_e \\ &= \eta f B_m^2 A_i l_i + (\pi^2 t^2/6\rho) f^2 B_m^2 A_i l_i \\ &= B_m^2 [\eta f A_i l_i + (\pi^2 t^2/6\rho) f^2 A_i l_i] \end{aligned}$$

$$B_m^2 = \frac{P_i}{\left[\eta\, f\, A_i\, l_i + (\pi^2 t^2/6\rho) f^2 A_i\, l_i\right]}$$
$$\therefore B_m = \sqrt{\frac{6\, P_i\, \rho}{\left[(\eta\ \ 6\,\rho + \pi^2 t^2 f)\, f\, A_i\, l_i\right]}} \quad (5)$$

Also we know $P_c = \delta^2 \rho_c$.volume
$= \delta^2 \rho_c . A_c L_{mt}$

$$\delta^2 = \frac{P_c}{\rho_c A_c L_{mt}} \quad \text{and}$$
$$\therefore \delta = \sqrt{\frac{P_c}{\rho_c A_c L_{mt}}} \quad (6)$$

Substituting for B_m and δ as obtained from Eq. 5 and 6 in Eq. 4,

$Q = 2.22fB_m\delta A_c A_i \times 10^{-3}$

$$Q = 2.22\,f\left[\sqrt{\frac{6\,P_i\,\rho}{f(6\,\eta\,\rho + \pi^2\,t^2\,f)A_i\,l_i}}\right]\left[\sqrt{\frac{P_c}{\rho_c\,A_c\,L_{mt}}}\right]A_c\,A_i \times 10^{-3}$$

$$Q = \left[\sqrt{\frac{29.6\,f\,\rho\,Ac\,Ai \times 10^{-6}}{(6\,\eta\,\rho + \pi^2\,t^2\,f)\rho_c\,L_{mt}\,l_i}}\right]\sqrt{P_i\,P_c}$$

$$\sqrt{P_i\,P_c} = \frac{Q}{\left[\sqrt{\frac{29.6\,f\,\rho\,Ac\,Ai \times 10^{-6}}{(6\,\eta\,\rho + \pi^2\,t^2\,f)\rho_c\,L_{mt}\,l_i}}\right]}$$

$$= \sqrt{\frac{Q^2(6\,\eta\,\rho + \pi^2\,t^2\,f)\rho_c\,L_{mt}\,l_i}{29.6\,f\,\rho\,Ac\,Ai \times 10^{-6}}}$$

$$\therefore\ P_i\,P_c = \frac{Q^2(6\,\eta\,\rho + \pi^2\,t^2\,f)\rho_c\,L_{mt}\,l_i}{29.6\,f\,\rho\,Ac\,Ai \times 10^{-6}} \qquad (7)$$

The above Eq. 7 is the constraint or performance equation. Thus the optimization problem is formed.

GRAPHICAL OPTIMIZATION

Note that the constraint Eq. 7 may be made as a function in l_i, where l_i is treated as a parameter (PEDES, 2006). The constraint function is drawn as a graph between P_i and P_c. Since the equation is non-linear, the constraint equation surface will be in the shape of parabola. It is as shown in Fig. 1. The objective function

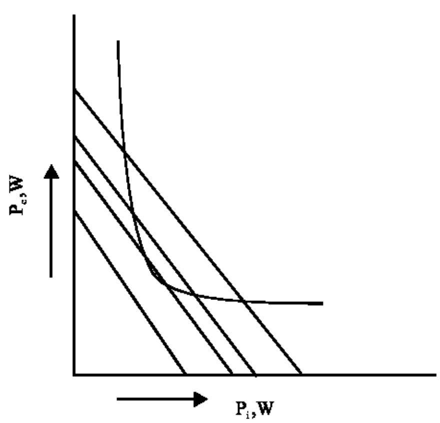

Fig. 1: Graphical optimization

(Eq. 3) is a linear equation .For a series of assumed values of total full load losses P_T, straight line surfaces are drawn between P_i and P_c on the same graph as shown in Fig. 1. Straight line graphs are drawn till it becomes tangent to the parabolic curve of constraint equation. The tangent point gives the values for P_i and P_c simultaneously. The values are unique and accurate. It may be remembered that since the constraint is taken at full load ie. Rated output, the fraction of full load x = 1. Hence the values obtained from graph are corresponding to full load and must be equal for optimum full load losses.

DESCRIPTION OF TRANSFORMER

Design a 5kVA, 11000/400V, 50Hz, single phase core type distribution transformer for optimum full load losses for different mean lengths of flux path in the core, l_i. Given that net core area, $A_i = 0.00729\ m^2$; Area of copper in window, $A_c = 0.6A_i$. The mean length of turn of winding, $Lmt = 0.4l_i$. The hysteresis coefficient, η=191J/m^3. The resistivity of silicon steel ρ is 0.6×10^{-6} Ωm. The core is made up of thin laminations of silicon steel of grade 92 having 0.35mm thick. The resistivity of copper conductor is 0.021×10^{-6} Ω-m.

GRAPHICAL ESTIMATION

While going through the description of the transformer, it may be observed that all structural data of the transformer are given. Further the magnetic and electric data, namely B_m and δ are not given. Hence we can use Eq. 5 and 6, where they are in terms of P_i and P_c, respectively in Eq. 7.

The constraint equation is

$$P_i\,P_c = \frac{Q^2(6\,\eta\,\rho + \pi^2\,t^2\,f)\rho_c\,L_{mt}\,l_i}{29.6\,f\,\rho\,Ac\,Ai \times 10^{-6}}$$

Substituting the values, the above equation becomes

$$P_iP_c = 5548l_i^2 \qquad (8)$$

For l_i= 1m; the points for P_i and P_c are obtained and tabulated as shown in Table 1.

The objective function is

$$P_T = P_i+P_c$$

For a series of assumed values for P_T, the points for Pi and Pc are obtained and tabulated as shown in Table 2-4.

Table 1: Constraint characteristics for l_i=1m

P_i,W	0	10	20	30	40	50
P_c,W	∞	554.8	277.4	185	139	111
P_i,W	60	70	80	90	100	110
P_c,W	92.5	79	69.35	61.6	55.48	50.4

Table 2: Objective function for P_T =120W

P_i,W	0	30	60	90	100	120
P_c,W	120	90	60	30	20	0

Table 3: Objective function for P_T=148.96W(≈149W)

P_i,W	0	30	60	90	120	148.96
P_c,W	148.96	118.96	88.97	58.96	28.96	0

Table 4: Objective function for P_T=160W

P_i,W	0	30	60	90	120	160
P_c,W	160	130	100	70	40	0

Graphs of constraint characteristics and objective functions are drawn on the same graph and the shape will be as shown in Fig. 1. It is observed that graph of objective function for P_T=149W becomes tangent to the constraint characteristics. The tangent points for P_i, P_c are 74.5W,74.5W.

The values are equal as expected and give the optimum full load losses for the transformer and satisfies the condition.

ANALYSIS

- Variation of full load losses, P_T with variation in l_i
  (a) For l_i = 1m; the total losses P_T = 149W. The full load efficiency at UPF,η_{fl} = 97.1%
  (b) If l_i = 2m; the total losses P_T = 298W The full load efficiency at UPF,η_{fl} = 94.4%
  (c) If l_i = 3m; ; the total losses P_T = 447W
  The full load efficiency at UPF, η_{fl} = 91.79%
  (d) If l_i = 4m; the total losses P_T = 596W.
  The full load efficiency at UPF, η_{fl} = 89.34%
  (e) If l_i = 5m; the total losses P_T = 745W.
  The full load efficiency at UPF, η_{fl} = 87%

A graph is drawn between l_i and total full load losses P_T and it is as shown in Fig. 2. It is a linear graph .At this juncture, it is reminded that the variation of losses with linear dimensions are cubic times the linear dimension. But the graph obtained is linear because the areas are kept constant (i.e., A_i and A_c)

- Efficiency at half full load (x = 0.5). Copper loss = $(0.5)^2 P_c$ = 18.6 W
  We know, P_i = 74.5 W

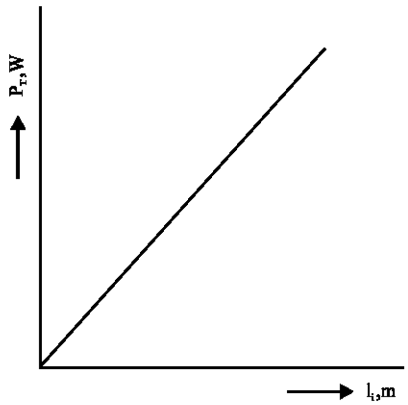

Fig. 2: l_i Vs P_T

∴ For l_i = 1m case; the total losses,
P_T = 74.5+18.6 = 93.1W
η at UPF = 98.17%

Fairly a good percentage of efficiency at 50% of full load, since it is a distribution transformer.

CONCLUSION

The optimum full load losses are obtained graphically and simultaneously and found satisfying the condition. Since the tangent point is giving the values, it is unique and accurate. Further it is observed that variations of total full load losses are linear with variation in mean length of flux path. It is understood that it is due to constant value of areas. Also the efficiency at half full load is calculated and it is satisfactory. It is concluded that this method may be adopted for estimating directly and simultaneously the full load losses of a given transformer.

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