

Observation Techniques for Sensorless Control in Induction Motor

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Abstract: This study presents the application of the techniques of the observer in sensorless control of inductor motor. Sensorless control requires the estimate in real time of the non measurable quantities. The observer of Luenberger and Kalman makes it possible to solve this problem thanks to the use of a model of state of the induction motor. In this research we use the model having for variables of state the current and stator flux. The technique of sensorless control using the observer of a complete nature is presented in simulation. Linear and non linear variables of Luenberger observer are also discussed in theoretical overview.

Key words: Induction motor, sensorless control, Kalman and Luenberger observers, techniques

INTRODUCTION

Sensorless control aims to decrease the number of sensors to be used in an industrial application which will increase the reliability and the robustness of the system of drive while decreasing its cost. The obstruction and the preventive maintenance are also reduced. The modern techniques of control concentrate, currently, their efforts on this problem. The basic idea consists in replacing a certain number of material sensors by sensors known as software sensors, i.e., programme or algorithms. It is obvious that implementing this type of sensors will decrease the number of failures of the systems of drive. In this research we will try out the simple case of these techniques of sensorless control by highlighting the difficulties which can be encountered.

ASYNCHRONOUS MACHINE MODELING

Induction motor is a nonlinear and no stationary system. The complexity of its model can be simplified by using the transformation of PARK and the technique of the orientation of flux. Various forms of models exist in the literature. To be able to apply modern techniques of control and monitoring of the systems, the representation by model of state is more suitable. The model of induction motor in the state space and an arbitrary reference which is given by Maouche *et al.* (2003).

$$\frac{d}{dt}\psi = \Omega\psi - RI + I_0U \quad (1)$$

$$\psi = L_M I \quad (2)$$

With: $\psi = \begin{bmatrix} \Phi_s \\ \Phi_r \end{bmatrix}$, is the vector of flux, $I = \begin{bmatrix} i_s \\ i_r \end{bmatrix}$, is the vector

of current, $U = \begin{bmatrix} u_s \\ u_r \end{bmatrix}$, is the vector of voltage.

The matrix Ω , R , I_0 and M are defined by:

$$\Omega = \begin{bmatrix} \Omega_a & 0_2 \\ 0_2 & \Omega_{ar} \end{bmatrix} \text{ with } \Omega_a = \begin{bmatrix} 0 & \omega_a \\ -\omega_a & 0 \end{bmatrix}$$

$$\Omega_{ar} = \begin{bmatrix} 0 & (\omega_a - \omega_r) \\ -(\omega_a - \omega_r) & 0 \end{bmatrix}, R = \begin{bmatrix} R_s & 0_2 \\ 0_2 & R_r \end{bmatrix}$$

With

$$R_s = \begin{bmatrix} r_s & 0_2 \\ 0_2 & r_s \end{bmatrix}, R_r = \begin{bmatrix} r_r & 0_2 \\ 0_2 & r_r \end{bmatrix},$$

$$L_M = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix} \text{ with } L_s = \begin{bmatrix} l_s & 0_2 \\ 0_2 & l_s \end{bmatrix},$$

$$L_r = \begin{bmatrix} l_r & 0_2 \\ 0_2 & l_r \end{bmatrix}, M = \begin{bmatrix} m & 0_2 \\ 0_2 & m \end{bmatrix} \text{ and finally } I = \begin{bmatrix} I_2 & 0_2 \\ 0_2 & I_2 \end{bmatrix}$$

The lemma of matrix inversion is used to simplify calculations and facilitates the transformation of the various dynamic models of the asynchronous machine according to the choice of the variables of state. Matrix L_m given by:

$$L_M = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix}$$

has as an inverse:

$$L^{-1}_M = \begin{bmatrix} (L_s - M L_r^{-1} M^T)^{-1} & -(L_s - M L_r^{-1} M^T)^{-1} M L_r^{-1} \\ -(L_r - M^T L_s^{-1} M)^{-1} M^T L_s^{-1} & (L_r - M^T L_s^{-1} M)^{-1} \end{bmatrix}$$

Various models corresponding to various references and various choices of the variables of states are possible. Among these models we consider the case of the stator current and rotor flux as variables of state. The model is written then:

$$\frac{d}{dt} \begin{pmatrix} i_s \\ \Phi_r \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ -R_s & \Omega_s \end{bmatrix} \begin{pmatrix} i_s \\ \Phi_r \end{pmatrix} + \begin{bmatrix} b_{11} \\ I_2 \end{bmatrix} u_s \quad (3)$$

$$\frac{d\omega_r}{dt} = \frac{1}{J} p (T_e - T_l) \quad (4)$$

$$T_e = p i_s^T \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \Phi_s \quad (5)$$

$$\begin{pmatrix} i_r \\ \Phi_s \end{pmatrix} = \begin{bmatrix} -M^{-1} L_s & M^{-1} \\ (M - L_r M^{-1} L_s) & L_r M^{-1} \end{bmatrix} \begin{pmatrix} i_s \\ \Phi_s \end{pmatrix} \quad (6)$$

The parameters of the matrix evolution are:

$$\begin{aligned} a_{11} &= \Omega_{ar} + (M - L_r M^{-1} L_s)^{-1} (R_r M^{-1} L_s + L_r M^{-1} R_s) \\ a_{12} &= (M - L_r M^{-1} L_s)^{-1} [(\Omega_s L_r - R_r) M^{-1} - L_r M^{-1} \Omega_s] \\ a_{21} &= -R_s, \quad a_{22} = \Omega_s, \\ \text{and } b_{11} &= -(M - L_r M^{-1} L_s)^{-1} L_r M^{-1} \end{aligned}$$

TECHNIQUE OF OBSERVATIONS

Kalman observator: The filter of Kalman gives a realization of the random $\hat{X}(t)$ variable representing the state of the system at the moment $X(t)$, which minimizes the priori estimation error, knowing the vector of measurement $Y(t) = (y_1, y_2, y_3, \dots, y_n)$. $\hat{X}(t)$. $Y(t) = (y_1, y_2, y_3, \dots, y_n)$. For that it uses a stochastic model of a system of a nature n defined by the following equations of observation and states:

$$Y(t) = AX(t) + BU(t) + V(t) \quad (7)$$

$$\dot{\hat{X}}(t) = A \hat{X}(t) + B U(t) + G W(t) \quad (8)$$

Where $U(t)$ and $Y(t)$ are the input signals and outputs, $X(t)$ is the vector of state of the system, $W(t)$ and $V(t)$ are the noises of state of the system and

measurement of the sensors. The matrix A, B, C, D and G are appropriately dimensioned (Bensaker, 1999). The noises $W(t)$ and $V(t)$ are generally defined by their statistically properties in the following way:

$$\begin{aligned} E[W(t)] &= E[V(t)] = 0, \quad \text{Var}[W(t)] = Q, \\ \text{Var}[V(t)] &= R \text{ and } \text{Cov}[W(t) V^T(t)] = S. \end{aligned}$$

Where $E[.]$ the expectation represents the average value of the variable considered, $\text{Var}[.]$ represents its variance and $\text{Cov}[.]$ represents its covariance. The problem of the observation is to rebuild all or part of the vector of state $X(t)$ when all or part of this vector of state is not measurable. This technique reduces the cost of the systems of the drive and increases their reliability by using less sensors. If a part only of the vector of state is not measurable the observer is known as of a reduced nature. The system's observability can be seen as the possibility measurement on the real system. Knowing the matrix A and C and if a system is observable, it is possible to rebuild its state from inputs ($U(t)$) and outputs ($Y(t)$) measurement. The interest of this reconstruction is multiple, it consists to obtain some state's components for which measurement is difficult, impossible or too expensive.

The observation of the system using the Kalman filter consists to estimate the stochastic system's state, modelled by the Eq. 7 and 8, recursively, in order to be real time implemented. In its extended version, it also allow to consider the variable or unknown system's parameters.

Under these conditions the estimator Kalman equations are (Metatla *et al.*, 2003; Bodson and Chiasson, 2002):

$$\frac{d\hat{X}(t)}{dt} = A\hat{X}(t) + BU(t) + K(t)[Y(t) - \hat{Y}(t)] \quad (9)$$

$$\hat{Y}(t) = C\hat{X}(t) + DU(t) \quad (10)$$

$$K(t) = (AP(t)C^T + S) [CP(t)C^T + R]^{-1} \quad (11)$$

$$\begin{aligned} \frac{dP(t)}{dt} &= AP(t)A^T + Q - K(t)[CP(t)A^T + S^T] \\ &\quad - [CP(t)C^T + R]K(t) \end{aligned} \quad (12)$$

Where $K(t)$ is the profit of the filter, $P(t)$ the matrix of variance of the error in estimation. Q , R and S are the matrices variances of the noises of state, measurement

and inter-correlation of the noises of state and measurement. The disadvantage of the filter of Kalman is that it requires knowledge of a prior statistical properties of the noises.

Observer of Luenberger: The observer or deterministic estimator of Luenberger makes it possible to reconstitute the state of an observable system starting from the measurement of outputs and the inputs. Its principle can be represented by simply removing the terms representing the noises. The system is then modelled in a deterministic way. It is widely used in the control of the systems by return of state when all or part of the vector of state cannot be measured. Under these conditions the model of the system is represented as follows:

$$Y(t) = CX(t) + DU(t) \quad (13)$$

$$\dot{X}(t) = AX(t) + BU(t) \quad (14)$$

The observer of Luenberger is defined by the two following equations

$$\frac{d\hat{X}(t)}{dt} = A\hat{X}(t) + BU(t) + L[Y(t) - \hat{Y}(t)] \quad (15)$$

$$\hat{Y}(t) = C\hat{X}(t) + DU(t) \quad (16)$$

Where L is the profit of the observer of Luenberger. It is to be determined by a technique of placement of poles while being based on the fact that the dynamics of the observer must be faster than that of the system to be observed (Bensaker, 1999).

These two preceding relations can be written in the following from highlighting the inputs of the observer.

$$\frac{d\hat{X}(t)}{dt} = [A - LC]\hat{X}(t) + [B - LD]U(t) + LY(t) \quad (17)$$

$$\hat{Y}(t) = C\hat{X}(t) + DU(t) \quad (18)$$

The dynamic of the observer is controlled by the eigenvalues of the matrix $[A - LC]$ and consequently it can be also determined by the use of a technique of placement of poles. The observer of Luenberger can be used to reconstitute any variable of state of the machines, for example the number of revolutions of the induction motor from the electromagnetic torque and of the angular position (measured or estimated). When the resistive

torque is known, the electromagnetic torque is estimated indirectly from the estimate of the flow and the measurement of the stator currents. The quality of the results is sensitive, directly or indirectly (via the estimate of flux) with variations of the parameters of the machine. The choice of the reference must be adapted to the terms available (according to the type of control and the simplicity of calculations).

PRINCIPLE OF CONTROL WITHOUT SENSORS

That is to say a system modelled in the space of state by:

$$X(k+1) = AX(k) + BU(k) \quad (19)$$

$$Y(k) = CX(k) \quad (20)$$

If the moment K corresponds to the order n of the system then the system of equations to solve comprises n unknown allowing to determine the n control signals $U(l)$ which passes the system of the state $X(0)$, to the state $X(n)$. The solution of the state equation of the system is given by:

$$X(k) = A^k X(0) + \sum_{l=0}^{k-1} A^{k-l-1} BU(l) \quad (21)$$

It can be also written in the following matrix form:

$$X(k) = A^k X(0) + [A^{k-1} B A^{k-2} B \dots A B B] \begin{pmatrix} U(0) \\ U(1) \\ \vdots \\ U(k-1) \end{pmatrix} \quad (22)$$

Where in an equivalent way:

$$X(k) - A^k X(0) = P \cdot U \quad (23)$$

Thus the order U is given by :

$$U = P^{-1} [X(k) - A^k X(0)] \quad (24)$$

Where and $P = [A^{k-1} B A^{k-2} B \dots A B B]$ and $U = [U(0) U(1) \dots U(k) U(k+1)]$

The relation (6) shows that the control relation when the initial conditions are null is of the form:

$$U = -K X \quad (25)$$

Ie. an order of function of the state variables or control by return of state. It should be noticed here that the matrix P is only the matrix of commandability of the system.

Constraint on the control: If one then imposes on the control signal U one or more constraints, to force it to vanish very quickly, which means optimising a criterion of performance. This constraint forces also the control signal to take very large values which cannot be limited and consequently not realizable. The introduction of a matrix of weighting Q makes it possible to obtain a law of order softer and more precise than in the case of the preceding order. The criterion to be minimized takes the form:

$$J = U^T Q U \quad (26)$$

Where the matrix Q is a positive definite symmetrical matrix. The resolution of this problem by using the method of the multipliers of Lagrange gives the relation of required control:

$$U = (Q^{-1} P^T) (P Q^{-1} P^T)^{-1} X(k) \quad (27)$$

Which is also a function of the state variables (control by return of state)?

Constraint on the state of the system: The criterion to be minimized in this case is: $J = X^T X$, While taking, $X = RU = AkX(0) + \sum Ak^{i-1}BU(1)$, with $X(0) = 0$, The criterion becomes: $J = U^T R^T RU = U^T QU$, which gives the preceding case whose solution is: $U = (Q^{-1}P) (P^T Q^{-1}P)^{-1} X$, The constraint can also be imposed on the output of the system Y(k). Under this condition the criterion is:

$$\begin{aligned} J &= Y^T Y = X^T C^T C X = X^T M X \\ &= U^T R^T M R U = U^T Q U \end{aligned} \quad (29)$$

Constraint on the state and the control of the system: The case of a constraint on the control and the state of the system can be treated by combining the two preceding cases. The criterion to be minimized will be as following:

$$J = X^T X + U^T U \quad (30)$$

By using the change of variable one can write:

$$J = U^T (Q+I)U \quad (31)$$

The solution using the multiplier of Lagrange is:

$$U = [Q+I]^{-1} P [P^T (Q+I)^{-1} P]^{-1} X \quad (32)$$

THE CONTROL WITHOUT SENSORLESS BY RETURN OF STATE

The sensorless control is used in the case where all or part of the vector of state is not measurable. Under this condition the non measurable part of the vector state must be estimated by using a technique of estimate. The most used technique is based on the concept of observer. The observer of state which is called also estimator or reconstructed of state. The sensorless control by return of state consists thus in replacing the non measurable components of the vector state by their estimated values. The relation of control is then written:

$$U(t) = -K\hat{X}(t) \quad (33)$$

Or the profit K is also determined by a technique of placement of poles. In the case where the system is also deterministic, i.e., where the noises have not an influence on the system, the observer of Luenberger is used to rebuild for example the number of revolutions of an electric motor or the flux of its rotor starting from measurements of the stator current and by the same way one also consider couple electromagnetic torque.

THE NUMERICAL SIMULATION

The principle of simulation is identical for all the model of the asynchronous machine. Under these conditions we will try out by simulating the case of the model having for vector state the current and the flux of the stator. As the stator currents are measurable and generally available practically in the regulation of the systems of drives, we will estimate the other variables of state of the asynchronous machine in particular stator and rotor flux and the rotor current. The stator flux as variable of state estimated is used to carry out the sensorless control of the asynchronous machine considered. Language MATLAB is used for the simulation. The characteristics of 11KW, bipolar motors. The simulations are presented in the Table 1.

Table 1: The characteristics of 11KW, bipolar motors

Sizes (terms)	Values	Units
Frequency	50	Hz
Tension three -phase current	380	Volts
Resistance of the stator	5.850	Ohms
Resistance of the rotor	3.650	Ohms
Inductance of the stator	0.6758	Henry
Inductance of the rotor	0.6758	Henry
Mutual inductance	0.6750	Henry
Coefficient of friction	1	N _r /rd
Moment of inertia	2.960.10 ⁻³	Kg ² /s
Rotor pulsation	303.01	rd/s
Scatter coefficient	0.067	

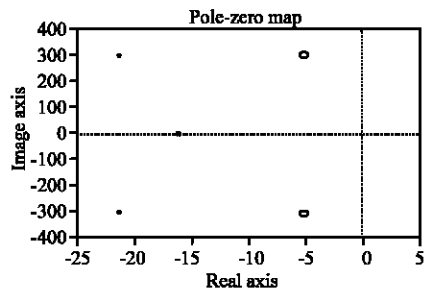


Fig. 1: Position of the pole and the zero

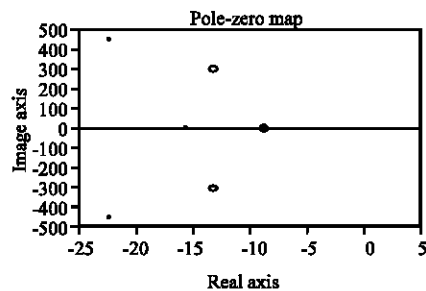


Fig. 2: Poles of the observer/machine

DYNAMIC STUDY OF THE MACHINE

Before passing to the technique of observation and estimate of the not measurable sizes of our machine, it is necessary to make a study on its dynamics. Figure 1 presents the position of the poles and the zeros of the asynchronous machine in the complex plan. One can see that there are 4 combined complex poles and 2 zero combined complexes. The real parts of the poles are negative, thus the system is stable.

SYNTHESIS OF THE OBSERVER

Figure 2 presents the position of the poles of the observer compared to the poles of the system which shows that the dynamics of the observer gives an answer faster than the dynamics of the machine.

The figure thus of follows present the sizes of estimated state.

CONTROL BY RETURN OF ESTIMATED STATE

According to the use of the control by return of state: $U(t) = -K\hat{X}(t)$. The numerical value of the profit K , determined by the technique of placement of poles, Fig. 4 present the new control.

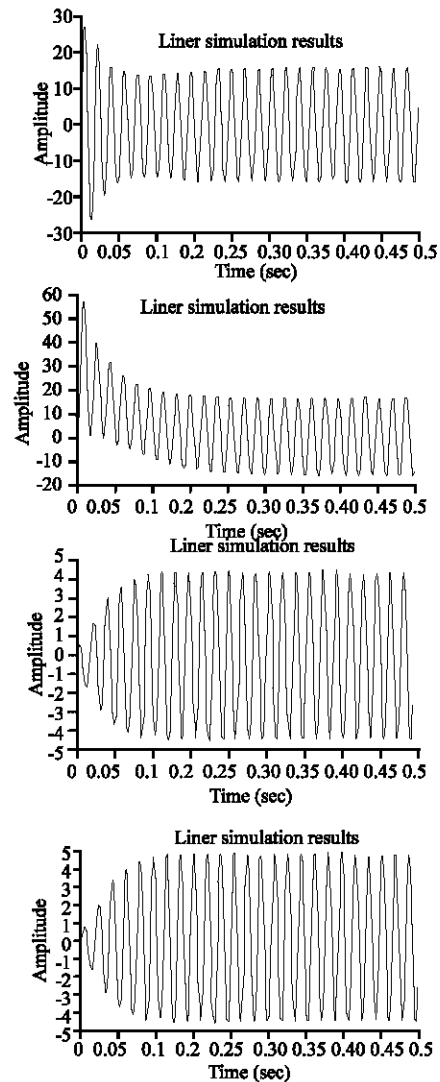


Fig. 3: Sizes estimated

Figure 5 and 6 show, respectively the electromagnetic torque and the number of revolutions.

ORDER BY QUADRATIC LINEAR REGULATOR

Figure 7 illustrate the new control related to the regulator.

The electromagnetic torque and the corresponding number of revolutions are presented by the Fig. 8 and 9.

Comparison of the errors in estimation: The two figures give a comparison between the errors in estimation of the torques and speeds estimated in the case of the order by return of estimated state and control by quadratic linear

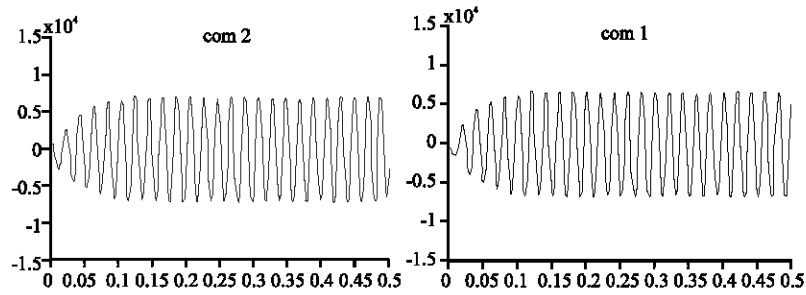


Fig. 4: New order

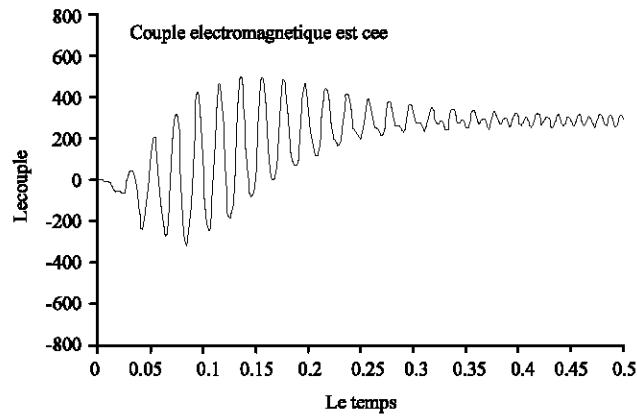


Fig. 5: Electromagnetic torque

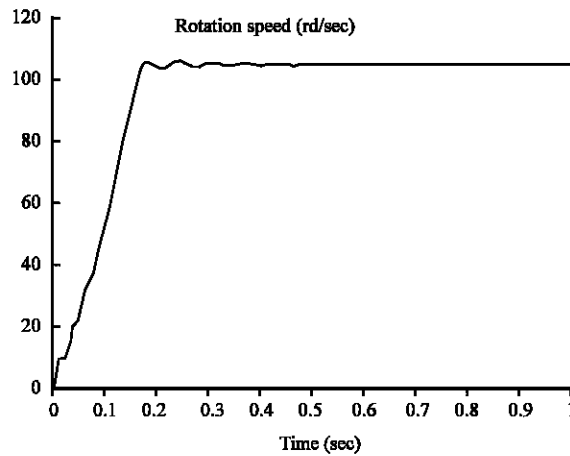


Fig. 6: Speed

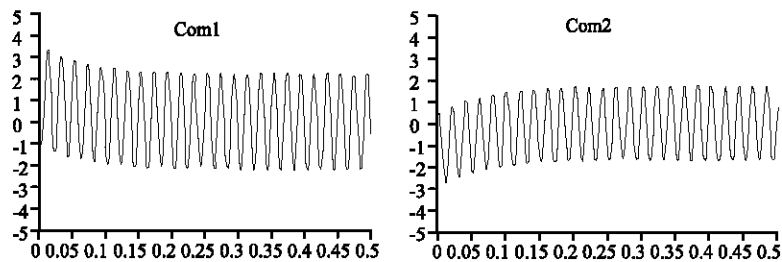


Fig. 7: New control

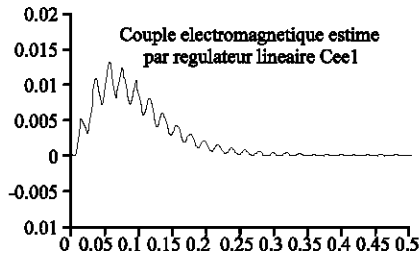


Fig. 8: Estimated electromagnetic torque

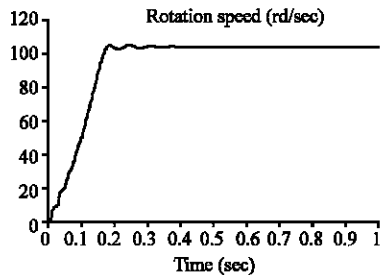


Fig. 9: Estimated speed

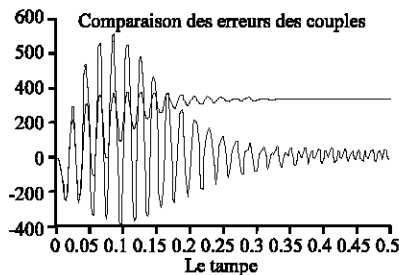


Fig. 10: Comparison of the errors in estimation of the torque

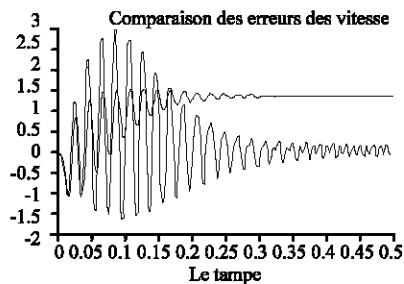


Fig. 11: Comparison of the error in estimation speeds

regulator, the colour black presents the torque and speed in the case of the control by linear regulator the colour blue, the torque and speed in the case of the control by return of estimated state, according to these results one can select the best method of order.

CONCLUSION

The sensorless control is a modern technique of a closed loop control of systems modelled in spaces of state; it aims to decrease the number of sensors to be used in which industrial applications make it possible to increase the reliability and the robustness of the system of drive while decreasing its cost. The basic idea of the control consists in replacing a certain number of sensors by sensors known as software ones, i.e., programs or algorithms. It is obvious that this decreases the number of the failure of the systems.

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