Loss Minimization by Incorporation of UPFC in Load Flow Studies

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Abstract: One of the most comprehensive FACTS devices is Unified Power Flow Controller (UPFC), which has been introduced by Gyugiy in 1991. The UPFC consists of two voltage-source converters, which are connected back-to-back though a DC link. The series voltage converter is connected to the transmission line by means of a series transformer and the shunt voltage converter by means of shunt transformer. The series voltage converter injects an AC voltage into the transmission line with controllable magnitude and phase angle. The shunt converter can exchange active and reactive powers with the system, which enables the system to do shunt compensation independently. To investigate the effect of the UPFC on the steady state condition of the system and load flow, different models have been introduced. These models are usually based on modification of traditional load flow methods. But, in this study, a mathematical model for UPFC which will be referred as UPFC injection model is derived. The UPFC injection model is easily incorporated in Newton-Raphson power flow model to study its effect for power flow control and losses minimization in the power system. A five-bus sample system i.e., Hale network has been taken as a case study and programme is written using MATLAB Software. Finally it is shown that a UPFC has the capability of regulating the power flow and minimizing the power loss simultaneously.

Key words: FACTS, UPFC, injection model, power flow control, losses minimization

INTRODUCTION

The concept of Flexible Alternating Current Transmission Systems (FACTS) was introduced first by Hingorani (1988). Since then, different kinds of FACTS devices have been proposed. One of the most comprehensive FACTS devices is Unified Power Flow Controller (UPFC), which has been introduced by Gyugyi (1991) and Hingorani (1988). The unified power flow controller in its general form can provide simultaneous, real time control of all basic power system parameters (transmission voltage, impedance and phase angle), or any combinations thereof, determining the transmitted power. It has been developed the generalized nodal admittance models are presented for Series Phase Shifters, Compensators, Interphase-Power Controllers and Unified Power Flow Controllers. All models have been included in a Newton-Raphson load flow algorithm, which is capable of solving large power networks. The algorithm retains Newton method's quadratic convergence (Gotham and Heydt, 1998; Dazhong et al., 2000). Three main generic types of FACTS devices are suggested and the integration of those devices into power flow studies, studies relating to wheeling and interchange power flow control are illustrated (Sun et al., 2000). According to typical control objectives of UPFCs, several sets of calculation formulas for the equivalent loads are deduced. Using the proposed

model, a Newton-Raphson algorithm in rectangular form is developed to deal with power flow analysis with UPFCs. The numerical examples of IEEE bus system are provided to demonstrate the effectiveness of proposed method (Keri *et al.*, 1999).

MATLAB modeling tool to determine the impact of UPFC on a two bus system has been used to give an overview of the versatile UPFC capabilities. Models suitable for incorporation in power flow programs are developed and analyzed. The application of UPFC for optimal power flow control is demonstrated through numerical examples. It is shown that a UPFC has the capability of regulating the power flow and minimizing the power losses simultaneously. An algorithm is proposed for determining the optimum size of UPFC for power flow applications (Noroozian et al., 1997). Since FACTS devices especially i.e., UPFC facilitate economy and efficiency in power transmission systems in an environmentally optimal manners performance of UPFC for variation of series voltage coefficient has been presented.

LOAD FLOW ANALYSIS

The load flow analysis in power system parlance is the steady state solution of the power system network. In this analysis, the power system network is modeled as an electric network and solved for the steady state powers, voltages at various buses and hence the power at the slack bus, power flows through inter connecting power channels. Power flow studies form an integral part of the analysis of the power systems as it the basis for many studies like security analysis, stability analysis and power flow analysis. Hence, power flow analysis assumes a very significant and important role for designing a new power system and for planning expansion of the existing one to meet increased load demand. In load flow analysis, single-phase representation is adequate since power systems are usually balanced.

Newton-raphson method: Among the numerous solution methods available for load flow analysis, the Newton-Raphson method is considered to be the most important. Many advantages are attributed to the Newton-Rapson (NR) approach. It's convergence characteristics are relatively powerful compared with other alternative processes and the reliability of N-R approach is comparatively good, since it can solve cases that lead to divergence with other popular processes. Failures do occur on some ill-conditional problems.

The power at bus p is

$$P_p - jQ_p = E_p^* I_p$$

The current at bus p is

$$\begin{split} I_p &= \sum\limits_{q=1}^n \mathrm{YpqEq} \\ P_p & \text{-jQ}_p = & \mathrm{E}_p * \sum\limits_{q=1}^n \mathrm{YpqEq} \end{split}$$

In polar coordinates, The voltage at bus p and q is

$$E_{\scriptscriptstyle p} = \mid E_{\scriptscriptstyle p} \mid e^{j\delta p}$$

$$E_\alpha \equiv \mid E_\alpha \mid e^{-j\delta q}$$

The impedance between p and q is

$$Y_{pq} = |\ Y_{pq}\ |\ e^{-j\theta pq}$$

$$P_p - jQ_p = \sum_{q=1}^{n} |\operatorname{EpYpqEq}| e^{-j(\theta pq + \delta p - \delta q)}$$

$$e^{i(\theta_{pq}+\delta_{p}-\delta_{q})} = \cos(\theta_{pq}+\delta_{p}-\delta_{q}) - i\sin(\theta_{pq}+\delta_{p}-\delta_{q})$$

the real and imaginary components of power are

$$P_p = \sum_{q=1}^{n} |E_p Y_{pq} E_q| \cos(\theta pq + \delta p - \delta q)$$

$$Q_{p} = \sum_{q=1}^{n} |EpYpqEq| \sin (\theta pq + \delta p - \delta q)$$
$$p = 2,3...n.$$

Given an initial set of bus voltages, the real and reactive powers are calculated from above equations and these complex nonlinear simultaneous algebraic equations will be solved iteratively as follows.

Let for the unknown variables be $(x_1, x_2, ----- x_n)$ and the specified quantities $(y_1, y_2, ---y_n)$. These are related by the set of non-linear equations:

$$Y_1 = f_1 (x_1, x_2, -----x_n)$$

 $Y_2 = f_2 (x_1, x_2, ------x_n)$
 $Y_n = f_n (x_1, x_2, ------x_n)$

$$\begin{bmatrix} Y_1 - f_1(x_1^0, ----x_n^0) \\ Y_2 - f_2(x_1^0, -----x_n^0) \\ Y_n - f_n(x_1^0, -----x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} \frac{\partial f_1}{\partial x_2} - - - \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} \frac{\partial f_2}{\partial x_2} - - - \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_n}{\partial x_1} \frac{\partial f_n}{\partial x_2} - - - \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ - \\ - \\ \Delta x_n^0 \end{bmatrix}$$

$$B = J*C$$

Here J is the first derivative matrix known as the jacobian matrix. The solution of the equations requires calculation of left hand vector B that is the difference of the specified quantities and calculated quantities at $(x_1^0, x_2^0, x_2^0, \dots, x_n^0)$. The next better solution obtained as follows:

$$\begin{array}{l} x_{1}^{1} \!\!\!\!= x_{1}^{0} \!\!\!\!+ \! \Delta \, x_{1}^{0} \,, \ \, x_{2}^{1} \!\!\!\!= x_{2}^{0} \!\!\!\!+ \! \Delta x_{2}^{0} \\ x_{n}^{1} \!\!\!\!\!= x_{n}^{0} \!\!\!\!\!+ \! \Delta x_{n}^{0} \,, \end{array}$$

Similarly, load flow set of linearized equations becomes as follows and we can calculate diagnol and off-diagnol elements of J_1 , J_2 , J_3 and J_4 from partial differentation as by Taylor expansion

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 ... J_2 \\ J_2 ... J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$$

We calculate the residual column vector consisting of ΔP , ΔQ . Let P_{sp} and Q_{sp} be the specified quantities at bus i. Assuming a suitable value of the solution (flat voltage profile in our case) the values of P and Q at various buses are calculated. Then,

$$\begin{split} & \Delta p_p = P_{p \text{ (scheduled)}} - P_{p \text{ (calculated)}} \\ & \Delta Q_p = Q_{p \text{ (scheduled)}} - Q_{p \text{ (calculated)}} \end{split} \qquad p = 2,3 \dots n. \end{split}$$

Where superscript zero means the value calculated corresponding to initial guess i.e., zeroth iteration. Having calculated the jacobian matrix and the residual column vector corresponding to the initial guess (initial solution) the desired increment voltage vector

$$\begin{bmatrix} \Delta \mathbf{e} \\ \Delta \mathbf{f} \end{bmatrix}$$

can be calculated by using any standard technique. The next better solution will be

$$e_{p}^{1} = e_{p}^{0} + \Delta e_{p}^{0}$$
 $f_{p}^{1} = f_{p}^{0} + \Delta f_{p}^{0}$

These values of voltages will be used in the next iteration. The process will be repeated and in general the new better estimates for bus voltages will be

$$\begin{split} e_p^{\ k+1} &= e_p^{\ k} + \Delta e_p^{\ k} \\ f_p^{\ k+1} &= f_p^{\ k} + \Delta f_p^{\ k} \end{split}$$

UNIFIED POWER FLOW CONTROLLER (UPFC)

Fast progress of power electronics has made Flexible AC Transmission Systems (FACTS) as a promising concept. Researches on FACTS technologies are being performed very actively. Along with advanced control techniques on FACTS devices, power flow among transmission networks is more and more controllable. Among a variety of FACTS controllers, the Unified Power Flow Controller (UPFC) is a new device in FACTS family, which has been introduced by Gyugiy (1991). It can be used in power systems for several purposes, such as shunt compensation, series compensation, phase shifting, power flow control and voltage control. With the adoption of UPFCs in power systems, the traditional power flow analysis will face new challenges in modeling and solution techniques.

Operating principle of UPFC: The unified power flow controller consists of two switching converters. These converters are operated from a common dc link provided by a dc storage capacitor as shown in Fig. 1.

Converter 2 provides the main function of the UPFC by injecting an ac voltage with controllable magnitude and phase angle in series with the transmission line via a series transformer. The basic function of converter 1 is to supply or absorb the real power demand by converter 2 at the common dc link. It can also generate or absorb

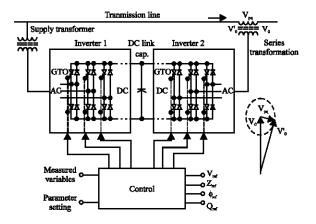


Fig. 1: Basic circuit arrangement of UPFC

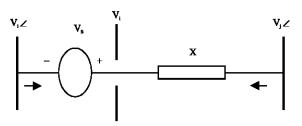


Fig. 2: Representation of a series connected VSC

controllable reactive power and provide independent shunt reactive compensation for the line. Converter 2 supplies or absorbs locally the required reactive power and exchanges the active power as a result of the series injection voltage.

UPFC injection model for power flow studies: In this study, a model for UPFC, which will be referred as UPFC injection model is derived. This model is helpful in understanding the impact of the UPFC on the power system in the steady state. Furthermore, the UPFC injection model can easily be incorporated in the steady state power flow model. Since the series voltage source converter does the main function of the UPFC, first derive the modeling of a series voltage source converter.

Series connected voltage source converter model: Suppose a series connected voltage source is located

between nodes i and j in a power system. The series voltage source converter can be modeled with an ideal series voltage V_s in series with a reactance X_s as shown in Fig. 2.

$$V_i = V_s + V_i$$

V_i = Fictitious voltage behind the series reactance.

V_s = Series source voltage.

 V_i = Voltage at ith node.

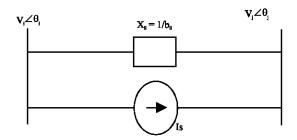


Fig. 3: Equivalent Norton's circuit of a series connected VSC

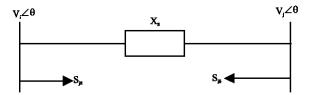


Fig. 4: Injection model for a series connected VSC

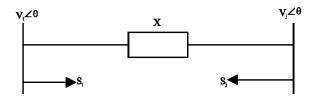


Fig. 5: Complete UPFC model

The series voltage source V_s is controllable in magnitude and phase, i.e.,

$$V_s = rV_i e^{j\gamma}$$

Where

 $r = Series voltage source co-efficient. (0 < r < r_{max})$

 γ = Series voltage source angle. $(0 < \gamma < 2\Pi)$

The injection model is obtained by replacing the equivalent circuit of series connected voltage source as Norton's equivalent circuit as shown in Fig. 3

The current source $I_s = -jb_sV_s$

Where $b_s = 1/X_s$

The power injected into the ith bus

$$\begin{split} & \overline{S}_{is} = \overline{V}_i (-\overline{I}_s)^* \\ & S_{is} = V_i [j b_s r \overline{V}_i e^{j \gamma}]^* \\ & S_{is} = -b_s r V_i^2 \sin(\gamma) - j b_s r v_i^2 \cos(\gamma) \end{split}$$

The power injected into the jth bus

$$\begin{split} & \overline{S}_{js} = \overline{V}_{j} (-\overline{I}_{s})^{*} \\ & S_{js} = V_{j} [-jb_{s}r\overline{V}_{i}e^{j\gamma}]^{*} \\ & S_{is} = b_{s}rV_{i}V_{i}sin(\theta_{ii} + \gamma) + jb_{s}rV_{i}V_{i}cos(\theta_{ii} + \gamma) \end{split}$$

Where
$$\theta_{ii} = \theta_i - \theta_i$$

From above equations, the injection model of series connected voltage source can be seen as two dependent loads as shown in Fig. 4.

Shunt connected voltage source converter model: In UPFC, the shunt connected voltage source (converter 1) is used mainly to provide the active power, which is injected to the network via the series connected voltage source. When the losses are neglected

$$P_{conv1} = P_{conv2}$$

The apparent power supplied by the series voltage source converter is

$$S_{conv2} = \overline{V}_s \overline{I}^*_{ij} = re^{j\gamma} \overline{V}_i \left[\frac{\overline{V}_i - \overline{V}_j}{jX_s} \right]^*$$

After simplication, the active and reactive power supplied by converter 2 is

$$\begin{split} &P_{conv2} = rb_sV_iV_jsin(\theta_i - \theta_j + \gamma) - rb_sV_i^2sin(\gamma) \\ &Q_{conv2} = -rb_sV_iV_jcos(\theta_i - \theta_j + \gamma) + rb_sV_i^2cos(\gamma) + r^2b_sV_i^2 \end{split}$$

The reactive power delivered or absorbed by converter 1 is independently controllable by UPFC and can be modeled as a separate controllable shunt reactive source. In view of above, it is assumed that $Q_{CONVI} = 0$.

The UPFC injection model is constructed from the series connected voltage source model with the addition of a power equivalent to P_{CONVI} +j0to node i. Thus, the complete UPFC injection model is shown in Fig. 5

$$\begin{split} &Q_{sj} = -rb_sViV_j\cos(\theta_{ij} + \gamma) &P_{si} = rb_sV_iV_j\sin(\theta_{ij} + \gamma) \\ &Q_{si} = rb_sV_i^2\cos(\gamma) + Q_{shunt} &P_{si} = -rb_sV_iV_i\sin(\theta_{ii} + \gamma) \end{split}$$

Modification of Jacobian matrix: The UPFC injection model can easily be incorporated in a load flow program. If a UPFC is located between node i and node j in a power system, the Jacobian matrix is modified by addition of appropriate injection powers.

The linearized load flow model is

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H....N \\ J....L \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V/V \end{bmatrix}$$

The Jacobian matrix is modified are as given below

and the superscript '0' denotes the Jacobian elements without UPFC.

$$\begin{split} J_{(i,i)} &= J^0_{(i,i)} & H_{(i,i)} = H^0_{(i,i)} - Q_{sj} \\ J_{(i,j)} &= J^0_{(i,j)} & H_{(i,j)} = H^0_{(i,j)} + Q_{sj} \\ J_{(j,i)} &= J^0_{(j,i)} - P_{sj} & H_{(i,i)} = H^0_{(j,i)} + Q_{sj} \\ J_{(j,j)} &= J^0_{(j,j)} + P_{sj} & H_{(j,j)} = H^0_{(j,j)} - Q_{sj} \\ N_{(i,i)} &= N^0_{(i,i)} - P_{sj} & L_{(i,i)} = L^0_{(i,i)} + 2Q_{si} \\ N_{(i,j)} &= N^0_{(i,j)} - P_{sj} & L_{(i,j)} = L^0_{(i,j)} \\ N_{(j,i)} &= N^0_{(j,i)} + P_{sj} & L_{(j,i)} = L^0_{(j,i)} + Q_{sj} \\ N_{(j,j)} &= N^0_{(j,j)} + P_{sj} & L_{(j,j)} = L^0_{(j,j)} + Q_{sj} \end{split}$$

Hale network: The single line diagram of a 5-bus system is shown in Fig. 6. The magnitude of voltage at bus 1 is adjusted to 1.06 per unit.

Table 1: Power flows and line losses for 5-bus system when iteration count = 0

Line	Power flows	Line losses
1-2	0.886-0.2174i	0.014573-0.023381i
1-3	0.41675-0.023331i	0.012368-0.0176i
2-3	0.2622+0.031744i	0.0038629-0.031964i
2-4	0.2972+0.024078i	0.004877-0.028905i
2-5	0.58371+0.056874i	0.012438+0.0048121i
3-4	0.20341-0.063883i	0.00041532-0.020036i
4-5	0.067424-0.027752i	0.000342-0.05191i

Total line losses: 4.8877-16.8985i

Table 2: Power flows and line losses for 5-bus system when iteration count = 8

Line	Power flows	Line losses
1-2	0.88864-0.085795i	0.014105-0.024308i
1-3	0.40723+0.011584i	0.01192-0.018555i
2-3	0.24694+0.035464i	0.0035152-0.032376i
2-4	0.27936+0.02962i	0.0044134-0.029656i
2-5	0.54823+0.07343i	0.011252+0.0017577i
3-4	0.18874-0.052022i	0.00035605-0.019898i
4-5	0.06333-0.022848i	0.00030711-0.051176i

Total line losses: 4.5868-17.4211i

Power flow control losses minimization without UPFC results are shown in Table 1 and 2.

Power flow control losses minimization with UPFC results: UPFC is incorporated between line 4 and 5 (Table 3-7).

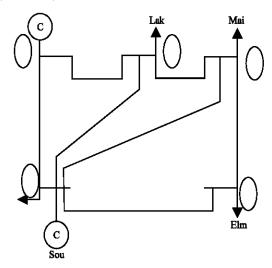


Fig. 6: Sample system (with out UPFC)

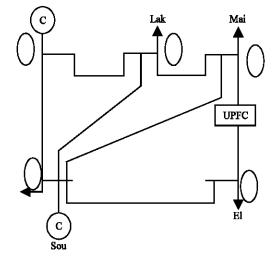


Fig. 7: Sample system (with UPFC)

Table 3: Power flows,	line losses and total	losses when $y = 0^{\circ}$
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r	Power flow	Line losses	Total losses
0	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.03	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.06	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.09	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.3	0.063327-0.022868i	0.00030707-0.051176i	4.5868-17.4213i
0.3125	0.063313-0.022974i	0.00030687-0.051179i	4.5866-17.4224i
0.325	0.063227-0.023599i	0.00030574-0.051194i	4.5857-17.4288i
0.335	0.062908-0.025916i	0.00030207-0.051251i	4.5822-17.4522i
0.338	0.062689-0.027501i	0.00030003-0.051288i	4.5799-17.4679i
0.34	0.062484-0.028979i	0.00029846-0.05132i	4.5779-17.4822i
0.35	0.060055-0.046204i	0.00030477-0.05164i	4.5609-17.6299i
0.4	0.011424-0.37299i	0.0086981-0.033139i	6.9184-12.4486i

Table 4: Power flows, line losses and total losses when $y = 90^{\circ}$

r	Power flow	Line losses	Total losses
0	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.03	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.06	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.09	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.3	0.06333-0.022848i	0.0003071-0.051176i	4.5863-17.4232i
0.34	0.063267-0.02288i	0.00030648-0.051179i	4.5868-17.4214i
0.35	0.063097-0.022965i	0.00030479-0.051186i	4.5868-17.4223i
0.4	-0.066207-0.08657i	0.00060024-0.052379i	4.9125-17.0457i

Table 5: Power flows, line losses and total losses when $\gamma = 180^{\circ}$

r	Power flow	Line losses	Total losses
0	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.03	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.06	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.09	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.3	0.06333-0.022848i	0.0003071-0.051176i	4.5868-17.4211i
0.34	0.063271-0.022846i	0.00030654-0.051177i	4.5867-17.4214i
0.35	0.063141-0.022873i	0.00030528-0.051181i	4.5864-17.4220i
0.4	0.08345-0.096936i	0.0009518-0.048467i	4.7903-16.4208i

Table 6: Power flows, line losses and total losses when $y = 270^{\circ}$

r	Power flow	Line losses	Total losses
0	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.03	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.06	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.09	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.3	0.063397-0.022886i	0.00030773-0.051175i	4.5863-17.4232i
0.34	0.084096-0.034478i	0.00054092-0.050929i	4.4299-18.0524i
0.35	0.14171-0.065713i	0.0016022-0.049013i	4.1605-19.3086i
0.4	0.79194-0.36693i	0.045811+0.071126i	14.5009+7.3509i

Table 7: Power flows, line losses and total losses when $v = 360^{\circ}$

r	Power flow	Line losses	Total losses
0	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.03	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.06	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.09	0.06333-0.022848i	0.00030711-0.051176i	4.5868-17.4211i
0.3	0.063327-0.022868i	0.00030707-0.051176i	4.5868-17.4213i
0.34	0.062484-0.028979i	0.00029846-0.051321i	4.5779-17.4822i
0.35	0.060055-0.046204i	0.00030477-0.05164i	4.5609-17.6299i
0.4	0.011424-0.37299i	0.0086981-0.033139i	6.9184-12.4486i

CONCLUSION

A steady state mathematical model for the UPFC was proposed. The proposed model can easily be incorporated in existing power flow programs. The capability of UPFC in optimal power flow applications was demonstrated through numerical example i.e. Hale Network. It was shown that by controlling the magnitude and angle of the series voltage source in a UPFC to satisfy the following objectives simultaneously:

- Regulating power flow through a transmission line.
- Minimisation of power losses without generation rescheduling.

The results indicate that the proposed UPFC injection model can perform power flow analysis accurately and efficiently. It is observed that power flow control and losses minimization are appreciable for a typical values of $\gamma = 270^{\circ}$ and r = 0.34. Also, there is a rapid change in power flow control and losses minimization for a typical values of $\gamma = 270^{\circ}$ and series voltage source coefficient r = 0.4, which indicates there is a certain limitation for these values.

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