International Journal of Electrical and Power Engineering 1 (3): 314-320, 2007

Vibratory Phenomena of Magnetic Origin in Electrical Machines: Study and Modeling Using FEM Techniques

Nassira Ferkha, M. Rachid Mekideche and Tarik Hacib
Laboratoire LAMEL, Université de Jijel-BP 98, Ouled Aissa-18000 Jijel, Algérie

Abstract: This study presents a theoretical study of the coupled magneto-elastic phenomena, with a view to modeling the vibratory behavior of squirrel cage asynchronous machine. A coupled model is developed to calculate the mechanical deformations of this machine in magnetic field. The two equations governing magnetic and mechanical phenomena are solved using finite element method. The magnetic force distribution is calculated through a local application of virtual work principle. A spectral analysis of this force made it possible to determine the dynamic response of the stator for each harmonic force. This response has been calculated only after a modal analysis which could determine the resonance frequencies and its proper associated modes.

Key words: Electrical machines, finite element, magnetic force, modal analysis, vibrations

INTRODUCTION

The electrical machine behaviour in generating vibrations and noise is determined by the electromagnetic field in the airgap and the mechanical structure of the machine. The link between the magnetic and the mechanical analysis is the electromagnetic force exerted by the magnetic field on stator and rotor. To predict stator deformations caused by magnetic field distributions occurring during operation, a local force formulation is needed. The magnetic field as well as the force distributions are considered not influenced by the deformation.

MAGNETO-MECHANICAL SYSTEM

Magnetic field calculation: In this study, the magnetic field is calculated using the finite element method, together with the time stepping scheme. According to Maxwell's equation, the governing equation is expressed in terms of magnetic vector potential A. The variational formulation is obtained by solving the following equation:

$$\text{rot}\left(\frac{1}{\mu}\,\text{rot A}\right) = J \qquad (1)$$

Where μ is the magnetic permeability, J is the current density. Solving Eq. 1 by finite element discretization leads to the following algebraic system:

$$[M]\,[A] = [J] \qquad (2)$$

Where [M] represents the magnetic stiffness matrix and [J] the source vector.

Magnetic force calculation: The calculation of the local force distribution in the electrical machine is a most important step for analysing its mechanical behaviour.

The magnetic force computation method is based on a local application of virtual work principle (Ferkha, 2004; Belahcen, 2000; Benhama *et al*., 2000). This nodal force has been calculated on each node, using the following equation:

$$F = -\int_0^A [A]^T \frac{\partial [M]}{\partial s}\, dA \qquad (3)$$

For a given element, the magnetic stiffness matrix is

$$[M_{ij}^e] = \frac{\nu}{4\Delta}[b_i b_j + c_i c_j] \qquad (4)$$

Derivation and integration are made at the element level. The derivatives of the stiffness matrix with respect to the x and y coordinates of the nodes of the element are

$$\frac{\partial [M^e]}{\partial x_1} = \frac{\nu}{4\Delta}\begin{bmatrix} 0 & c_1 & -c_1 \\ c_1 & 2c_2 & c_3 - c_2 \\ -c_1 & c_3 - c_2 & -2c_2 \end{bmatrix} - \frac{b_1}{2\Delta}[M^e] + \frac{[M^e]}{\nu}\frac{\partial \nu}{\partial x_1} \qquad (5)$$

$$\frac{\partial [M^e]}{\partial x_2} = \frac{\nu}{4\Delta}\begin{bmatrix} -2c_1 & -c_2 & c_1 - c_3 \\ -c_2 & 0 & c_2 \\ c_1 - c_3 & c_2 & 2c_3 \end{bmatrix} - \frac{b_2}{2\Delta}[M^e] + \frac{[M^e]}{\nu}\frac{\partial \nu}{\partial x_2} \qquad (6)$$

Corresponding Author: Nassira Ferkha, Laboratoire LAMEL, Université de Jijel -BP 98, Ouled Aissa-18000 Jijel, Algérie

$$\frac{\partial[M^e]}{\partial x_3}=\frac{\nu}{4\Delta}\begin{bmatrix} 2c_1 & c_2-c_1 & c_3 \\ c_2-c_1 & -2c_2 & -c_3 \\ c_3 & -c_3 & 0 \end{bmatrix}-\frac{b_3}{2\Delta}[M^e]+\frac{[M^e]}{\nu}\frac{\partial \nu}{\partial x_3} \quad (7)$$

$$\frac{\partial[M^e]}{\partial y_1}=\frac{\nu}{4\Delta}\begin{bmatrix} 0 & -b_1 & b_1 \\ -b_1 & -2b_2 & -b_3+b_2 \\ b_1 & -b_3+b_2 & 2b_3 \end{bmatrix}-\frac{c_1}{2\Delta}[M^e]+\frac{[M^e]}{\nu}\frac{\partial \nu}{\partial y_1} \quad (8)$$

$$\frac{\partial[M^e]}{\partial y_2}=\frac{\nu}{4\Delta}\begin{bmatrix} 2b_1 & b_2 & b_3-b_1 \\ b_2 & 0 & -b_2 \\ b_3-b_1 & -b_2 & -2b_3 \end{bmatrix}-\frac{c_2}{2\Delta}[M^e]+\frac{[M^e]}{\nu}\frac{\partial \nu}{\partial y_2} \quad (9)$$

$$\frac{\partial[M^e]}{\partial y_3}=\frac{\nu}{4\Delta}\begin{bmatrix} -2b_1 & b_1-b_2 & -b_3 \\ b_1-b_2 & 2b_2 & b_3 \\ -b_3 & b_3 & 0 \end{bmatrix}-\frac{c_3}{2\Delta}[M^e]+\frac{[M^e]}{\nu}\frac{\partial \nu}{\partial y_3} \quad (10)$$

Where

$$b_i = y_j - y_k \quad (11)$$

$$c_i = x_k - x_j \quad (12)$$

Here x_i, y_i i = 1,2,3 are the coordinates of the nodes i of the element and the index i,j and k are circular indices 1, 2, 3, 1, 2, 3 etc. Δ is the element area.

The integral in Eq. 3 is going achieve using the coordinate transformation $A = A_0$ t and $A = A_0$ dt.

From the point of view of vibration analysis, it is important to know the spatial distribution and the time dependence of the force. For this purpose, the force is developed into a two dimensional Fourier series. In our case, we have study the effect of the spatial harmonic on the vibratory behaviour of the stator.

Vibration calculation: The vibration computation is based on the use of the mode summation method where the dynamic Eq. 13 is solved in the space of eigen vectors (Belahcen, 2000; Ren *et al.*, 1995). This approach, associated to the use of the FEM, assumes the knowledge of the force distribution and it can advantageously determine the response at different points of the structure mesh.

$$[M]\,[\ddot{d}]+[C]\,[\dot{d}]+[K]\,[d]=[F] \quad (13)$$

[M], [C] and [K] are, respectively mass, damper and stiffness matrix of the structure, [d] is displacement vector.

Determination of resonant frequencies and modes: In the case of the studied machine, the stator is considered to be a free structure and the damper is considered to be negligible (Ferkha, 2004; Ren *et al.*, 1995) therefore the equation of movement becomes:

$$[M]\,[\ddot{d}]+[K]\,[d]=[F] \quad (14)$$

To determine the vector of the displacements [d] to the nœuds of the stator, it is necessary to determine its matrix of mass and its matrix of rigidity then, its resonance frequencies and their proper associated modes.

$$[M]=\sum_e [B]_e^T\,[M]_e\,[B]_e \quad (15)$$

$$[K]=\sum_e [B]_e^T\,[K]_e\,[B]_e \quad (16)$$

$$[F]=\sum_e [B]_e^T\,[F]_e \quad (17)$$

$$[M]_e=\int_{V_e} \rho\,[N]^T[N]\;dV_e \quad (18)$$

$$[K]_e=\int_{V_e} [N]^T[D]^T[H]\,[D]\,[N]\,dV_e \quad (19)$$

$[M]_e$ and $[K]_e$ are, respectively mass and stiffness matrix of an element, $[F]_e$ is magnetic forces vector applied on an element, $[B]_e$ is the matrix of localization of every element [N] are the shape functions and ρ is the mass density.

$$[H]=\begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

λ and μ are the Lame coefficients

$$[D] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix}$$

In this study the calculation of the resonance frequencies and their proper associated modes is going to be done from the Eq. 20. This frequency equation is derived as a function of parameters which describe the stator's geometry, material composition and elastic properties.

$$([K] - \omega_i^2 [M])[X_i] = 0 \qquad (20)$$

Where $f_i = \omega_i / 2\pi$ are the natural (resonance) frequencies and $[X_i]$ is the vector of proper modes.

Determination of the vibratory response: The vibratory response of the structure (stator), for each harmonic force, is equal to a linear combination of associated mode shapes as follows (Ferkha, 2004).

$$[d] = \sum_{i=1}^{NT} \eta_i X_i \qquad (21)$$

Where η_i are the modal coordinates associated to each mode.

Since we remain in the linear domain during the answer, only changes the amplitude of the mode during the time. We can write therefore:

$$[\dot{d}(t)] = \sum_{i=1}^{NT} \dot{\eta}_i(t) X_i \qquad (22)$$

$$[\ddot{d}(t)] = \sum_{i=1}^{NT} \ddot{\eta}_i(t) X_i \qquad (23)$$

As using this decomposition the general equation to solve becomes

$$\sum_{i=1}^{NT} \left[\ddot{\eta}_i(t)[M]X_i + \eta_i(t)[K]X_i \right] = [F] \qquad (24)$$

In premultipliant the equation above by X_j^T and while using the relations of orthogonally between modes (Ferkha, 2004) we gets a set of equations so-called normal equations. The system of the modal equations of the movement amounts to NT uncoupled differential equations, either:

$$[m]\eta + [k]\ddot{\eta} = \varphi^T [F] = f(t) \qquad (25)$$

$$\varphi = [X_1\ X_2\ X_3 \ldots X_{NT}] \qquad (26)$$

$$\eta = \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \\ \eta_3(t) \\ \ldots \\ \eta_{NT}(t) \end{bmatrix} \qquad (27)$$

Where

φ : Modal matrix of which its components are the proper mode of the system,
η : Vector of the modal coordinates,
$[m]$: Diagonal matrix (NT´NT) of the generalized masses,
$[k]$: Diagonal matrix (NT´NT) of generalized rigidities,
$f(t)$: Vector of the modal forces,
NT : No. of degrees of freedom (ddl) of the structure.

We solved the Eq. 25 in the harmonic regime. The considered harmonic and the dynamic response on the space of the eigenmode, are expressed, respectively as following:

$$F(t) = F_0 e^{j\omega t} \qquad (28)$$

$$\eta_j(t) = y_j e^{j\omega t} \qquad (29)$$

Where F_0 and y_j are, respectively, the amplitudes of the considered harmonic and the considered mode.

In this case, the equation of movement for every mode can be written as:

$$-\omega^2 y_j + \omega_j^2 y_j = \frac{f_j}{m_j} \qquad (30)$$

Where ω and ω_j are the pulsations of the considered harmonic and the considered mode, respectively. Finally, we lead to the following expression:

$$y_j = \frac{f_j}{m_j\left(\omega_j^2 - \omega^2\right)} \qquad (31)$$

The answer of the structure, in term of displacements, in each node is then:

$$d(t) = \varphi\, y\, e^{j\omega t} = \sum_{j=1}^{NT} y_j X_j e^{j\omega t} \qquad (32)$$

RESULTS AND DISUSSION

The elaborated model has been applied for the modeling of the vibratory behavior of the squirrel cage asynchronous machine presented in the Fig. 1 (Saitz, 2001). The main parameters of the studied motors are summarized in Table 1.

The present model is applied to a 2D magnetic field and an elastic problem (Tang *et al*., 2006). In this study, the deformations are small and they can not affect the machines, only one pole of the motor can be modelled magnetic field distribution. According to the symmetry (periodicity or anti-periodicity) presented in the electrical with adequate boundary conditions. Figure 2 shows the meshes used for magnetic and elastic field problems.

In Fig. 3-5 we have presented respectively the evolutions of the magnetic vector potential and the magnetic force, according to the time, in a point of the stator. We remark that the frequency of the magnetic force is the double of the frequency of the vector potential.

The Fig. 6, shows the spatio-temporal evolution of the magnetic force acting on the stator. While considering the spatial distribution of this force, we can remark that it is not uniformly distributed. In another manner, the variation of this force according to the mechanical angle

Table 1: Main parameters of the studied motors

No. of phases	3
No. of pairs of poles	2
Outer diameter of the stator [mm]	310
Air gap diameter [mm]	199
Core length [mm]	249
No. of stator slots	36
No. of rotor slots	28
Connection	Star
Rated current [A]	50
Electric frequency [Hz]	50

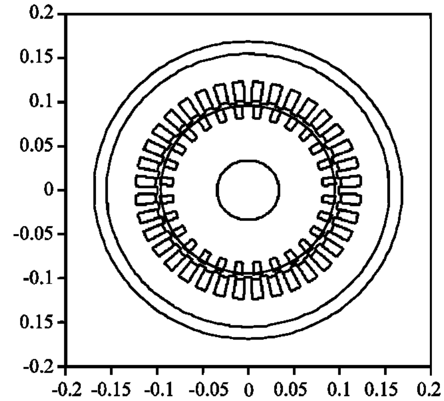

Fig. 1: Structure of the studied machine

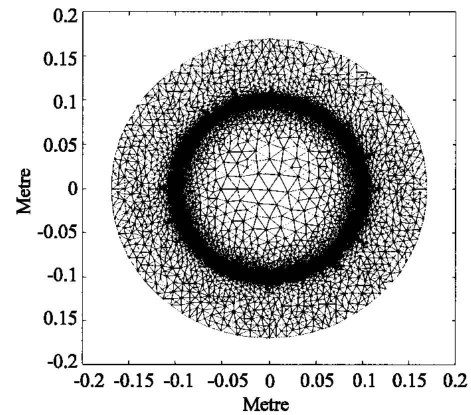

Fig. 2: Mesh of magnetic problem

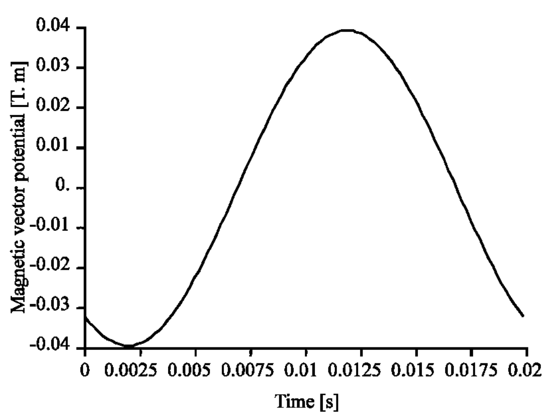

Fig. 3: Evolutions of the magnetic vector potential according to the time

is not uniform, because of the presence of the slots. It puts in evidence that, to study the vibratory behavior of the machine, it is necessary to do a spectral analysis of

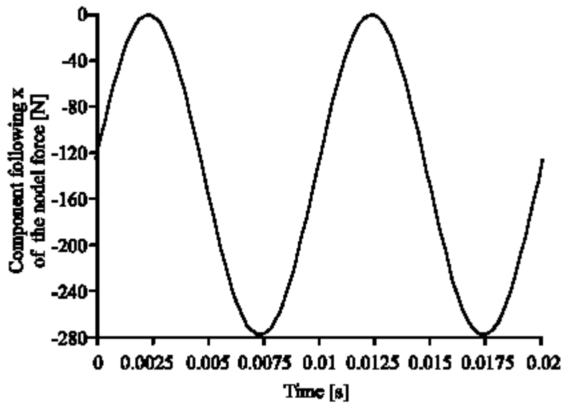

Fig. 4: Evolutions of the component following x of the magnetic force according to the time

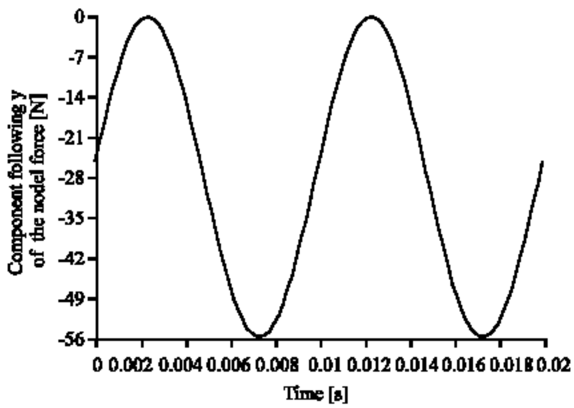

Fig. 5: Evolutions of the component following y of the magnetic force according to the time

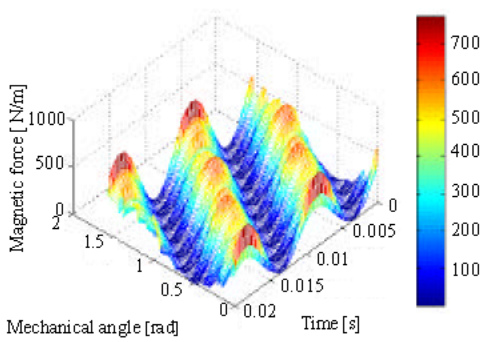

Fig. 6: Spatio-temporal evolution of the magnetic force

the magnetic force that is rich in harmonic. Figure 7 and 8 shows the spectral decompositions of this force.

The natural frequencies corresponding to the six first modes of the stator that are gotten from the mechanical

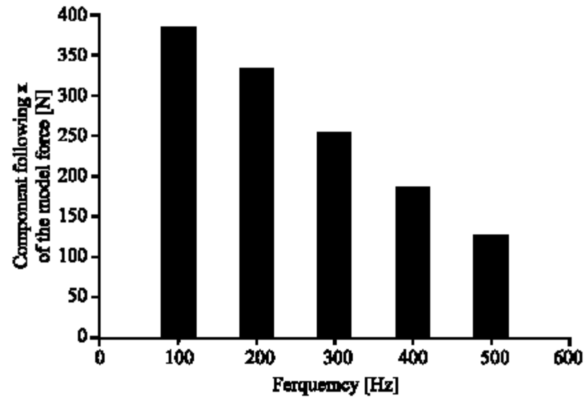

Fig. 7: Spectra of the component following x of the magnetic force

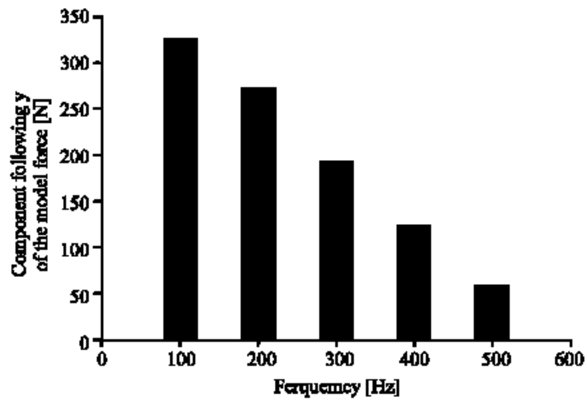

Fig. 8: Spectra of the component following y of the magnetic force

Table 2: Natural frequencies of the stator

Propre modes	Natural frequencies [Hz]
1	163870
2	177370
3	219920
4	307230
5	310410
6	437440

calculation code, are recapitulated in Table 2. These frequencies have very high values and very far from those corresponding to the first harmonics. What diminishes the risk of a resonance. It is for this reason that, we always search to make increase the natural frequencies of the electrical machines.

The dynamic answers (displacement) of the stator for the three first harmonics, gotten from the mechanical calculation code, are presented in Fig. 9-11.

From these result we can note that, the gotten displacements, for the different harmonic, are the same order of highness. Besides, we remark that below the resonance, the displacement is proportional to

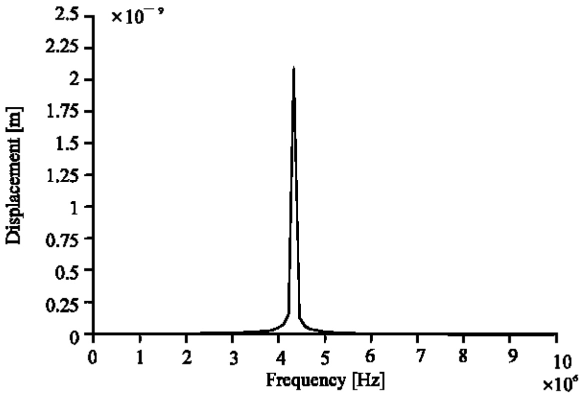

Fig. 9: Dynamic answer of the stator for the first harmonic, gotten from the mechanical calculation code

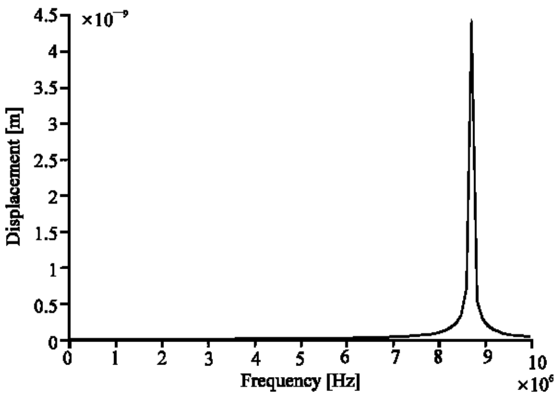

Fig. 10: Dynamic answer of the stator for the second harmonic, gotten from the mechanical calculation code

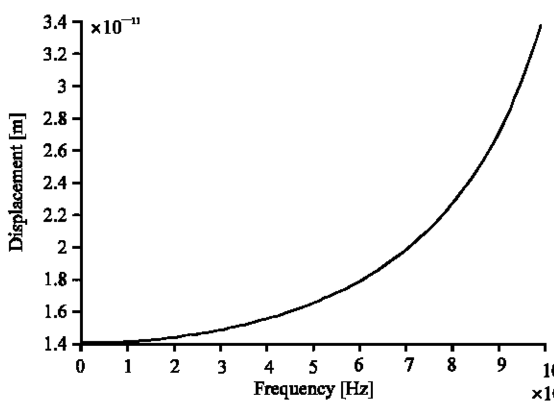

Fig. 11: Dynamic answer of the stator for the third harmonic, gotten from the mechanical calculation code

the frequency, while it is inversely proportional in over of this resonance. It can be justified, by the influence of the frequency on the dynamic resistance of the stator, that is given by this matrix: $[KD] = [K] - \omega^2 [M]$.

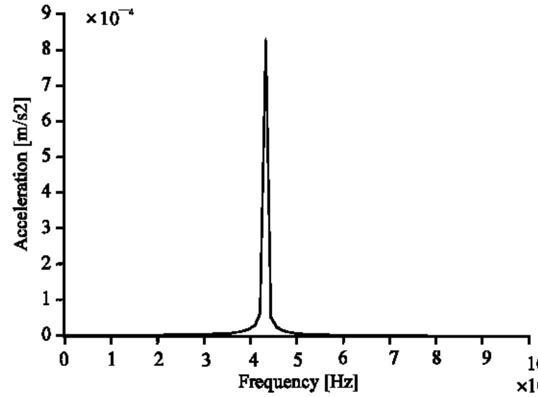

Fig. 12: Acceleration in a point of the stator for the first harmonic, gotten from the mechanical calculation code

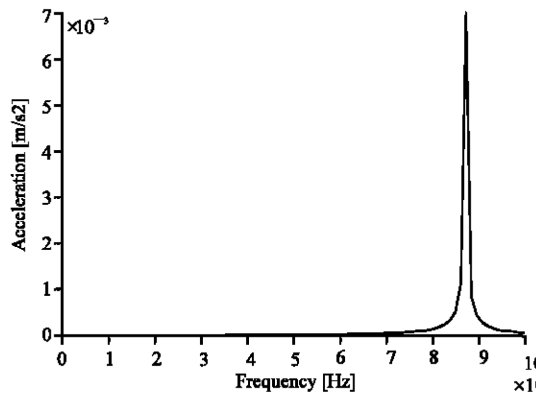

Fig. 13: Acceleration in a point of the stator for the second harmonic, gotten from the mechanical calculation code

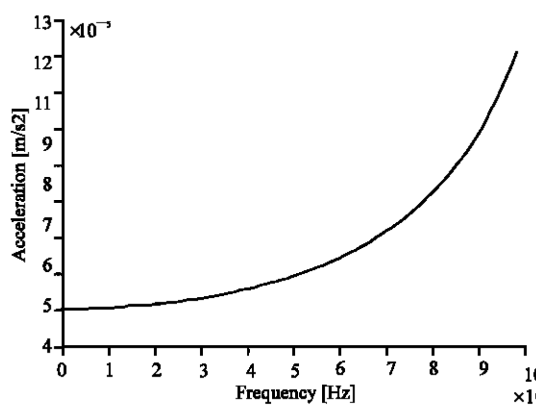

Fig. 14: Acceleration in a point of the stator for the third harmonic, gotten from the mechanical calculation code

The accelerations are gotten from the seconds derivatives of the displacements as following:

$$[\ddot{d}] = -\omega^2 [d]$$

Figure 12-14 shows the accelerations gotten in the same precedent point of the stator.

CONCLUSION

In this research, we introduced in a calculation code of the magnetic field, based in a 2D finite element method, a modulus which allows the determination of the local magnetic force distribution, to calculate the resulting mechanical deformations. This response has been calculated only after a modal analysis which could determine the resonance frequencies and its proper associated modes.

REFERENCES

Belahcen, A., 2000. Methods of calculating the magnetic forces for vibration and noise analysis in electrical machines, ACTA Polytechnica Scandinavia, Electrical engineering series, n°103, Finish Academies of Technology. Espoo, Finland.

Benhama, A., A.C. Williamson and A.B.J. Reece, 2000. Virtual work approch to the computation of magnetic force distribution from finite element field solutions, IEE E. Proc. Elect. Power Applied, 31: 437-442.

Ferkha, N., 2004. Modelisation des phénomènes vibratoires d'origine magnétique dans une machine électrique, Thèse de Magister, Université de Jijel.

Ren, Z., B. Ionexu, M. Besbes and A. Razek, 1995. Calculation of mechanical deformation of magnetic materials in electromagnetic devices, IEEE Trans. Magnetics, 31: 1873-1876.

Saitz, J., 2001. Magnetic field analysis of induction motors, combining Preisach hysteresis modeling and finite element techniques», IEEE. Trans. Magnetics, 37: 3693-3697.

Tang, Z., P. Pillay, A.M. Omekanda, C. Li and C. Cetinkaya, 2004. Young's modulus for laminated machine structures with particular reference to switched reluctance motor vibrations», IEEE Transactions on Industry, Vol. 40.