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Parameter Identification of a DC Motor Using Moments Method

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Abstract: Time moments have been introduced in automatic control because of the analogy between the impulse response of a linear system and a probability function. This basic idea has generated applications in identification, model order reduction and controller design. In this study, a newly developed identification algorithm, called moments method, is introduced and applied to the parameter identification of a dc motor. The simulation and experimental results are presented and compared.

Key words: Identification, moments method, DC motor

INTRODUCTION

DC motors are widely used as actuating elements in many industrial applications for their advantages of easy speed and position control and wide adjustable range. Consequently, examination of dc motor behavior constitutes a useful effort for analysis and control of many practical applications (Rubaai and Kotaru, 2000; Basilio and Moreira, 2004).

Mathematical modeling is one of the most important and often the most difficult step towards understanding a physical system. Although models of some system can be constructed from the physical laws, most often systems are too complex to be modeled this way. A model is usually constructed from a set of input-output data which is obtained experimentally and is represented in the form of a table or a graph (Basilio and Moreira, 2004; Touhami *et al*., 1994). So for such systems it is required to build an accurate model as possible to get a suitable dynamic performance from the speed control system (Weerasooriya and Sharkawi, 1991). In the case of modeling dc motors, the physical information are nearly sufficient for building the model structure. The second part of the modeling is to determine the parameters of the model. Parameter estimation of dynamical system has attractive applications in the control field (Rubaai and Kotaru, 2000; Basilio and Moreira, 2004; Touhami *et al*., 1994; Weerasooriya and Sharkawi, 1991). Identification of dynamical system consists of two main parts. Determination of model structure and estimation of the model parameters. The structure of model can be determined using a priori physical information. In many cases there is no sufficient information for building the model structure or it is too complex to deal with. In such cases only a global approximation of the dynamic system can be obtained. However in other cases some physical information is available which can lead to some structured information.

System identification of dc motors is a topic of great practical importance for almost every servo control design a mathematical model is needed. In the literature there are many classical methods to identify dc motors parameters (Rubaai and Kotaru, 2000; Basilio and Moreira, 2004; Weerasooriya and Sharkawi, 1991; Louis *et al*., 1998). Time moments have been introduced in automatic control because of the analogy between the impulse response of a linear system and a probability function (Coirault *et al*., 1995; Etien *et al*., 2000). Thus, an impulse response is characterized by an infinity of moments, practically, only the first ones are necessary as for a probability density function. This basic idea has generated applications in identification, model order reduction and controller design, known as the method of moments.

DC MOTOR MODEL

The dynamic of the separately excited dc motor may be expressed by the following equations

$$K\omega(t) = -R_a i_a(t) - L_a \frac{di_a(t)}{dt} + U_a(t) \qquad (1)$$

$$Ki_a(t) = J\frac{d\omega(t)}{dt} + f\omega(t) + T_L(t) \qquad (2)$$

where K, R_a, L_a, J and f are respectively, the torque and back-EMF constant, the armature resistance, the

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armature inductance, the rotor mass moment of inertia and the viscous friction coefficient. • (t), $i_a(t)$, $U_a(t)$ and $T_L(t)$, respectively denote the rotor angular speed, the armature current, the terminal voltage and the load torque.

METHOD OF MOMENTS

The moments constitute the basis for a non classical representation of linear systems. The characterization of an impulse response by its moments is equivalent to the moment characterization of a probability density function (Etien *et al*., 2000). Impulse response moments are system invariants. Like for a probability density function, it is not necessary to compute an infinity of moments to characterize with a good approximation the shape of the impulse response only the first ones are necessary to perform this characterization.

Temporal moment of a function: Let us consider a stable linear system, characterized by its impulse h(t) then,

$$H(s) = \frac{B(s)}{A(s)} \tag{3}$$

H(s) can be expanded in Taylor series in the vicinity of s = j• 0

$$H(s) = \sum_{n=0}^{\infty} (-1)^n (s - j\omega_0)^n \bar{A}_{n,\omega_0} \tag{4}$$

where

$$\bar{A}_{n,\omega_0} = \int_0^{\infty} \frac{t^n}{n!} h(t) e^{-j\omega_0 t} dt$$

is the n th order frequency moment of h(t) for • = • $_0$, notice that A_{n,ω_0} is complex. In the particular case • $_0$ = 0, frequency moments correspond to classical time moments

$$A_n(h) = \int_0^{+\infty} \frac{t^n}{n!} h(t) dt \tag{5}$$

They permit the characterization of H(j•) around • $_0$ = 0, as well as that of the impulse response h(t). $A_0(h)$ is the area of h(t), $A_1(h)$ defines mean time of h(t) and $A_2(h)$ deals with the dispersion of h(t) around its mean time (Etien *et al*., 2000). Equation 4 is rewritten as

$$H(s) = \sum_{n=0}^{\infty} (-1)^n s^n A_n(h) \tag{6}$$

Let $H(s) = \sum_{n=0}^{\infty} \frac{S^n}{n!} \left[\frac{d^n H(s)}{ds^n} \right]_{s=0}$

Then, time moments can be expressed as

$$A_n(h) = \frac{(-1)^n}{n!} \left[\frac{d^n H(s)}{ds^n} \right]_{s=0} \tag{7}$$

Moments and parameters of a transfer function: Let y(t) the step response of the studied system. We proposes to identify the system by the model

$$H(s) = \frac{Y(s)}{E(s)} = K_1 . \frac{1 + b_1 s + b_2 s^2 + \ldots.. + b_m s^m}{1 + a_1 s + a_2 s^2 + \ldots.. + a_n s^n} \tag{8}$$

from the final value theorem, as time approaches infinity for a stable linear system, the system response approaches a steady state value K_1 given by

$$K_1 = \lim_{t \to \infty} y(t) = y(\infty) \tag{9}$$

If a step input is applied to the system described in Eq. 8, by taking the Laplace transform of the normalized response gives

$$H(s) = s.y(s) \tag{10}$$

Let us consider •(t) an error function with

$$\varepsilon(t) = K_1 - y(t) \tag{11}$$

By introducing the Laplace transform in Eq. (11), (8) can be written as

$$\varepsilon(s) = K_1 \left[1 - \frac{1}{s} . \frac{1 + b_1 s + b_2 s^2 + \ldots.. + b_m s^m}{1 + a_1 s + a_2 s^2 + \ldots.. + a_n s^n} \right] \tag{12}$$

The development of (12) gives

$$\varepsilon(s) = K_1 \left[\frac{(a_1 - b_1) + \ldots..(a_m - b_m) s^{m-1} \ldots. a_n s^{n-1}}{1 + a_1 s + a_2 s^2 + \ldots\ldots\ldots\ldots. + a_n s^n} \right] \tag{13}$$

Then, using (6) we can write

$$\varepsilon(s) = \sum_{n=0}^{\infty} (-1)^n s^n A_n(\varepsilon) \tag{14}$$

According to (6) and (14) we can deduce the coefficients of the transfer function *H(s)* by solving the following matrix system

$$\begin{bmatrix} K_1(a_1 - b_1) \\ K_1(a_2 - b_2) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ K_1(a_{n+1} - b_{n+1}) \end{bmatrix} = \begin{bmatrix} A_0(\varepsilon) & 0 & \dots & 0 \\ -A_1(\varepsilon) & A_0(\varepsilon). & \dots & 0 \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ A_n(\varepsilon) & . & \dots & . \end{bmatrix} . \begin{bmatrix} 1 \\ a_1 \\ . \\ . \\ . \\ . \\ . \\ a_n \end{bmatrix} \quad (15)$$

where $A_n(\bullet)$ is the nth order temporal moment.

DC motor transfer function and its moments: For our cases, when n = 2 and m = 1, the transfer function (8) becomes

$$H(s) = K_1 . \frac{1 + b_1 s}{1 + a_1 s + a_2 s^2} \quad (16)$$

system (15) is reduced to the following matrix system

$$\begin{bmatrix} K_1(a_1 - b_1) \\ K_1 a_2 \\ 0 \end{bmatrix} = \begin{bmatrix} A_0 & 0 & 0 \\ -A_1 & A_0 & 0 \\ A_2 & -A_1 & A_0 \end{bmatrix} . \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} \quad (17)$$

the resolution of this matrix system (17), gives the following coefficients:

$$a_1 = \frac{A_1 A_0 - K_1 A_2}{A_0^2 - K_1 A_1} \quad a_2 = \frac{-A_1 + a_1 A_0}{K_1} \quad b_1 = a_1 - \frac{A_0}{K_1} \quad (18)$$

Parametric identification: After having deduced the mathematical forms which are used for calculation of the transfer function coefficients and which enable us at the same time to calculate the electric and mechanical motor parameters, we present here, the stages to be followed at the time of the determination of these parameters.

It is possible from a terminal voltage step $\bullet U_a$ to determine the majority of the dc motor parameters, that is also possible from the abrupt terminal voltage variation. We measure the initial and final currents and speeds values. At the steady state we can write

$$U_{a0} = R_a i_{a0} + K\omega_0 \quad (19)$$

$$U_{a1} = R_a i_{a1} + K\omega_1 \quad (20)$$

Where U_{a0}, i_{a0} and $\bullet_0$, respectively denote the terminal voltage armature current and the rotor speed at initial regime subscripted "0". U_{a1}, i_{a1} and $\bullet_1$, respectively denote the terminal voltage, armature current and the rotor speed at final regime subscripted "1".

According to (19) the torque and back-EMF constant K can be written as

$$K = \frac{U_{a1} - \frac{i_{a1}}{i_{a0}} U_{a0}}{\omega_1 - \frac{i_{a1}}{i_{a0}} \omega_0} \quad (21)$$

the steady state check

$$\Delta U_a = R_a \Delta i_a + K\Delta\omega \quad (22)$$

where $\bullet U_a$, $\bullet i_a$ and $\bullet\bullet$, respectively denote the terminal voltage variation, armature current variation and rotor speed variation.
The armature resistance is given from (22) as

$$R_a = \frac{\Delta U_a - K\Delta\omega}{\Delta i_a} \quad (23)$$

According to Eq. 24 and 25 we can obtain the two transfer functions of the armature current and rotor speed

$$\Delta U_a = R_a \Delta i_a + L_a \frac{di_a}{dt} + K\Delta\omega \quad (24)$$

$$K\Delta i_a = J\frac{d\Delta\omega}{dt} + f\Delta\omega \quad (25)$$

The armature current transfer function is given as

$$H_1(s) = \frac{\Delta i_a(s)}{\Delta U_a(s)} = \frac{\frac{f}{K^2 + R_a f}\left(1 + \frac{Js}{f}\right)}{1 + \tau_m \tau_e s^2 + (\tau_m + \mu\tau_e)s} \quad (26)$$

The rotor speed transfer function is given as

$$H_2(s) = \frac{\Delta\omega(s)}{\Delta U_a(s)} = \frac{\frac{K}{K^2 + R_a f}}{1 + \tau_m \tau_e s^2 + (\tau_m + \mu\tau_e)s} \quad (27)$$

Where

$\tau_e = \frac{L_a}{R_a}$ Electrical time constant

$\tau_m = \frac{R_a J}{K^2 + R_a f}$ Mechanical time constant

$$\mu = \frac{R_a f}{K^2 + R_a f} \quad \text{Usually small coefficient}$$

The calculation of K_i and K_v gains of the tow outputs $i_a(t)$ and • (t), respectively, by taking account Eq. 24 and 25 gives

$$K_i = \Delta i_a(\infty) = \frac{f \Delta U_a}{K^2 + R_a f} \quad (28)$$

$$K_v = \Delta\omega(\infty) = \frac{K \Delta U_a}{K^2 + R_a f} \quad (29)$$

According to (26) and (27) we deduce f and μ

$$f = \frac{K \Delta i_a(\infty)}{\Delta\omega(\infty)} \quad (30)$$

$$\mu = \frac{R_a f}{K^2 + R_a f} \quad (31)$$

By identification of $H_1(s)$ and $H_2(s)$ denominators with H(s) denominator we obtain

$$a_1 = \tau_m + \mu\tau_e \quad (32)$$

$$a_2 = \tau_m \tau_e \quad (33)$$

According to (32) and (33) we can obtain a second order equation

$$\mu\tau_e^2 - a_1\tau_e + a_2 = 0 \quad (34)$$

The resolution of the equation (34) gives two roots one is positive, the other is negative (rejected).

- According to (32) and (33) we deduce $•_m$
- The deduction of $•_m$ and $•_e$ gives L_a and J.
- The static torque can be calculated from steady state as

$$T_{st} = K i_{a0} - f\omega_0 \quad (35)$$

EXPERIMENTAL RESULTS

The separately excited dc motor used for experimental tests, has the nominal characteristics shown on Table 1. The first experiment is the determination of electric dc motor parameters according to direct tests, like armature resistance R_a and armature inductance L_a, as well as the back-EMF constant K. Mechanical parameters are also determined by direct tests (static torque T_{st} and viscous friction coefficient f). The deceleration test, enables us to determine the moment of inertia J. The second experiment to be carried out is to identify the dc motor parameters from the dynamic test. According to a step amplitude • U_a of terminal voltage applied to the armature circuit of the dc motor. The initial and final values of the armature current and the angular speed obtained from this test, are shown on Table 2. The back-EMF constant K, armature resistance R_a and the viscous friction coefficient f, can be determined by using, respectively Eq. 21, 23 and 30. Table 3 shows the first, second and third order moments values ,as well as the transfer function coefficients values successively calculated from the trapezoids method.

Let us know a_1, a_2 and μ then, we have the second order equation $0.00874•_e^2$-$0.0556•_e$ + 0.00144 = 0 the resolution of this equation gives $•_{e1}$ = 0.026s and $•_{e2}$ = 6.34s ($•_{e2}$ is a rather large time constant, is thus rejected).

Table 1: Specification of experimental DC motor

Rated power	180 W
Rated speed	1500 rpm
Armature voltage	270 V
Field voltage	220 V
Armature current	1.1 A
Field current	0.4 A

Table 2: Dynamic test

	U_a(V)	i_a(A)	• (rpm)
Initial regime	60	0.113	400
Final regime	248	0.167	1745

Table 3: 1,2 and 3 order moments and transfer function coefficients values

	A_0	A_1	A_2	a_1	a_2	b_1
• (t)	6.121818	0.14034940	-0.00094	0.055600	0.001440	0.012524
i_a(t)	-0.29657	-0.152533	-0.000504	0.470565	0.240323	-5.962602

Table 4: Comparison between direct tests and moments method for parameter identification of DC motor

	R_a (•)	L_a (H)	K (N.m.A•^{1})	J (Kg.m^2)	f (N.m.s rad•^{1})	T_{st} (N.m)
Direct tests	28	0.820	1.34	0.0028	0.00054	0.127
Moments method	30.9	0.803	1.323	0.0031	0.0005	0.128

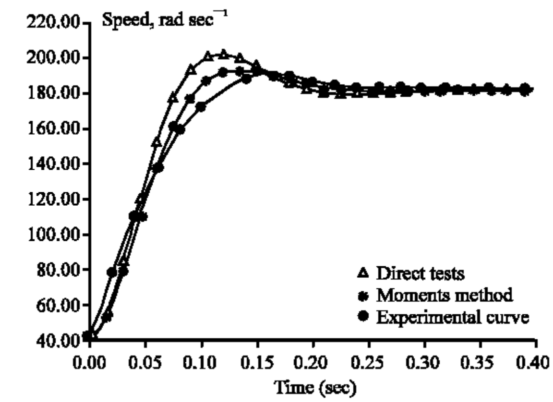

Fig. 1: Rotor speed angular response

The deduction of $\bullet_e$ and $\bullet_m$ enables to calculate J and L_a. Table 4 summarizes the values of the parameters calculated from the two identification methods (direct tests and moments method). Finally to check the precision of each method, we have simulated the dynamic test applying a step amplitude of terminal voltage $\bullet u_a$ = 188V, to the armature circuit of dc motor, as well as, deceleration test and mechanical characteristic. According to Fig. 1-3 the curves simulated from the dynamic test parameters with the moment method are close to the real curve

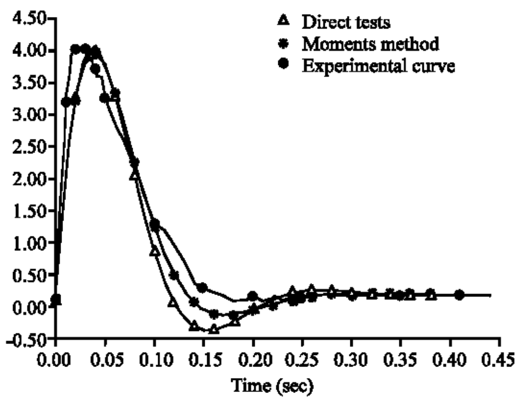

Fig. 2: Armature current response

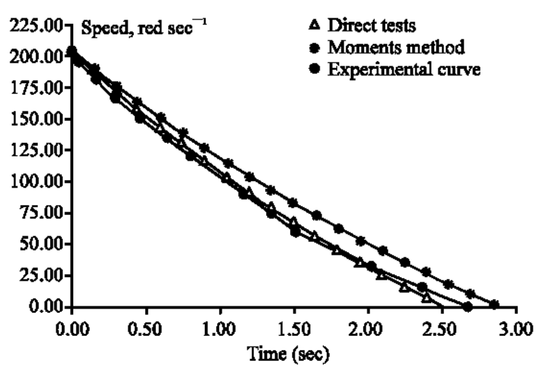

Fig. 3: Deceleration test

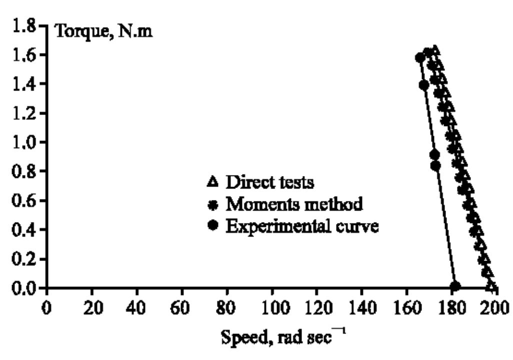

Fig. 4: Mechanical characteristic

(experimental curve) that those simulated from the direct tests parameters. With regard to the steady state (Fig. 4) the curves simulated from the dynamic test and direct tests parameters are almost identical and close to real measurement (experimental curve).

CONCLUSION

In this research, we tried to contribute our share in the discipline of the dc motors modeling. This contribution, which can be classified in a very wide field of identification methods can be summarized in the following results: We have developed a dynamic model based on the moments method, this method especially makes it possible to have a model closer to reality in transitory mode. We have proposed a comparison between various models based on the identification methods (direct tests and moments method), this comparison is made on the basis of real measurements taken in laboratory on a separately excited dc motor, with 180 W of rated power. It shows the advantage of the only dynamic test for identification, coupled to the moments method

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