A Hybrid Method for Calculating Very Close Electromagnetic Fields Due to Inclined Lightning Strokes

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Abstract: A hybrid method is presented for evaluating the electromagnetic fields very close to the lightning strokes. The vertical channel has been widely studied, unlike the inclined channel particularly the very near fields, the litterature show poor approach for such problems. Using a simple method for calculating the magnetic field by considering the lightning stroke as an infinitely long filament. The electric field is then calculated by simply discretising the Ampere's law. The results show that the channel inclination affects markedly the fields at close distances.

Key words: Hybrid method, electromagnetic, strokes, fields vertical, filament

INTRODUCTION

Lightning is a major natural source of EM radiation that interferes with modern electronics and communication systems. Lightning EM fields that causes overvoltages and overcurrents are very important factors in selecting the insulation level of the conductors in the design of effective and economic protection devices (Chandima and Vernon, 2000). Knowledge of Electromagnetic Pulse (LEMP) distribution at a given location is essential in determining the threat level for a sensitive system. In recent years, numerous theoretical analysis and experiments have been carried out to provide insight into the problem of lightning induced voltages (Chandima and Vernon, 2000; Gerhard, 1990; Chowdhuri et al., 2001). The various theoretical analysis are often quite different in their calculated voltage waveshape, polarity and distribution of the voltage along the line (Gerhard, 1990). The return stroke models can be defined as mathematical construction that can represent the properties of the lightning return stroke model (Chandima and Vernon, 2000). One of the basic assumptions in most of the models is that the channel is vertical, while the channel inclination effects have shown to be of great importance when calculating the lightning induced effects on overhead lines (Gerhard, 1990; Lupo, 2000). In this study we propose a simple way based on a hybrid method for evaluating LEMP in the vicinity of the lightning. The geometry to explain a model phenomenon is shown in Fig. 1. The return stroke is inclined and makes an angle • with the Z axis. Unlike the vertical stroke, the inclined return stroke produces a vector potential that has a horizontal component in the direction of overhead lines (Carlos et al., 2000). Such a component has large effects on the voltage induction on the line.
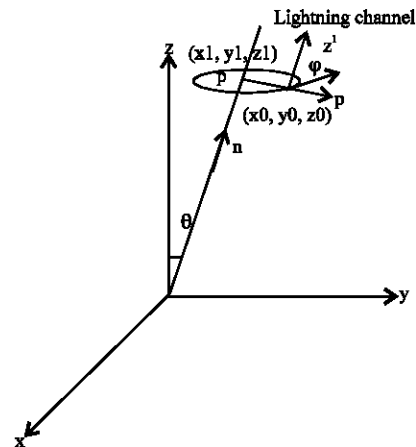

Fig. 1: The inclined channel and the observation poit (x_0, y_0, z_0)

FORMULATION OF THE METHOD

We consider a constant current through the inclined infinitely long channel, the current is given by the following relation (Yang and Zhou, 2004):

$$i(t) = I_{01} \frac{(t/\tau_1)^2}{(t/\tau_1)^2 + 1} e^{-t/\tau_2} + I_{02}\left(e^{-t/\tau_3} - e^{-t/\tau_4}\right) \quad (1)$$

where I_{01} and I_{02} are amplitudes of the current. $\bullet_1$, $\bullet_2$, $\bullet_3$ and $\bullet_4$ are time constant.

In the very near field and considering the lightning channel as an infinitely long filament with an arbitrary orientation defined by the unit vector n:

$$n = \cos\bullet\, i + \cos\bullet\, j + \cos\bullet\, k \quad (2)$$

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the magnetic field H reduces to the well known Ampere's law:

$$H(x, y, z, t). = I(t)/(2 \bullet \bullet) \qquad (3)$$

where $\bullet$ is the azymuthal direction in cylindrical coordinates centered on the current path, $\bullet$ is the cylindrical radial direction. The validity of this approximation is satisfactory for low frequency and very near fields (Sabrina, 2001; Baba and Rakov, 2003).

ELECTROMAGNETIC FIELDS CALCULATION

In the rectangular coordinate system the x, y and z components of the magnetic field in a point (x_1, y_1, z_1) are derived from (Lupo, 2000):

$$H_x(x,y,z) = \left(\frac{z_0 - z_1}{\rho} \cos\beta - \frac{y_0 - y_1}{\rho} \cos\gamma \right) H_\varphi \qquad (4)$$

$$H_y(x,y,z) = \left(\frac{x_0 - x_1}{\rho} \cos\gamma - \frac{z_0 - z_1}{\rho} \cos\alpha \right) H_\varphi \qquad (5)$$

$$H_z(x,y,z) = \left(\frac{y_0 - y_1}{\rho} \cos\alpha - \frac{x_0 - x_1}{\rho} \cos\beta \right) H_\varphi \qquad (6)$$

Based on Ampere's law in an isotropic medium the electric field is obtained by the following relations:

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} \qquad (7)$$

where $\bullet$ and $\bullet$ are, respectively the conductivity and the permitivity. We write out the vector components of the curl operator to yield the following system of scalar equations:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x \right) \qquad (8)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y \right) \qquad (9)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z \right) \qquad (10)$$

Using the Yee method FDTD (Allen, 1995) to discretise the above system of equations, we obtain for the space partial derivative:

$$\frac{\partial F}{\partial x}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) =$$

$$\frac{F^n_{i+1/2,j,k} - F^n_{i-1/2,j,k}}{\Delta x} + O\left[(\Delta x)^2 \right] \qquad (11)$$

The ±1/2 increment in the i subscript (x-coordinate) of F denotes a space finite-difference over ±1/2x. The same thing will be done for the first time partial derivative of F, evaluated at a fixed space point (I,j, k), follows by analogy:

$$\frac{\partial F}{\partial x}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) =$$

$$\frac{F^{n+1/2}_{i,j,k} - F^{n-1/2}_{i,j,k}}{\Delta t} + O\left[(\Delta t)^2 \right] \qquad (12)$$

The ±1/2 increment in the n subscript (time-coordinate) of F denotes a time finite-difference over ±1/2t. This notation is chosen to interleave the $\vec{E}$ and $\vec{H}$ components in time and space at intervals of ±1/2t and ±1/2x, respectively for purposes of implementing a leapfrog algorithm.

Applying Eq. 8-12, the following expressions are obtained for each E-field component:

$$E_x^{n+1/2}(i,j,k) = \left(1 - \frac{\sigma(i,j,k)}{\varepsilon(i,j,k)}\Delta t \right) E_x^{n-1/2}(i,j,k) +$$

$$\frac{\Delta t}{\varepsilon(i,j,k)\Delta} \left| H_z^n(i,j+1/2,k) - H_z^n(i,j-1/2,k) \right.$$

$$\left. H_y^n(i,j,k-1/2) - H_y^n(i,j,k+1/2) \right| \qquad (13)$$

$$E_y^{n+1/2}(i,j,k) = \left(1 - \frac{\sigma(i,j,k)}{\varepsilon(i,j,k)}\Delta t \right) E_y^{n-1/2}(i,j,k) +$$

$$\frac{\Delta t}{\varepsilon(i,j,k)\Delta} \left| H_x^n(i,j,k+1/2) - H_x^n(i,j,k-1/2) \right.$$

$$\left. H_z^n(i-1/2,j,k) - H_z^n(i+1/2,j,k) \right| \qquad (14)$$

$$E_z^{n+1/2}(i,j,k) = \left(1 - \frac{\sigma(i,j,k)}{\varepsilon(i,j,k)}\Delta t \right) E_z^{n-1/2}(i,j,k) +$$

$$\frac{\Delta t}{\varepsilon(i,j,k)\Delta} \left| H_y^n(i+1/2,j,k) - H_y^n(i-1/2,j,k) \right.$$

$$\left. H_x^n(i,j+1/2,k) - H_x^n(i,j-1/2,k) \right| \qquad (15)$$

For the air region the E-field components become:

$$E_x^{n+1/2}(i,j,k) = E_x^{n-1/2}(i,j,k) +$$

$$\frac{\Delta t}{\varepsilon(i,j,k)\Delta}\left|H_z^n(i,j+1/2,k) - H_z^n(i,j-1/2,k) + \right.$$

$$\left. H_y^n(i,j,k-1/2) - H_y^n(i,j,k+1/2) \right| \qquad (16)$$

$$E_y^{n+1/2}(i,j,k) = E_y^{n-1/2}(i,j,k) +$$

$$\frac{\Delta t}{\varepsilon(i,j,k)\Delta}\left|H_x^n(i,j,k+1/2) - H_x^n(i,j,k-1/2) \right.$$

$$\left. + H_z^n(i-1/2,j,k) - H_z^n(i+1/2,j,k) \right| \qquad (17)$$

$$E_z^{n+1/2}(i,j,k) = E_z^{n-1/2}(i,j,k) +$$

$$\frac{\Delta t}{\varepsilon(i,j,k)\Delta}\left|H_y^n(i+1/2,j,k) - H_y^n(i-1/2,j,k) \right.$$

$$\left. + H_x^n(i,j+1/2,k) - H_x^n(i,j+1/2,k) \right| \qquad (18)$$

The E-field at the instant n+0.5 is calculated by using the information of both the electric field at the instant n-0.5 and the magnetic field at the instant n. Although this method of precalculating the magnetic field at a certain point does not need meshing the entire region of the configuration,we can say that the method is a Yee FDTD method.

The numerical algorithm for the curl Eq. 7 requires that the time increment • t have a specific bound relative to the lattice space increment • . This bound is necessary to avoid numerical instabillity. The temporel discrete interval • t and spatial discrete interval • must satisfy the courant stability condition Yang and Zhou (2004) that is:

$$\Delta t \leq \frac{\Delta}{2c} \qquad (19)$$

where C is the velocity of light and the space lattice is taken constant for the three coordinates • x = • y = • z = • . For the proposed case • t is taken as 0.0167 ns and • =10 cm. But due to the caracteristics of this one, some additional considerations should be taken into consideriation.

RESULTS AND DISCUSSION

A simple model is proposed to calculate the EM fields due to an inclined stroke. It is shown that the channel inclination affects more markedly the fields at close
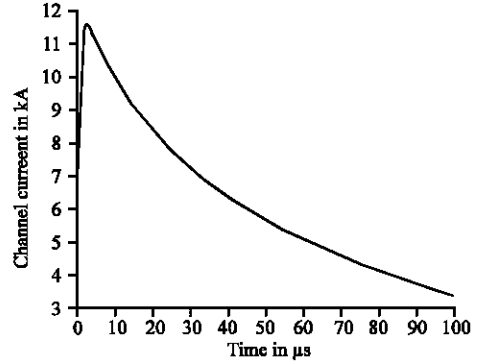

Fig. 2: Channel current I_{01}=3.25kA, I_{02}=8.95kA, •$_1$=0.072 µs, •$_2$ = 16.67µs, •$_3$ = 100 µs, •$_4$ = 0.5µs
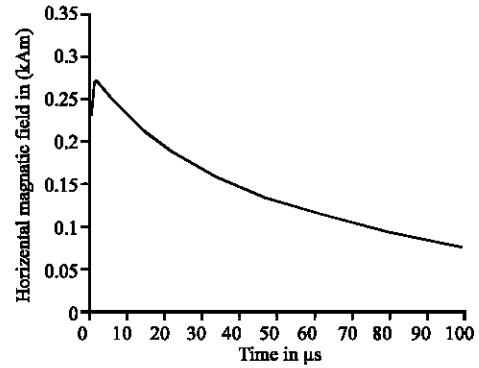

Fig. 3: Horizontal magnetic field intensity 5 m from the channel
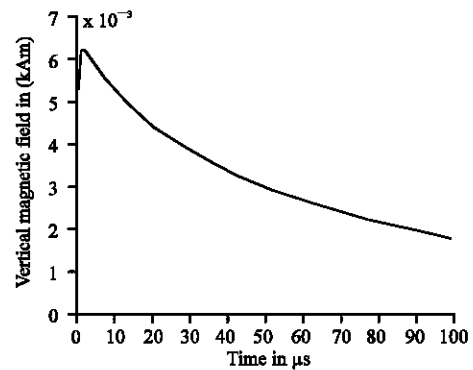

Fig. 4: Vertical magnetic field intensity 5m from the channel

distances Fig. 2-6. Due to the lack of theoritical and experimental results regarding near fields and especialy inclined stroke the comparison with other results was too difficult to be done. The drawback of the proposed method consists in the fact that the distance from the
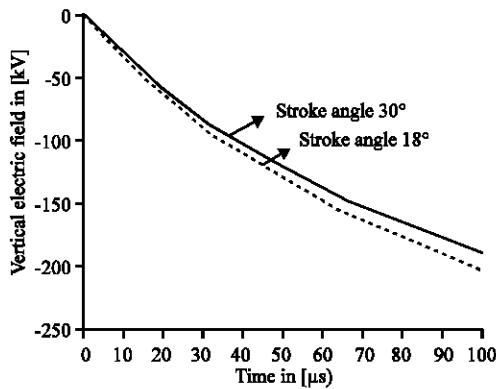
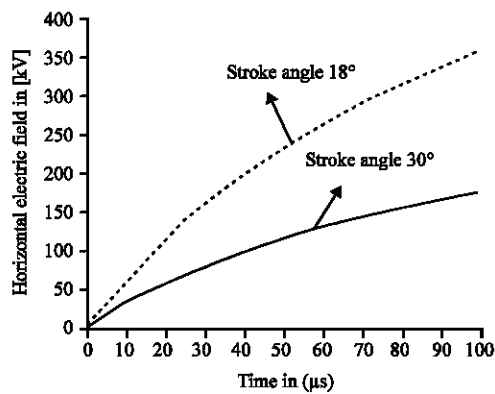Fig. 5: Vertical electric field at height 5 and 5 m from the channel

Fig. 6: Horizontalelectric field at height 5 and 15 m from the channel

channel must be very close and the validity of the approximation of the EM fields is limited to a few megahertz.

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