Medwell Online International Journal of Electrical and Power Engineering 1 (2): 189-195, 2007

Comparative Study of Direct Torque and Nonlinear Controls of a Permanent Magnet Synchronous Motor

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Abstract: We present in this study, a comparative study between 2 structures of control, the Direct Torque Control (DTC) and the Non-Linear Control (NLC) of the Permanent Magnet Synchronous Motor (PMSM). To illustrate the performance and the robustness of these two control techniques, simulation results are presented.

Key words: Direct Torque Control (DTC), Permanent Magnet Synchronous Motor (PMSM), Nonlinear Control (NLC), input-output linearization, differential geometry theory, robustness

INTRODUCTION

Non-Linear Control (NLC) relies on the input-output linearization principle (Takahashi and Noguchi, 1986; Canudas, 2000). This non-linear approach, which does not make any a priori assumptions about flux orientation, is an interesting alternative to vectorial control.

The principles of Direct Torque Control (DTC) have been elaborated in the second half of the nineteen hundred eighties (Akin, 2003). This type of control has been introduced as an alternative to vectorial control by orientation of the rotor flux, which presents the major disadvantage of being relatively sensitive to the variations of the parameters of the machine.

DTC singles itself by a simplified structure, minimizing the influence of the parameters of the machine, in particular by the fact that it requires neither a speed measurement in real time, nor a complex control by Pulse Width Modulation (PWM) of the inverter (Bolognani *et al*., 2003; Leite *et al*., 2004).

MODELLING OF THE PMSM

The electrical and mechanical equations of the PMSM in the rotor reference frame (d,q) are expressed as follows (Canudas, 2000):

$$\begin{cases} \dfrac{dI_d}{dt} = -\dfrac{R_s}{L_d}I_d + p\dfrac{L_q}{L_d}I_q\Omega + \dfrac{1}{L_d}U_d \\ \dfrac{dI_q}{dt} = -\dfrac{R_s}{L_q}I_q - p\dfrac{L_d}{L_q}I_d\Omega - p\dfrac{\Phi_f}{L_q}\Omega + \dfrac{1}{L_q}U_q \\ \dfrac{d?_r}{dt} = -\dfrac{f}{J}\Omega + \dfrac{3p}{2J}[(L_d - L_q)I_qI_d + \Phi_f I_q] - \dfrac{1}{J}C_r \end{cases} \quad (1)$$

The state-space- equations of the PMSM can be written as follows:

$$\frac{d}{dt}[X] = [A]\,[X] + [B]\,[U] \quad (2)$$

With:

$$[X] = [I_d \;\; I_q]^T \qquad [U] = [U_d \;\; U_q \;\; \Phi_f]^T \quad (3)$$

And by defining:

$$[A] = \begin{bmatrix} -\dfrac{R_s}{L_d} & \omega_r\dfrac{L_q}{L_d} \\ -\omega_r\dfrac{L_d}{L_q} & -\dfrac{R_s}{L_q} \end{bmatrix} \quad [B] = \begin{bmatrix} \dfrac{1}{L_d} & 0 & 0 \\ 0 & \dfrac{1}{L_q} & -\dfrac{\omega_r}{L_q} \end{bmatrix} \quad (4)$$

The electromagnetic torque and the mechanical equations are given by:

$$\begin{cases} C_e = \dfrac{3p}{2}[(L_d - L_q)I_qI_d + \Phi_f I_q] \\ J\dfrac{d\Omega}{dt} + f\Omega = C_e - C_r \\ \omega_r = p\Omega \end{cases} \quad (5)$$

BASIC DTC PRINCIPLES

DTC is a control philosophy exploiting the torque and flux producing capabilities of ac machines when fed

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by a voltage source inverter that does not require current regulator loops, still attaining similar performances to that obtained by a vector control drive.

Behavior of stator flux: In the reference (d, q), the stator flux can be obtained by the following equation:

$$\bar{V}_s = R_s \bar{I}_s + \frac{d}{dt}\bar{\Psi}_s \tag{6}$$

By neglecting the voltage drop due to the resistance of the stator to simplify the study (for high speeds), we find:

$$\bar{\Psi}_s \approx \bar{\Psi}_{s0} + \int_0^t \bar{V}_s dt \tag{7}$$

During one sampling period, the voltage vector applied to the PMSM remains constant, we can write:

$$\bar{\Psi}_s(k+1) \approx \bar{\Psi}_s(k) + \bar{V}_s T_e \tag{8}$$

Then: $\Delta\bar{\Psi}_s \approx \bar{V}_s T_e$

$\bar{\Psi}_s(k)$: is the stator flux vector of the current sampling step;

$\bar{\Psi}_s(k+1)$: is the stator flux vector of the next sampling step;

T_e: is the period of sampling.

$\Delta\bar{\Psi}_s$: is the variation of the stator vector flux;

For a constant sampling period, is proportional to the voltage vector applied to the stator of the PMSM.

Behavior of the torque: The electromagnetic torque is proportional to the vector product between the stator and rotor flux according to the following expression (Canudas, 2000):

$$C_e = k(\bar{\Psi}_s \times \bar{\Psi}_r) = k\left|\Psi_s\right|\left|\Psi_r\right|\sin(d) \tag{9}$$

With:

$\bar{\Psi}_s$: is the stator flux vector;

$?_r$: is the rotor flux vector;

$$K = {}^{p}\!/_{L_q}$$

•: is the angle between the 2 flux vectors.

Table 1: Selection for basic direct torque control

• • s	• C_e	S_1	S_2	S_3	S_4	S_5	S_6
1	1	V_2	V_3	V_4	V_5	V_6	V_1
	0	V_7	V_0	V_7	V_0	V_7	V_0
	-1	V_6	V_1	V_2	V_3	V_4	V_5
0	1	V_3	V_4	V_5	V_6	V_1	V_2
	0	V_0	V_7	V_0	V_7	V_0	V_7
	-1	V_5	V_6	V_1	V_2	V_3	V_4

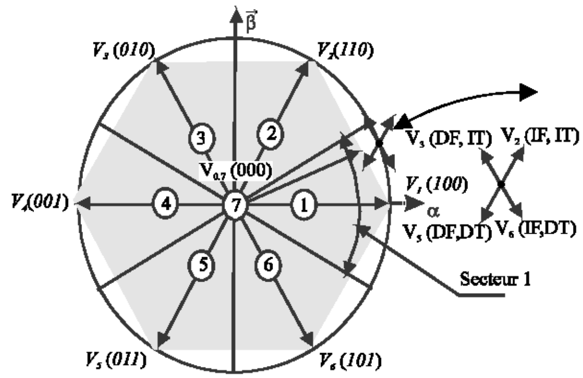

Fig. 1: Partition of the complex plan in 6 angular sectors IT: Increas the torque, DT: Decrease the torque, IF: Increase the flux, DF: Decrease the flux

Commutation strategy elaboration: Table 1 shows the commutation strategy suggested by Takahashi (1986) to control the stator flux and the electromagnetic torque of the PMSM.

Figure 1 gives the partition of the complex plan in the 6 angular sectors S_i, i = 1 to 6.

NON-LINEAR MODEL OF THE PMSM

With the simplifying assumptions relating to the PMSM, the model of the motor expressed in the Park reference frame is given in the following suitable state form (Isidori, 1989; Slotine and Li, 1991):

$$\begin{aligned} \dot{X} &= F(X) + GU \\ Y &= H(X) \end{aligned} \tag{10}$$

where:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} I_d \\ I_q \\ \Omega \end{bmatrix} \quad U = \begin{bmatrix} U_d \\ U_q \end{bmatrix} \quad G = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix}$$

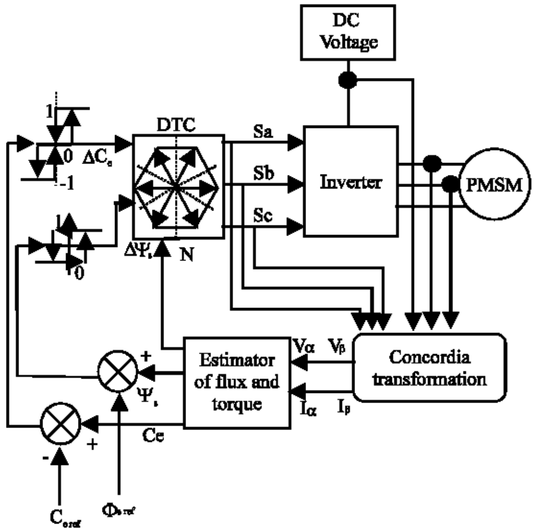

Fig. 2: Bloc diagram of aPMSM drive with DTC

$$F(X)=\begin{bmatrix} f_1(x)\\ f_2(x)\\ f_3(x)\end{bmatrix}=\begin{bmatrix} a_1x_1+a_2x_2x_3\\ b_1x_2+b_2x_1x_3+b_3x_3\\ c_1x_2+c_2x_1x_2+c_3x_2-\dfrac{C_r}{J}\end{bmatrix}$$

$$a_1=\frac{-R_s}{L_d}\quad a_2=\frac{pL_q}{L_d}$$

$$b_1=\frac{-R_s}{L_q}\quad b_2=\frac{-pL_d}{L_q}\quad b_3=\frac{-p\Phi_f}{L_q}$$

$$c_1=\frac{-f}{J}\quad c_2=\frac{3p(L_d-L_q)}{2J}\quad c_3=\frac{3p\Phi_f}{2J}$$

In $f_3(x)$ the load torque C_r is removed from the state equations and will be considered as a perturbation (Fig. 2).

INPUT-OUTPUT LINEARIZATION CONTROL OF THE PMSM

Principle: The input-output linearization technique uses a nonlinear change of coordinates and feedback to transform the nonlinear system (10) into a decoupled linear one. The control gol is twofold, first to regulate the rotor speed and second to control the axis component of the stator current to be zero which insure a maximum torque operation (Kaddouri *et al.*, 1994; Kwanty and Blankenship, 2000):

$$Y(X)=\begin{bmatrix} y_1(x)\\ y_2(x)\end{bmatrix}=\begin{bmatrix} h_1(x)\\ h_2(x)\end{bmatrix}=\begin{bmatrix} x_1\\ x_2\end{bmatrix}=\begin{bmatrix} I_d\\ \Omega\end{bmatrix} \quad (11)$$

Control linearization: The linearizing condition permits to verify if the non linear system admits an input-output linearization is the order of the relative degree of the system (Kaddouri *et al.*, 1994; Kwanty and Blankenship, 2000). To obtain the nonlinear control law, we calculate the output relative degree (i.e., the number of possibilities which is necessary to derive the output in order to obtain the input U).

The relative degree of the d-axis current $I_d = y_1$:

$$\dot{y}_1(x)=L_fh_1(x)+L_gh_1(x)U_d \quad (12)$$

with:

$$L_fh_1(x)=f_1(x)$$
$$L_gh_1(x)=(g_1\ \ 0)$$

The relative degree of $y_1(x)$ is $r_1 = 1$

The relative degree of the mechanical speed • = y_2

$$\dot{y}_2(x)=L_fh_2(x)+L_gh_2(x)U_q \quad (13)$$

with:

$$L_fh_2(x)=f_3(x)$$
$$L_gh_2(x)=0$$

We note that the inputs U do not appear in (13), a second derivative became then necessary:

$$\ddot{y}_2(x)=L_f^2h_2(x)+L_gL_fh_2(x)U_q \quad (14)$$

with:

$$L_f^2h_2(x)=c_2x_2f_1(x)+f_2(x)(c_3+c_2x_1)+c_1f_3(x)$$
$$L_gh_2(x)=\left[c_2x_2g_2\quad g_2(c_2x_2+c_3)\right]$$

The relative degree of $y_2(x)$ is $r_2 = 2$

The relative degree of the system is equal to the system order n ($r = r_1 + r_2 = n = 3$). The system is exactly linearizable. The vector defining the relation between the physical inputs (U) and the derivative outputs (Y(X)) is given by the expression:

$$\begin{bmatrix} \dot{y}_1(x) \\ \ddot{y}_2(x) \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} I_d \\ \frac{d^2}{dt^2}\Omega \end{bmatrix} = A(X) + D(X)\begin{bmatrix} U_d \\ U_q \end{bmatrix} \quad (15)$$

where:

$$A(X) = \begin{bmatrix} f_1(x) \\ c_2 x_2 f_1(x) + f_2(x)(c_3 + c_2 x_1) + c_1 f_3(x) \end{bmatrix}$$

$$D(X) = \begin{bmatrix} g_1 & 0 \\ c_2 x_2 g_1 & g_2(c_2 x_1 + c_3) \end{bmatrix}$$

To linearize the input-output comportments of the motor in closed loop, we apply the following nonlinear state feedback (Kaddouri *et al.*, 1994; Kwanty and Blankenship, 2000);

$$\begin{bmatrix} U_d \\ U_q \end{bmatrix} = D^{-1}(x)\left[-A(x) + \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \right] \quad (16)$$

The decoupling matrix $D^{-1}(x)$ must be invertible. The application of the linearizing law (16) on the system (15) allows obtaining two mono-variable, linear and decoupled sub-systems.

$$\begin{bmatrix} \dot{y}_1(x) \\ \ddot{y}_2(x) \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} I_d \\ \frac{d^2}{dt^2}\Omega \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (17)$$

Control algorithm by poles placement: The internal inputs (V_1, V_2) are calculated by imposing static modes ($I_{dref} = I_d$ and • $_{ref}$ = •) and an error dynamic (Kaddouri *et al.*, 1994; Azizun *et al.*, 2003; Zhou and Wang, 2002; Be labbes and Meroufel, 2002).

$$\begin{aligned} &\frac{d}{dt}e_1 + k_{11}e_1 = 0 \\ &\frac{d^2}{dt^2}e_2 + k_{11}\frac{d}{dt}e_2 + k_{22}e_2 = 0 \\ &e_1 = I_{dref} - I_d \\ &e_2 = \Omega_{ref} - \Omega \end{aligned} \quad (18)$$

The internal inputs (V_1, V_2) are defined as:

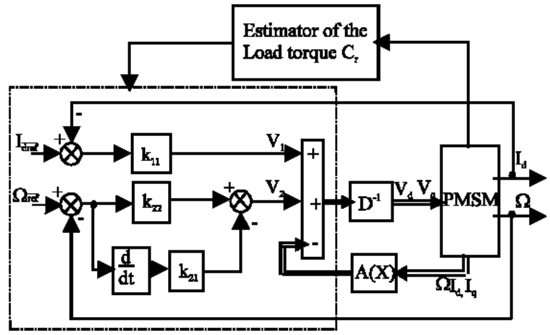

Fig. 3: Block diagram of the nonlinear controller

$$V_1 = k_{11}(I_{dref} - I_d) + \frac{d}{dt} I_{dref}$$

$$V_2 = k_{21}\left(\frac{d}{dt}\Omega_{ref} - \frac{d}{dt}\Omega\right) + k_{22}(\Omega_{ref} - \Omega)\frac{d^2}{dt^2}\Omega_{ref}$$

$$\dot{I}_{dref} = \dot{O}_{ref} = \ddot{O}_{ref} = 0$$

The gains k_{11}, k_{21}, k_{22} are chosen so that the following Hurwitz polynomial equation;

$$\begin{aligned} &s + k_{11} = 0 \\ &s^2 + k_{21}s + k_{22} = 0 \end{aligned} \quad (19)$$

The control diagram is given by Fig. 3.

COMPARISON IN THE SPEED REGULATION LEVEL

In order to have a better appreciation of the results obtained through the two studied control techniques, it is necessary to carry out a comparison of the static and dynamic characteristics of these 2 techniques under the same operating conditions (references, loads disturbances... etc.) and in the same configuration of simulation (no sampling, duration of simulation... etc). Thereafter we must make a choice of the type of control according to the specifications of considered application.

Step of the ioad torque: Figure 4 represents the speed and torque of the PMSM in the case of a startup without a load with a load torque step of 5.5 Nm after 0.2s.There is no overshoot for both cases (DTC and NLC). According to Fig. 4 we notice that during startup the increase of speed is done with an limited acceleration with an applied torque and a minimum response time for the NLC.

Speed inversion: In order to test the robustness of the complete drive system, we apply a change of the speed

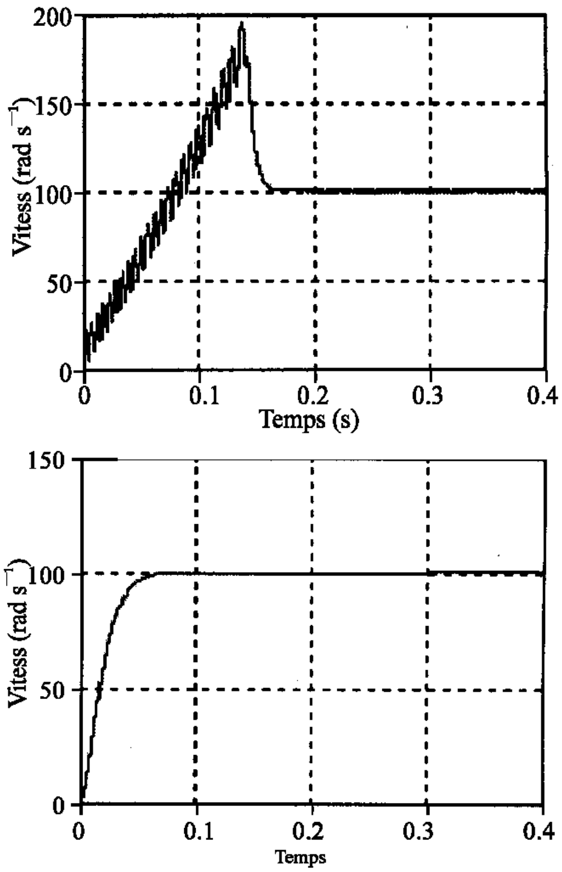

Fig. 4: Load torque performance comparison

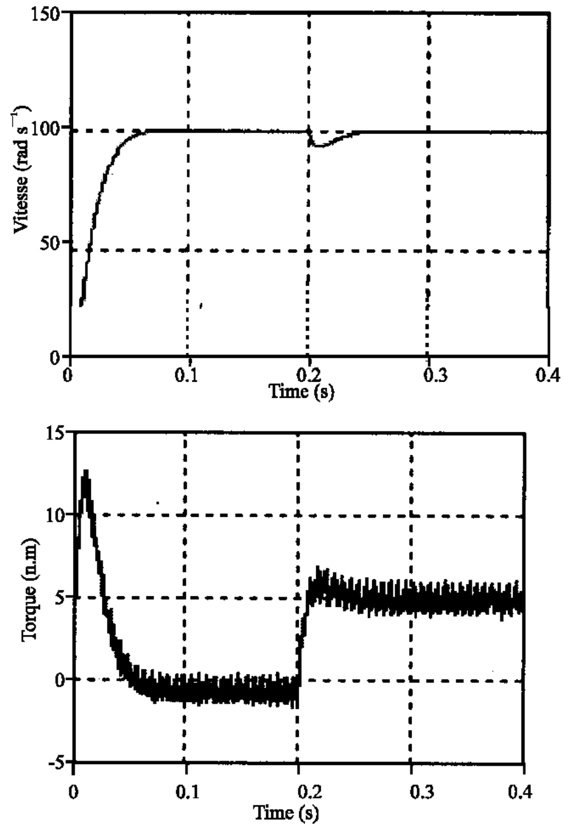

Fig. 5: Speed performance Comparison

reference from 100 to -100 rad/sec at time 0.2s, according to Fig. 5. For this case of the inversion of the direction of rotation, we can say that speed tracking is carried out normally and without overshoot for the two control techniques. It appears that the nonlinear control technique presents a good performance for the starting up and a fast rejection of disturbances. The results of simulation thus obtained for both controls are presented in Fig. 5. We notice that the NLC has a better response for speed inversion compared to the DTC.

Insensitivity to the variations of the parameters: We test the performances by simulation of both the Non-Linear Controller (NLC) and the Direct Torque Control (DTC). The tests consist of a variation of the stator resistance of 100%. The results are given by Fig. 6.

CHOICE OF THE CONTROL

The nonlinear control is Interesting because it works in abroad range of speed and has a perfect decoupling between control of flux and that of torque. However, many calculation functions, for example those for PWM (Pulse With Modulation) and the model of the machine present significant obstacles for complete integration and the correct operation of this control technique. In addition, the use of a mechanical sensor increases the cost of the control system and decreases reliability.

Direct Torque Control DTC, is much simpler and it does not require a mechanical sensor, like vector Control. Its algorithm is, in addition, simple because it is related to a model of the machine where the only parameter which intervenes is stator resistance. Moreover, the PWM is replaced, in this control, by a simple commutation table which makes it even simpler. We notice that the nonlinear control develops superior performance to that of the direct torque control DTC. We can easily observe it by applying a load torque disturbance. Finally, the technique of nonlinear control by linearization with the direction input-outputs is based on the idea to transform a nonlinear system into a linear system. It is well adapted to the problems of continuity of the trajectory and stability system. It is well adapted to the problems of continuation of the trajectory and stability.

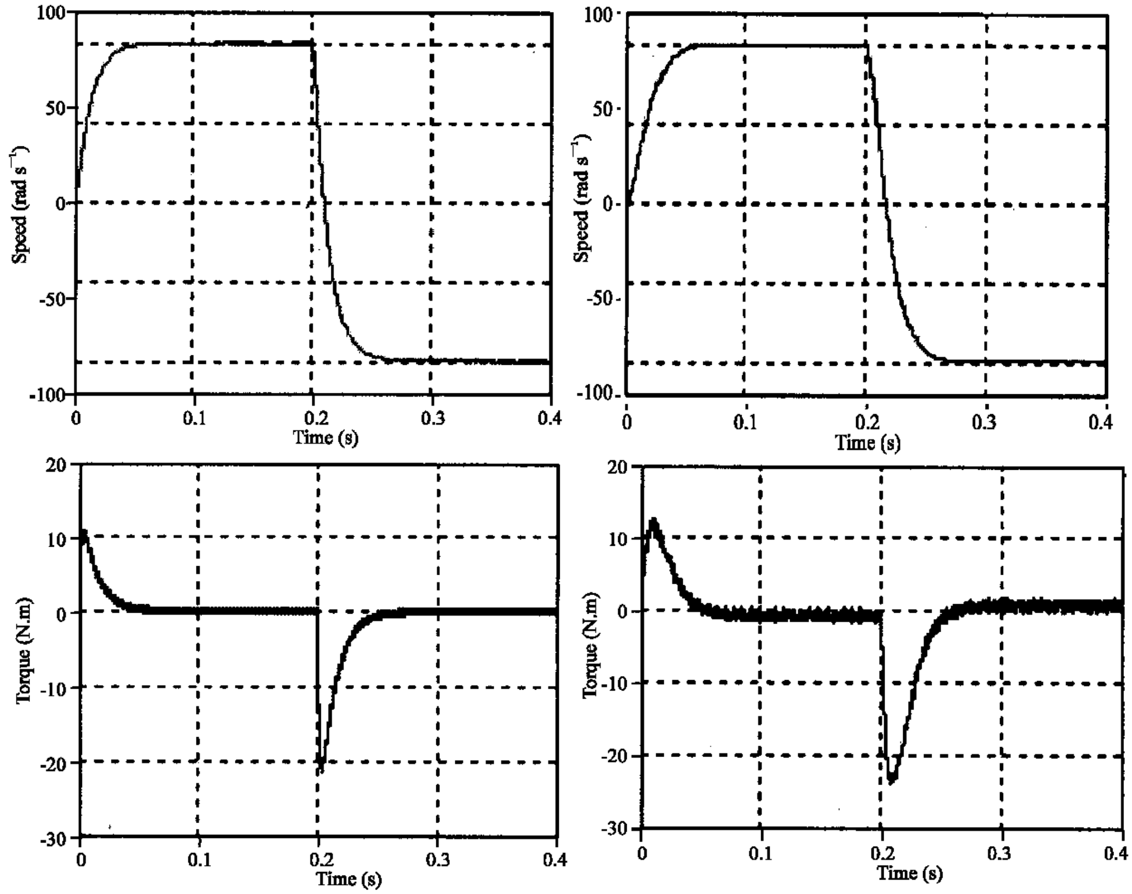

Fig. 6: Comparison of stability against resistance r variation

Table 2: Parameters of the PMSM

PMSM	Names	Value [unit]	Values [unit]
R_s	Stator resistance	1.4	•
L_d	d-axis inductance	0.0066	H
L_q	q-axis inductance	0.0058	H
J	Inertia of rotor	0.00176	kg.m2
f	Friction coefficient	0.00038818	N.m/Rad/s
p	Number of pole pairs	3	----
• f	Permanents magnets flux	0.1546	Wb

CONCLUSION

This study presents a comparative study of two PMSM control strategies: non-linear and direct torque control techniques.

This analysis enabled us to compare the static and dynamic performances of the non-linear and direct torque controls of a PMSM. The simulation results obtained for the speed control of the PMSM, whatever the studied operating ranges, show that the response with the nonlinear control is faster and more robust to disturbances of the load torque and of the parametric variations of the motor (Table 2).

List of principal symbols

U_d, U_q, I_d, I_q: Represent respectively the stator voltage and currents (d, q) axes components.

L_d, L_q: Stator inductance (d, q) axes components.

• : Rotor speed

V_1, V_2: Internal inputs

f_1, f_2, f_3: NL Functions

h_1, h_2: Outputs respectively 1, 2

k_{11}, k_{21}, k_{22}: NL gains controller

L_{fn}, L_{gh}: Lie derivative from dregs along f and g

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