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The Effect of Lateral Refraction on Angular Measurements in Bauchi, Nigeria

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Abstract: Angular measurements data were obtained using a manual glass scale Theodolite. Morning, afternoon and evening observations were made for the 3 months of November, December and January. Reductions and rigorous adjustments were made to obtain the best estimates for the observation analysis. Temperature and pressure measurements were also, taken alongside the angles so as to find the optimum period for best observations free of atmospheric refraction. The coefficient of lateral refraction determined for Bauchi was 0.0567, while the monthly means of the observations were subjected to standard statistical tests. From the result of these tests it is concluded that: the arithmetic mean of angular measurement was 87°17' 49.28". The best period for angular measurements in Bauchi is between 12:00 noon to 4:00 pm. The best month in the year for angular measurements is January. All daytime horizontal angles measured within the period must be subtracted by 1.5".

Key words: Refraction, angular measurements, refraction coefficient, control establishment, errors, horizontal angle

INTRODUCTION

Angular measurements are quite significant in determining the relative position of points on the surface of the earth. This is so because such methods of surveying like triangulation, traversing, resection and Intersection are still used in this part of the global community for surveying control establishment and extension schemes and for angular measurements to perform these roles effectively, they must be free from both systematic and random errors (Bomford, 1980). To strengthen this fact, Richardus (1981) identified the main sources of errors in the measurement of angles of this nature as:

- Axial errors of the Theodolites
- Centering error
- Leveling error
- Pointing error
- Reading error
- Error due to the Instability of the tripod/target or tower (sinking)
- Refraction error

Some of the errors aforementioned can be modeled out of the observation through an adjustment scheme, but not the error due to the effect of atmospheric refraction effectively. However, Bomford (1980) identified refraction as the most serious source of error to contend with using optical instruments and in any major triangulation and traverse scheme and further suggested a correction through the determination of a refraction coefficient in the study area. There is no known method of the exact determination of its magnitude in field conditions. Richardus (1981), seems to hold the same view, but added that some particular time of day or month (season) of the year be defined and recommended through empirical determinations to obtain the 'best' period for the angular measurements hence, the study.

Lateral refraction: Refraction is defined as the bending of light ray as it passes from one substance (medium) to another. Microsoft Encarta (2009), further stated that the angle the light ray bends depends on the difference between the speed of light in one substance and the next. Lateral refraction or curvature is a phenomenon that occurs in a horizontal plane when the decrease of air-density with height results in a bending of light ray curving in the vertical plane. This bending of light causes measured angular values to be in error. Refraction is maximum during the heat of the day when the air near the ground will be over-heated (Clark, 1983; Davies *et al*., 1981; Johnson, 1991). Refraction error is additive and its correction subtractive (Davies *et al*., 1981).

Observation location: Bauchi town is the capital of Bauchi state of Nigeria, which is 1 of the 6 states in the Northeast geopolitical zone of Nigeria. Bauchi lies on the Lat. 10°15'N and Long. 9°15'E. Dry and wet seasons are the 2 major seasons of Bauchi, which lies in the savannah belt of the Northeast region. Bauchi like most cities in this middle belt is characterized by extremely hot climate, which is not suitable for angular observation of this nature using the glass arc theodolite. Since, the use of this instrument for position fixing is still relevant particularly in this part of the world, there is need to determine refraction coefficient and the most convenient period for angular observations free from lateral refraction in Bauchi since no known attempt have in the past been made.

MATERIALS AND METHODS

The pre-analysis and design of the angle measurements took cognizance and care of the observation methods, number, models and conditions to isolate refraction error. Both temporary and permanent adjustments were made to contain the systematic errors. The wild T2 theodolite used was tested and found suitable. Ten measures of each angle were made for 30 days of each month of observation. After, the mean angular observations were obtained for the 3 months (November, December and January for 1992, 1997 and 2002, Table 1) together with temperature and pressure measurements. A television mast whose distance 755.320 m from the base length was measured with a wild Di 10 Distoma was used as a target for the angle measurements. The 3 months were chosen for the study based on some earlier studies by the researcher.

RESULTS AND DISCUSSION

Data: The field measurements yielded the following data as shown in Table 1.

Analysis of data: The angles showed some consistency and high degree of closeness (precision) to one another. This consistency further revealed the result of the pre-analysis design and the good working condition of the instrument used during the measurement.

Table 2 shows the deviations of the monthly means from the overall mean and it is clearly evident that January observations deviated least from the overall means as accuracy may be defined as the degree of closeness of observed value to the most probable value.

Analysis of graphs: Figure 1, which has the horizontal angles plotted against time of day reveals that midday is best for angular observations and free from refraction, while the early mornings and evenings are worst for the observations.

Also in Fig. 2, the angles were plotted against temperature; the Fig. 2 shows that midday readings were not affected by refraction as there was a steady value of angular measurement between 23.5 and 28.5°C.

Thirdly, two other classes of horizontal angles 74°01' and 40°24' measured with 87°17' series were all plotted together. The result is shown in Fig. 3. The 3 angles showed high degree of resemblance to the Fig. 1 thus, reinforcing the claim.

Determination of the refraction coefficient: To determine the refraction coefficient k, for Bauchi, we note that from Bomford (1980):

$$\Omega = k\theta \quad (1)$$

Where,

Ω = Angle of refraction at observation station

θ = Subtended angle at the center of earth, which is obtained from the relation

Table 1: Monthly means of field angular observations

Time of day	November	December	January	Mean
Morning	87°17'46.7"	87°17'51.4"	87°17'50.4"	87°17'49.50"
Afternoon	87°17'45.9"	87°17'51.7"	87°17'49.6"	87°17'49.06"
Evening	87°17'45.2"	87°17'52.4"	87°17'50.3"	87°17'49.30"
Mean	87°17'45.9'	87°17'51.8"	87° 17'50.3"	87°17'49.28"

The overall mean of the angles is derived as 87°17', 49.28"

Table 2: Deviations from the mean

November	December	January
+2.58"	-2.12"	-0.22"
+3.38"	-2.42"	+0.22"
+4.08"	-3.12"	-0.02"

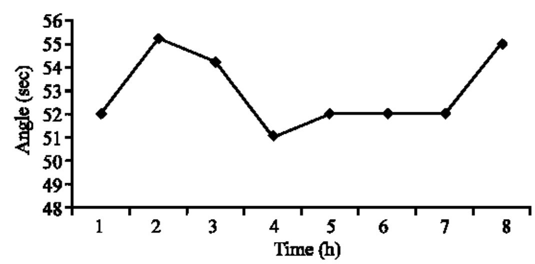

Fig. 1: Angle time graph of angular observations

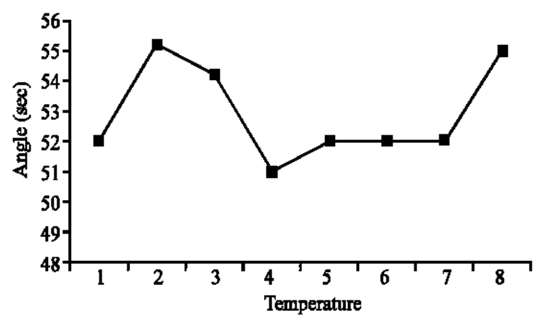

Fig. 2: Angle temperature graph

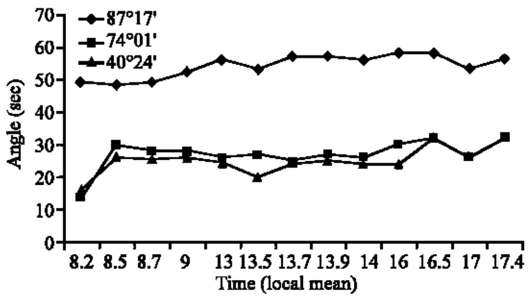

Fig. 3: Graph of the three horizontal angles against time of day

$$\theta = L/R \quad (2)$$

Where,

L = Ray path length (or measured distance to target mast)

R = Mean radius of earth. The measured distance to mast, L = 755.320 m; R = 6378135 m (parameters of geodetic reference system ellipsoid of 1980)

Therefore,

$$\theta = \frac{755.320}{6378135 \times \text{Sin}1''} = 24.4'' \quad (3)$$

If $\partial T/\partial h$ is a constant along the line of observation; where, terrain is relatively flat as the differential of temperature over height is small and constant.

But:

$$\Omega = \frac{L}{2\sigma} \equiv \frac{R\theta}{2\sigma} \quad (4)$$

where, σ is the radius of curvature of ray path.

Also by Bomford (1980):

$$\frac{1}{\sigma} = 16.3\frac{P}{T^2}(0.0342 + \frac{\partial T}{\partial h})\cos\beta \quad (5)$$

Where,

P = Pressure in millibar

T = Temperature in kelvin

β = Inclination of the ray path to the horizontal, for P = 710; T =2 8°C = 301^K

Therefore,

$$k = \frac{R}{2\sigma} \text{ and } \frac{\partial T}{\partial h} = -0.0055°\text{Cm}^{-1}$$

$$\therefore k = 252\frac{P}{T^2}(0.0342 + \frac{\partial T}{\partial h})$$

$$= 252 \times \frac{710}{301^2}\left[0.0342 + (-0.0055)\right]$$

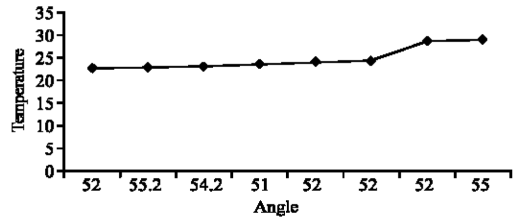

Fig. 4: Temperature angle graph

Therefore, the average angle of refraction Ω is

$$\Omega = k\theta$$
$$= 0.0567 \times 24.4''$$
$$= 1.38''$$

Statistical tests: The monthly means were considered for statistical test and analysis. The test of hypothesis at α = 0.05 confidence level showed that it is not necessary and enough to adopt a single mean of the angle measurements obtained in several months for computation of positions as the angles showed some significant differences (Fig. 4).

This has however, strengthened the aim of the study that there was need to get the 'best' time or period of observation of the horizontal angles 'free' from atmospheric refraction since the arithmetic mean was not enough estimate of the angles needed for position determinations using horizontal angles as components.

Furthermore, the monthly means X_1, X_2, X_3 and standard deviations δ_1, δ_2, δ_3 for 3 months of November, December and January were computed as:

$$X_1 = 87°17'45.9'' \quad X_2 = 87°17'51.5'' \quad X_3 = 87°17'50.3''$$
$$\delta_1 = 01.44'' \quad \delta_2 = 0.00'' \quad \delta_3 = 1.24''$$

where, N = 15 years and $\delta_1^2 = \delta_2^2 = \delta_3^2 = \delta$ be population variances.

From the 2 cases with degree of freedom as 28, a two-tailed hypothesis test was made. If we choose α = 0.05 confidence level and based on Ayeni (2001), we have:

$$S_P = \sqrt{\frac{(n_1 - 1)\delta_1^2 + (n_3 - 1)\delta_3^2}{n_1 + n_2 - 2}} = 1.343726163''$$

$$t = \frac{\left(\bar{X}_1 - \bar{X}_3\right) - (\mu_{\mu_3})}{S_P\sqrt{\frac{1}{n_1} + \frac{1}{n_3}}} = -8.97$$

Table 3: ANOVA

Source	Sum of squares	df	Mean squares	F-ratio
	BSS	V_1 = k-1	S^2_1 = BSS/k-1	
		= 3-1 = 2	= 10318.654	
	WSS	V = N-K	S^2_2 = WSS/N-K	
		= 9-3 = 6	= 40	
Total	TSS	N-1	S^2_3 = WSS/N-K	
			= 20640.010	

Table 4: ANOVA for orthogonal contrast

Source	Sum of squares	df	Mean squares	Computed ratio	Table F-ratio
SS	-8.85	2	-8.85	-0.0008	2.6
H: $C_1 = 0$	-	-	-	-	5.14
SS	-6.60	2	-6.60	-0.006	2.6
H: $C_2 = 0$	-	-	-	-	5.14
SS	2.25	2	2.25	+0.002	2.6
H: $C_3 = 0$	-	-	-	-	5.14
WSS	2.602	6	0.40	-	-

From the statistical table (Neave, 1978):

$$t_{0.05}(28) = -2.05$$

From these results, we reject H_0 and accept H_1 as $\mu_1 \neq \mu_3$ and therefore, it was significant. Based on this evidence therefore, it is believed that there is a better month (refraction-free wise) or time of day suitable for angular observation as there are some significant differences in the monthly means even when projected to 50 as observations in Fig. 1 and 2 have shown.

In addition, the result of the Analysis of Variance (ANOVA) Table 3 and the ANOVA Table 4 for the orthogonal contrast; using the Bartlett's test as illustrated by Ayeni (2001) was based on statistic whose sampling distribution provides exact critical values when the sampling sizes are equal has Bartlett's statistic B computed as -0.841. Since B <5.99, we accept: H_0: $\sigma^2_1 = \sigma^2_2 = \sigma^2_3$.

CONCLUSION

Therefore, there is homogeneity of population variance from the three samples. Further tests into the morning, afternoon and evening observations revealed results similar to the above, thus strengthening the earlier view and result.

- From the foregoing, therefore, the coefficient of refraction for Bauchi angular observations was computed as k = 0.0567. This value of k is necessary in quantifying refraction error, which can then through a least square adjustment scheme be modeled out of the readings
- Since, refraction error is additive, therefore, for any horizontal angle measured in Bauchi at a distance of 3 km and above must be subtracted by 1.5. This is for daytime measurements only
- Only the arithmetic mean of angular observations over several months and at any time of day should not be used for position determination or any other measurement requiring a high degree of accuracy
- For Bauchi, the best periods for angular measurements using glass arc Theodolites are between 12:00 noon to 4:00 pm and the month for such observation is January.

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