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On the Control and Eradication Strategies of Mathematical Models of the Tuberculosis in A Community

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Abstract: In this study the stability of equilibrium states of a model in an attempt to suggest control and eradication strategies to minimize the number of infectious tuberculosis (TB) cases in a Community. The model exhibits two equilibria namely, the disease-free and the endemic equilibrium. The model analyses show that combination of both treatment and preventive method can eradicate TB cases.

Key words: Equilibrium states, free equilibrium, endemic equilibrium, tuberculosis, treatment and preventive, control and eradication

INTRODUCTION

In Low income Countries, Tuberculosis (TB) has caused a great deal of ill health in their Population. Tuberculosis had been known from the time immemorial and has been present in the human population since 2400BC.

Tuberculosis is transmitted from one person to another by droplet spray. Tuberculosis can be spread just by sneezing, coughing, laughing or singing by some one with TB disease for the infection to take place. A patient with infectious tuberculosis, when he/she coughs out tuberculosis droplet (s) nuclei into the air, the droplet (s) nuclei are then inhale into the respiratory tract of another person. The likelihood of becoming infected with TB is related to the likelihood of coming in contact with the *bacillus* causing tuberculosis. Exposure to the TB is most likely when there is a high concentration of bacilli in the air. The *mycobacterium* or *bacillus* then starts to grow. If the organism grows, they cause a small area of *bronchopneumonia*, the organism then spread to the lymph node in the chest. They then carried by the lymph into the blood. The only infectious tuberculosis patients are those with Pulmonary tuberculosis PTB whom the sputum contains acid fast bacilli visible on direct microscopy of sputum (Louden *et al*., 1954).

In the Olden days, the infectious tuberculosis patients used to be isolated and then the traditional healers prepared local medicine to be taken by the patients. This is still practice in some remote areas these days. Evidence has shown that the local medicine proved ineffective in most cases and as a result many patients have died. Many researchers have put up mathematical models after the out break of tuberculosis, notably among them are Anderson *et al*., (1988) Anderson and May (1991), Blower *et al.* (1995, 1996), Ernason (1994), Ernason *et al.* (1996), Louden *et al*., (1954), Frost (1937), Ferebee (1970), Ssenoga (1998) and Adetunde (2008) to mention few. In Blower *et al.* (1996) discussed control and eradication of TB strategies using a simple model under which they gave a theoretical frame work for designing effective control strategies of TB.

In this study we attempt to study the stability of equilibrium states of the model in Adetunde (2008).

THE MATHEMATICAL MODEL

Variables and parameters: We use the following notations and/or definitions for the human populations.

Susceptibless S (t): The number of individuals who can be infected but have not yet contracted the tuberculosis but may contract it if exposed to any mode of its transmission.

Infectives I (t): The number of individuals who have contracted the tuberculosis and are actively or capable of transmitting it.

Latent l (t): The number of individuals who, although apparently healthy themselves, harbour infection which can be transmitted to others:

Recoveries R (t): The number of individuals who are recovered after treatment and are immune to the disease.

Assuming in the give population at time say t, S (t), I (t), C (t) and R (t) denote susceptible, infective carriers and recoveries, respectively.

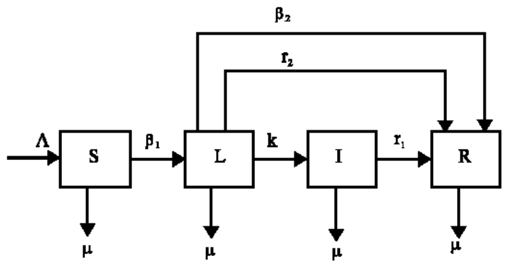

Fig. 1: A compartmental diagram for one strain-model of tuberculosis

We define the following parameters as follows:

- Λ : The recruitment rate.
- μ : The per capital natural mortality rate.
- d : The tuberculosis-induced mortality rate.
- β_1 : The probability that a susceptible individual becomes infected by one infectious individual per contact per unit time.
- β_2 : The probability that a recover individual becomes infected by one infectious individual per contact per unit time.
- k : The rate of progression to active tuberculosis.
- r_2 : The recovery rate of the latent class.
- r_1 : The recovery rate of the infectious class and
- c : The per capital contact rate.

Assumptions of the model: The following assumptions are taken into the consideration in the construction of the model:

- All the people in the community are likely to be infectious individual in case of contact.
- All immigrants and new born in the community are uninfected, hence they join the susceptible group.
- The community has a fixed area size and only the population size will be varying.
- The population size is N = s (t) + I (t) + R (t).

Compartmentalization of the model: The dynamic of a one strain model of tuberculosis can be described as in the compartmental model Fig. 1.

Equations of the model: In view of the above assumptions and inter-relations between the variables and parameters as described in the compartmentalized model in Fig.1, we have the following system of differential Equations:

$$\frac{dS}{dt} = \lambda - \mu S - \beta_1 CS \frac{1}{A} \quad (1)$$

$$\frac{dL}{dt} = \beta_1 CS \frac{1}{A} + \beta_2 CR \frac{1}{A} - (\mu + K + r_2) L \quad (2)$$

$$\frac{dI}{dt} = KL - (\mu + d + r_1) I \quad (3)$$

$$\frac{dR}{dt} = r_1 I + r_2 L - \mu R - \beta_2 CR \frac{1}{A} \quad (4)$$

ANALYSIS OF THE MODEL

In this study, we present the results of stability analysis of the equilibrium point.

Equilibrium of the model: The governing system of equations of the model-(i.e. Eq. 1-4) has two non-negative equilibrium points namely:

Disease free equilibrium:

$$E_0\left(\lambda/\mu, 0, 0, 0\right)$$

the disease-free equilibrium; Here we have I*= L*= R* = 0 and we define

$$S_0 = \lambda/\mu$$

as the asymptotic carrying capacity of the population.

Conditions for existence of S*: The following conditions must be noted for the conditions for the existence of S*:

- The susceptibles can not have a non-trivial equilibrium Population i.e Λ ≠ 0, when Λ = 0 this means that susceptible population goes to extinction.
- The susceptibles will have an equilibrium value when there is recruitment, i.e when Λ > 0.

Endemic equilibrium: E* (S*, L*, I*, R*) is the endemic equilibrium for the system of Eq. 1-4.

Stability of the equilibra: Now to determine the stability of E0 and E*, the following variational Jacobian matrices are computed corresponding to equilibrium points E_0 and E*:

$$M_0 = \begin{pmatrix} -\mu & 0 & -\beta_1\left(\frac{\lambda/\mu}{A}\right) & 0 \\ 0 & -(\mu + k + r_2) & \beta_1\left(\frac{\lambda/\mu}{A}\right) & 0 \\ 0 & k & -(\mu + d + r_1) & 0 \\ 0 & r_2 & r_1 & -\mu \end{pmatrix}$$

$$M^* = \begin{pmatrix} -\left(\mu+\beta_1 c\frac{I^*}{A}\right) & 0 & -\beta_1\frac{S^*}{A} & 0 \\ \beta_1 c\frac{I^*}{A} & -(\mu+k+r_2) & \beta_1 c\frac{S^*}{A}+\beta_2 c\frac{R^*}{A} & \beta_2 c\frac{I^*}{A} \\ 0 & k & -(\mu+d+r_1) & 0 \\ 0 & r_2 & r_1-\beta_2 c\frac{R^*}{A} & -\mu-\beta_2 c\frac{I^*}{A} \end{pmatrix}$$

From M_0 it is clear that E_0 is asymptotically stable provided

$$\frac{A}{\lambda/\mu} > \left(\frac{k}{\mu+k+r_2}\right)\left(\frac{\beta_1 c}{\mu+d+r_1}\right) \text{ i.e., } R_0 > 1,$$

the disease dies out and wipe out under condition the equilibrium E* does not exist. If $R_0 > 1$, then E* exists and the infection is maintained in the population; hence to minimize the disease the population of the infections and latently infected must be very small, this implies that

$$\frac{dL}{dt} < 0$$

and

$$\frac{dI}{dt} < 0$$

With these conditions we can minimize the disease if and only if

$$A > \left(\frac{\beta_1 cS+\beta_2 cT}{\mu+k+r_2}\right)\left(\frac{k}{\mu+d+r_1}\right)$$

CONCLUSION

In the study, we carry out analytical method of determine the population equilibriumof a community having TB, with reference to a disease free and endemic equilibrium. Further more, we investigated the conditions of existence and stability of the population equilibrium. By analyzing the model, we have found a threshold parameter Ro;. It is noted that when Ro < 1 then the epidemic will die out and when Ro > 1 the disease will persist in the population and become endemic. The model has two non-negative equilibria namely

$$E_o\left(\lambda/\mu, 0, 0, 0\right)$$

the disease free equilibrium are E* (S*, L*, I*, T*) the endemic equilibrium. Conclusively, it was observed that the non-trivial equilibrium is unstable. This indicates that it is very possible to control, or minimize and eradicate the TB in a Community. This can be achieve if the following recommendations are strictly adhere to:

- Increasing the community awareness on the modes of transmission of tuberculosis, transmission reduction practices and the home care strategies for tuberculosis patients.
- Discouraging overcrowding through restricting recruitment of individual into area and increasing the size of the area occupied by extending boundaries and encouraging emigration.
- Encouraging the medical personnel to keep their medical ethics and give the required treatment to patients this will help in quickening the recovery rate.
- People need to be equipped with abilities to recognize the signs and symptoms of tuberculosis and the relationship of tuberculosis and HIV/AIDS among others. So as to encourage them to go for TB test.
- Patients have to know the dangers with incomplete drug dosage, as a result we need to encourage them to complete their drug intake. this will help in improving on the recovery rate as well as reducing on the rate to of infectious tuberculosis.
- There should be high level of treatment for the people in a community with TB endemic.
- There should be high priority for all group (s), organization (s) and individual (s) who provide healthcare to include TB services as part of the health services they provided.
- The use of preventive Chemotherapy should be taken serious in every community with TB.
- Government should give it a priority to the allocation of the limited resources to the organization/body whose activities are aimed to fight against TB.
- Since TB is an opportunistic disease accompanying the uppression of the Immune-system by HIV infection all HIV positive individuals should be given

Preventive Chemotherapy so as to protect them from TB infection or prevent their Latent TB infections from progressing to TB.

- Public should be educated that TB is a curable and preventable disease, so that People with symptoms suggestion of TB can willingly report to medical centers to seek medical attention.
- Ministry of Health should sensitize the public, through the National TB Programmes and other NGO's who provide health care services about TB.

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