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A New Technique for Analysis of Extreme Rainfall for Nigeria

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Abstract: Experience and studies confirm that short duration rainfall of less than 2 or 3 h are of special significance for flood studies particularly with roads, highways and urban settlements in the high rainfall tropical regions of the world. The situation is compounded for Nigeria and some developing nations lying in the tropical zone where intense annual rainfall is high but scarcity of reliable hydrological data for planning and design of drainage works is scarce. For these peculiar situations, the need arises for development and use of statistical tools capable of extracting the maximum possible information content from data that are generally seen to be inadequate. In this study, rainfall Intensity-Duration-Frequency (IDF) studies were undertaken for Nigeria using annual extreme rainfall series available for 35 meteorological stations. In order to obtain floods of high return period with a good level of confidence that is needed in engineering practice, the whole country was grouped into 10 rainfall zones. The analysis was undertaken using the Extreme-Value Type 1 (EVI) distribution. For comparison purposes, the parameters of the (EVI) distribution were evaluated using the new concept of Maximum Entropy (ME) and compared with the results of the estimates obtained by the widely recommended Maximum Likelihood (ML) technique. The superiority of the new concept of Maximum Entropy (ME) for evaluation of the parameters of the EVI distribution is demonstrated in terms of its greater efficiency in the extraction of maximum possible information content from each group of data as well as its relative economic use of computer time. The IDF curves generated for different rainfall zones in Nigeria by the new method of Maximum Entropy (ME) are illustrated graphically.

Key words: Technique, extreme rainfall, intensity, frequency, distribution, Nigeria

INTRODUCTION

Nigeria with a total land surface of over 9000,000 km^2 lies roughly between 3° and 15° East of Greenwich and 4° and 14° North of the equator. Three basic types of rainfall are experienced in Nigeria. These are:

- Conventional rainfall due to excessive heating which generates vertical air currents. The belt of conventional rainfall is east to west following the overhead sun and shifting north and south across the whole country. Conventional rainfall is more pronounced in the coastal region where it is experienced all the year round.
- Orographic rainfall caused by highlands forming barriers in the path of moisture laden winds forcing the later to rise and in so doing to drop their moisture content in the form of rain. This type of rainfall is experienced in the hilly regions of the country, as for example in the windward sides of Jos, Obudu and Adamawa plateau. These 3 regions are shown in Fig. 1 as zones VIIb, Va and Vb, respectively.
- Frontal rainfall caused by frontal currents made up of cold air which undercuts warm wet lighter air causing rain. Frontal rainfall is associated with the inter-tropical front and line squalls.

The first comprehensive non-empirical IDF studies for Nigeria with a return period of 50 or more years was undertaken by Oyebande (1982). In those studies, the investigator employed the Maximum Likelihood technique to estimate the parameters of the EVI distribution for each of the rainfall durations. He also divided Nigeria into 10 principal rainfall zones with sub-zones as shown in Fig. 1. In deriving the rainfall zones which were based on topographic and climatic factors, Oyebande (1982) identified the IDF regimes as follows:

- Rainfall IDF regime characteristised by three indices namely, the ratio of:
  - The 10 year 10 min fall to that of 10 year 30 min fall.
  - 2 year 15 min fall to 2 year 60 min fall
  - 25 year 1 h fall to 25 year 24 h fall.

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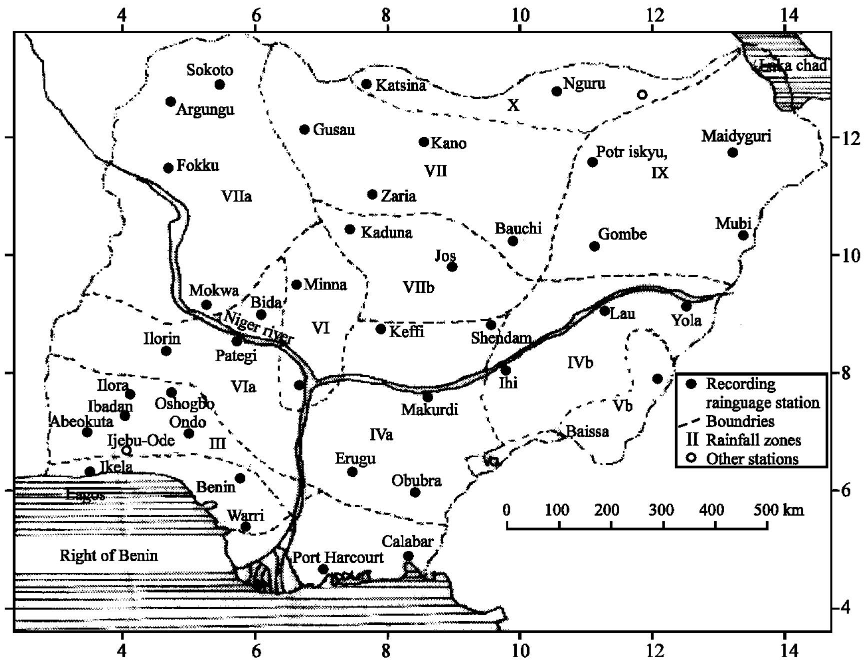

Fig. 1: The ten principal rainfall zones of Nigeria

Table 1: Ranges of characteristics of rainfall zones

Zone	A (%)	B (%)	C (%)	Altitude (M)	Annual mean rainfall (mm)	Annual mean no. of raindays
I	41-46	47-90	43-49	80	2150	170
II	50-54	50-55	50-65	80	1550-2900	125-190
III	52-54	56-57	52-57	225-305	1217-1600	106-151
IVa	48-49	54-55	54-60	113-307	1224-1800	94-109
IVb	50-51	52-54	54-57	150-190	900-1150	75-90
V(a and b)	-	-	-	460-2409	1400-3670	-
VI	40-44	50-52	50-52	63-260	1190-1320	98-101
VIIa	59-67	59-67	53-66	119-351	710-1070	58-55
VIIb	59-63	63-67	56-61	645-1285	1218-1400	108-126
VIII	46-48	50-56	56-60	460-750	840-1085	50-82
IX	45-53	58-59	74-83	350-415	651-776	62-64
X	65-69	52-73	68-72	325-520	525-600	49-61

A = 100 * (10 -year, 10 min. fall/10-year 30 min. fall); B = 100* (2-year, 15 min. fall/2-year 60 min. fall); C = 100 * (25-year, 1 h. fall/25-year 24 h. fall)

Table 2: Derived zonal rainfall records

Zone	Station names	No. of stations	Length of period (Station years)	Period
I	Port Harcourt, Calabar, Umudike	3	45	1951-78
II	Warri, Lagos, Ikeja, Oshodi, Benin, Aero and NIFOR	6	115	1948-78
III	Oshogbo, Ondo, Ilora, Ibadan	4	51	1956-75
IVa	Enugu, Ilorin, Bida, Makurdi	4	82	1956-78
IVb	Ibi, Yola	2	22	1956-73
V(a and b)	-	-	-	-
VI	Lokoja, Minna	2	34	1956-75
VIIa	Mokwa, Yelwa, Sokoto	3	35	1956-78
VIIb	Kaduna, Jos, Aero, Lamingo	3	60	1956-78
VIII	Bauchi, Zaria, Kano, Gusau, Samaru	5	61	1951-75
IX	Potiskum, Maiduguri, Aero and Water Works.	3	38	1956-74
X	Nguru, Katsina	2	20	1960-75
	Total	37		

- Topographical characteristic as expressed by the altitude of the station, in metres above the mean sea level.
- The average rainfall pattern of regime at the station as characterised by mean annual rainfall and mean number of rainy days per year. A summary of the criteria used to derive the zones are shown in Table 1 while the derived zones together with the zonal records are shown in Table 2.

The inadequacy of data on extreme rainfall in Nigeria as in several other regions of the world also makes imperative the search for techniques that can efficiently extract the maximum possible information content from a set of inadequate hydrological data. Several techniques are available in literature for estimating the parameters of the EVI distribution. These include the method of moments, the Maximum Likelihood Method (NERC, 1975) and others due to Kimball (1946) and Gumbel (1958). Unfortunately the different methods, in general, produce different estimates and confidence intervals. Estimates of the parameters as given by the method of moments are based on the sample mean and variance. These estimates are therefore inaccurate when the sample size is small. Jowitt (1979) has demonstrated the concept of Maximum Entropy (ME) as an alternative useful technique for estimating the parameters of the EVI distribution. He further showed that even the more widely accepted Maximum Likelihood Method (ML). provides the “correct” or unique estimates only when the sample size is large. For moderate sample sizes as are being used in this study, Jowit (1979) has used Kimball’s (1946) concept of “sufficient statistical estimation functions” to deduce that the parameters of the EVI distribution obtained by the concept of Maximum Entropy are better estimates than those obtained by the Maximum Likelihood (ML). More recent attempts to use the principle of Maximum Entropy (ME) to evaluate the parameters of other statistical distributions has been published by Oyegoke *et al.* (1983) and Singh (1997).

MATERIALS AND METHODS

The Extreme Value type 1 (EVI) distribution: For a random variable x, unrestricted in sign, the EVI distribution is given by the expression:

$$f(x) = \alpha^{-1} \exp[-\{(x - u)/\alpha\} - \exp\{- (x-u)/\alpha\}] \quad (1)$$

Where, u and α are the parameters of the distribution. The cumulative distribution function of the EVI distribution is given as:

$$F(x) = \exp [-\exp \{-(x-u)/\alpha\}] \quad (2)$$

Where, F(x) is the probability of an event not exceeding x. If x, u and α are expressed in terms of a reduced variate y such that

$$x = u+\alpha y \quad (3)$$

Equation 2 becomes:

$$F(x) = \exp [-(\exp-y)] \quad (4)$$

By further defining the return period T as the reciprocal of the exceedance probability, the reduced variable y can be expressed as:

$$y = -\log_e [\log e (T/ (T-1))]\ y = - \log_e [\log_e (T/ (T-1)] \quad (5)$$

The EVI distribution and the principle of maximum entropy: The expression for the entropy function, H, of a discrete probability function p (x_i) was defined by Shannon (1948) as:

$$H = -k\Sigma p (x_i) \log_e p (x_i) \quad (6)$$

Sonuga (1972) introduced the concept of Maximum Entropy to hydrologic frequency analysis. With Eq. 6 as the base he obtained the minimally biased probability distribution of x consistent with the information that the mean, x and standard deviation s, are known, as follows:

$$p (x_i/ x, s) = \exp (-a_0 - a_1x_1 - a_2x_i^2 \ldots\ldots) \quad (7)$$

Where, $a_{0,}$ a_1 and a_2 are the Lagrangian multipliers associated with the normality and main constraints, respectively.

Equation 7 can simply be written as:

$$p (xi/ x, s) = \exp [-a_0 - \Sigma a^m x^m_i] \quad (8)$$

Building on the work of Sonuga (1972) and Jowitt (1979) undertook rigorous analysis that deduced the EVI distribution from the principle of Maximum Entropy, when the given information relating to a random variable for unrestricted sense consists solely of the first two moments.

$$\frac{E (x-u)}{\alpha} \text{ and } \frac{E [\exp (- x-u)}{\alpha}$$

Such that the choice of u and α assume the properties of the EVI distribution which are mainly

$$\frac{E (x - u)}{\alpha} = 0.5772 \quad (9)$$

and

$$\frac{E (\exp- x - u)}{\alpha} = 1 \quad (10)$$

Where, E (.) is the expectation operator.

At this stage, the procedure for obtaining the parameters u and α of the EVI distribution by the Maximum Entropy method can be outlined.

The EVI distribution and the principle of maximum entropy: Oyegoke and Sonuga (1983) outlined the computational procedure for the use of the principle of Maximum Entropy as follows:

Step 1: Obtain initial estimates α_0 and u_0 using the simpler method of moments such that:

$$\alpha_0 = (\sqrt{6} / \prod)\ [\ \Sigma\,(xi - x\,)^2 / N\ ]^{1/2} = (\sqrt{6}/\prod)\ S \quad (11)$$

and

$$u_0 = x - 0.5772\alpha_0 \quad (12)$$

Where, mean

$$x = 1/N\sum_{i=1}^{N} x_i \quad (13)$$

and standard deviation

$$s = \sqrt{[\Sigma(x_i - x)^2 / (N\text{-}1)]} \quad (14)$$

Step 2: Transform to a new variate z such that:

$$z_i = (x_i - u_0\,)/\alpha_0 \quad (15)$$

and obtain sample moments $\overline{z}$ and $\overline{\epsilon}_z$ where

$$\overline{z} = 1/N\sum_{i=1}^{N} z_i \quad (16)$$

and

$$\overline{\varepsilon_z} = 1/N\sum_{i=1}^{N}\exp(-z_i) \quad (17)$$

Step 3: At this stage values of u and α are required satisfying two conditions viz:

$$\overline{z} = 0.5772 \quad (18)$$

and

$$\overline{\varepsilon_z} = 1 \quad (19)$$

This can be accomplished by introducing two new variables ν and β such that

$$z = 0.5772\beta + \nu \quad (20)$$

and

$$\log_e \varepsilon_z = -\nu + 0.4228(\beta - 1) \quad (21)$$

The values for β and ν are now used to obtain new estimates of α and u such as:

$$\alpha_1 = \alpha_0\beta \quad (22)$$

and

$$u_1 = u_0 + \alpha_0\nu \quad (23)$$

hence, new values of z are obtained such that

$$z_i = (x_i - u_1)/\alpha_1 \quad (24)$$

The process from Eq. 14-23 is repeated until the values of ν and β are sufficiently close to zero and unity, respectively.

Algorithm for the determination of u and α by the maximum likelihood method: This is readily available in literature as for example Clarke (1973) and Baghirathan and Shaw (1978). The procedure begins also with using the simpler method of moments to provide the initial estimates of α and u as given by Eq. 11 and 12 after which new estimates of α and u are obtained using Eq. 25 and 26 stated as follows:

$$\alpha_k = x - (\sum_{i=1}^{N} x_i \exp(-x_i/\alpha_{k\text{-}1})/(\sum_{i=1}^{N} x_i \exp(-x_i/\alpha_{k\text{-}1}) \quad (25)$$

and

$$u_k = -\alpha_{k\text{-}1}\ \text{loge}\ [\sum_{i=1}^{N} x_i \exp(-x_i/\alpha_{k\text{-}1})/N] \quad (26)$$

Updated values of u and α are obtained through an iterative procedure until the values obtained by successive iterations are practically the same.

DISCUSSION

The results obtained through the investigations outlined above are shown in Table 3-6 where the parameters of the EVI distribution are shown tabulated for each of the principal rainfall zones of Nigeria. The numbers of computational iterations necessary for accurate results by the two methods employed are also shown in the Table 3-6 for comparison purposes. The new technique employing the Maximum Entropy method (M.E.M) converges faster to a unique result and the computation time it uses for each iteration is less than half the time necessary for each iteration by the Maximum Likelihood (M.L.M). Hence, in using this relatively new

Table 3: A comparison of the number of Iterations necessary to yield accurate results by the Maximum Entropy Method (M.E.M) and Maximum Likelihood Method (M.L.M) for zones I, II and III

			Maximum entropy method			Maximum likelihood method		
Zone	Duration in hour	Sample size N	Iterations I	Value of α	Value of u	Iterations I	Value of α	Value of u
I	0.2	45	2	24.67	118.43	6	23.66	118.05
	0.4	45	3	19.34	98.48	7	19.08	98.38
	1.0	45	2	12.3	60.25	5	12.19	60.21
	3.0	45	3	6.64	26.26	5	6.7	26.29
	6.0	45	3	3.66	14.44	5	3.75	14.48
	12.0	45	2	1.84	7.63	8	1.86	7.64
	24.0	45	3	0.95	4.02	3	0.98	4.03
II	0.2	101	3	25.5	114.71	6	23.04	114.97
	0.4	101	5	19.48	88.05	6	20.49	88.47
	1.0	101	3	13.03	55.09	7	13.23	55.08
	3.0	101	3	6.59	23.64	5	6.78	23.83
	6.0	101	2	3.85	13.15	5	3.88	13.17
	12.0	101	1	2.21	6.96	4	2.23	6.97
	24.0	101	2	1.32	3.82	5	1.32	3.83
III	0.2	51	3	18.69	108.29	6	19.59	108.73
	0.4	51	2	14.95	78.27	6	15.17	78.37
	1.0	34	3	12.14	43.11	6	12.59	43.32
	3.0	51	2	5.49	17.39	3	5.58	17.43
	6.0	51	2	2.91	9.53	3	2.97	9.56
	12.0	51	2	1.48	4.87	3	1.49	4.85
	24.0	51	2	0.88	2.62	3	0.85	2.61

Table 4: A comparison of the number of Iterations necessary to yield accurate results by the Maximum Entropy Method (M.E.M) and Maximum Likelihood Method (M.L.M) for zones IVa, IVb and VI

			Maximum entropy method			Maximum likelihood method		
Zone	Duration in hour	Sample size N	Iterations I	Value of α	Value of u	Iterations I	Value of α	Value of u
IV(a)	0.2	62	3	21.89	108.21	3	21.97	108.15
	0.4	62	5	20.28	78.53	9	22.29	79.71
	1.0	63	4	12.64	46.52	6	12.96	46.69
	3.0	64	3	5.47	18.53	6	5.6	18.59
	6.0	62	3	3.11	9.95	3	3.19	9.99
	12.0	62	1	1.67	5.37	3	1.67	5.37
	24.0	62	1	0.93	2.66	3	0.93	2.66
IV(b)	0.2	22	7	41.1	93.94	12	46.03	97.06
	0.4	22	4	18.81	75.65	9	20.06	76.3
	1.0	22	3	12.06	46.13	6	12.62	46.41
	3.0	22	2	6.06	17.91	6	6.29	18.02
	6.0	22	3	3.4	10.19	6	3.59	10.31
	12.0	22	3	1.74	5.15	6	1.84	5.2
	24.0	22	3	0.88	2.61	6	0.93	2.64
IV	0.20	33	3	25.88	105.67	6	24.78	105.22
	0.40	34	1	22.44	77.37	6	22.49	77.39
	1.0	34	3	15.19	46.97	3	14.52	46.74
	3.0	34	1	6.01	19.94	3	5.96	19.92
	6.0	34	1	3.32	10.46	3	3.25	10.43
	12.0	34	1	2.36	5.01	3	2.36	5.01
	24.0	34	2	0.995	2.86	3	1.03	2.79

Table 5: A comparison of the number of iterations necessary to yield accurate results by the Maximum Entropy Method (M.E.M) and Maximum Likelihood Method (M.L.M) for zones VIIA, VIIB, and VIII

			Maximum entropy method			Maximum likelihood method		
Zone	Duration in hour	Sample size N	Iterations I	Value of α	Value of u	Iterations I	Value of α	Value of u
VII(a)	0.2	35	3	30.33	108.6	6	28.61	107.95
	0.4	35	2	18.62	71.14	6	18.79	71.21
	1.0	35	2	10.06	38.22	6	10.4	38.38
	3.0	35	3	4.88	13.7	6	5.25	13.9
	6.0	35	1	2.08	8.03	3	2.04	8.01
	12.0	35	1	1.13	4.25	3	1.10	4.24
	24.0	35	3	0.53	2.17	3	0.51	2.16

Continue Table 5

			Maximum entropy method			Maximum likelihood method		
Zone	Duration in hour	Sample size N	Iterations I	Value of α	Value of u	Iterations I	Value of α	Value of u
VII(b)	0.2	60	3	23.79	100.2	3	23.3	99.97
	0.4	60	2	17.76	72.62	3	17.96	72.71
	1.0	60	1	9.87	39.81	3	9.82	39.79
	3.0	60	1	3.89	15.73	3	3.87	15.72
	6.0	60	1	2.17	8.57	3	2.14	8.56
	12.0	60	1	1.14	4.48	3	1.14	4.49
	24.0	59	3	0.66	2.41	6	0.69	2.43
VIII	0.2	60	2	17.7	94.9	6	18.21	95.14
	0.4	60	4	16.96	67.98	6	17.99	68.51
	1.0	60	1	10.89	40.84	3	10.77	40.91
	3.0	60	3	4.89	15.97	3	4.81	15.94
	6.0	60	3	2.78	8.5	6	2.73	8.48
	12.0	60	2	1.47	4.43	3	1.47	4.43
	24.0	59	2	0.8	2.29	6	0.79	2.28

Table 6: A comparison of the number of Iterations necessary to yield accurate results by the Maximum Entropy Method (M.E.M) and Maximum Likelihood Method (M.L.M) for zones IX and X

			Maximum entropy method			Maximum likelihood method		
Zone	Duration in hour	Sample size N	Iterations I	Value of α	Value of u	Iterations I	Value of α	Value of u
IX	0.2	38	3	26.29	98.66	6	23.5	98.37
	0.4	38	3	22.5	73.35	6	22.67	73.48
	1.0	38	2	11.78	41.16	3	11.74	41.14
	3.0	38	4	4.44	15.74	6	4.59	15.81
	6.0	38	3	2.64	8.58	6	2.72	8.61
	12.0	38	3	1.37	4.35	6	1.41	4.37
	24.0	38	3	0.72	2.21	3	0.75	2.22
X	0.2	21	3	22.69	100.72	3	23.04	100.87
	0.4	21	4	20.17	65.62	6	21.09	66.05
	1.0	21	7	14.12	30.69	12	15.83	31.77
	3.0	21	2	3.8	12.38	6	3.89	12.43
	6.0	21	2	2.05	6.51	3	2.11	6.53
	12.0	21	2	1.07	3.3	3	1.1	3.31
	24.0	21	3	0.57	1.7	3	0.59	1.71

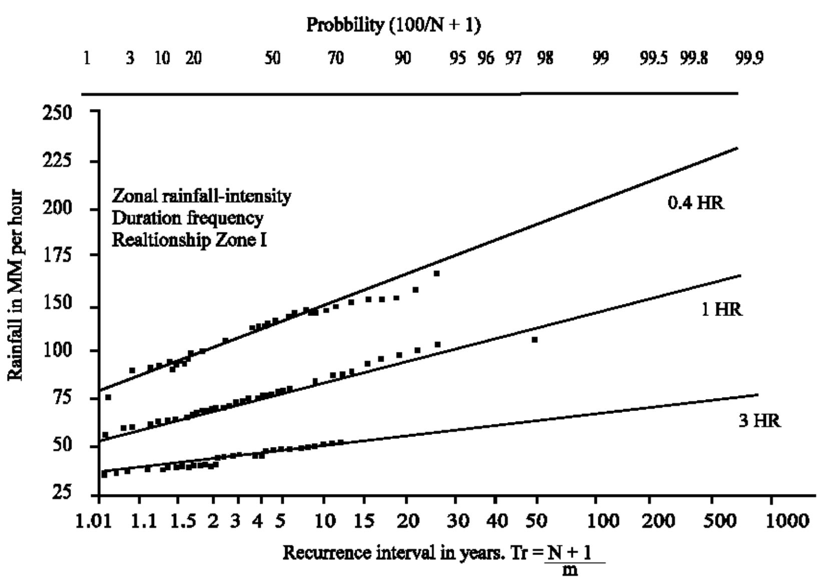

Fig. 2: Reginal intensity-duration-frequency relation-ships for zone I by the Maximum Entropy Method (M.E.M)

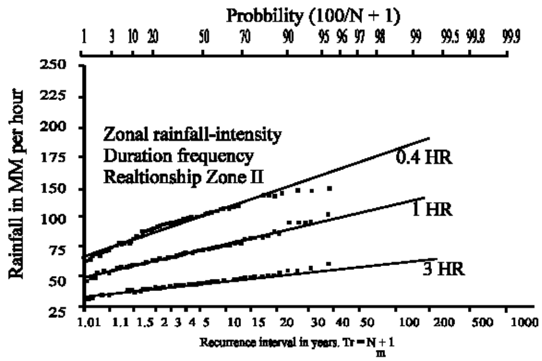

Fig. 3: Reginal intensity-duration-frequency relationships for zone II by the Maximum Entropy Method (M.E.M)

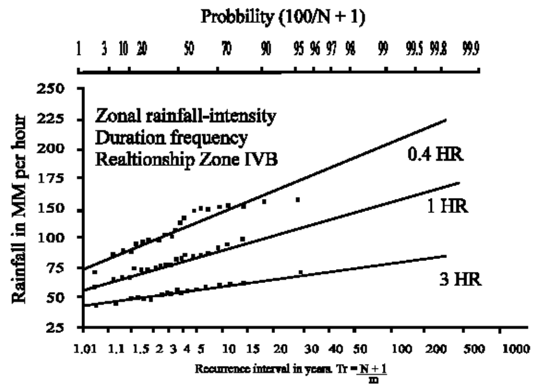

Fig. 6: Reginal intensity-duration-frequency relationships for zone IVB by the Maximum Entropy Method (M.E.M)

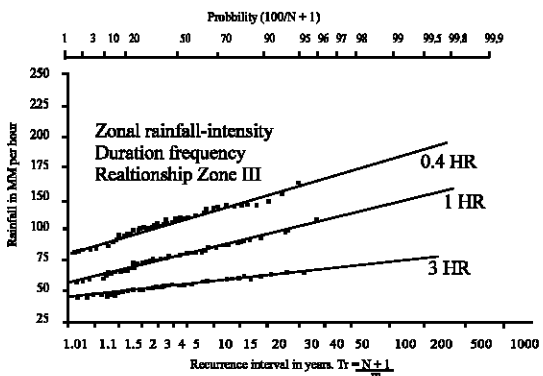

Fig. 4: Reginal intensity-duration-frequency relationships for zone III by the Maximum Entropy Method (M.E.M)

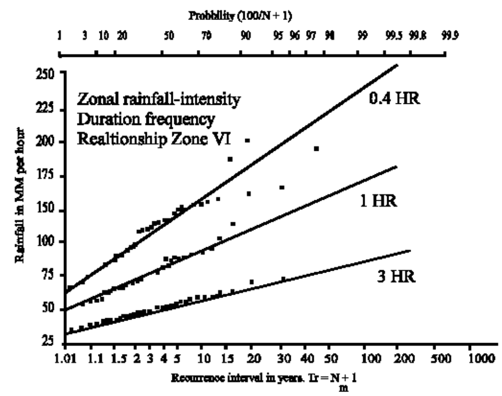

Fig. 7: Reginal intensity-duration-frequency relationships for zone VI by the Maximum Entropy Method (M.E.M)

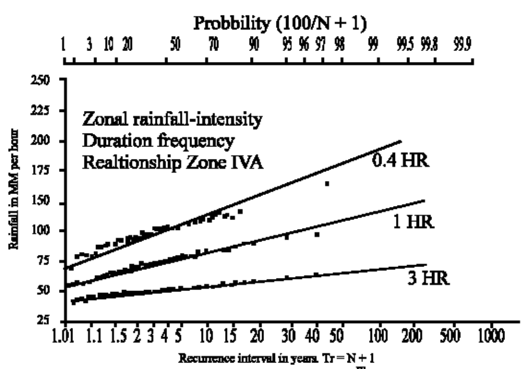

Fig. 5: Reginal intensity-duration-frequency relationships for zone IVA by the Maximum Entropy Method (M.E.M)

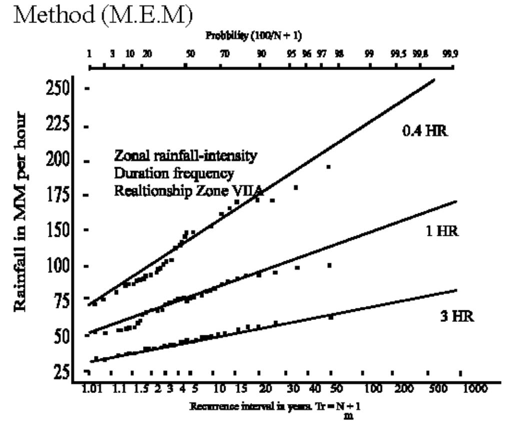

Fig. 8: Reginal intensity-duration-frequency relationships for zone VIIA by the Maximum Entropy Method (M.E.M)

parameter estimation technique, considerable savings can be made in the computation time. The results obtained are also shown graphically in Fig. 2-11 where the straight-line graph represents the theoretically fitted Gumbel distribution, while the dotted points about this line represent the historical data. With these results, Intensity-

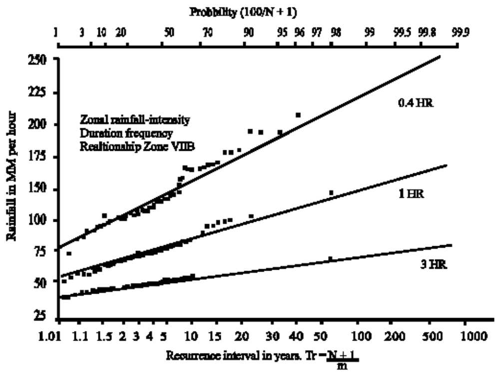

Fig. 9: Reginal intensity-duration-frequency relationships for zone VIIB by the Maximum Entropy Method (M.E.M)

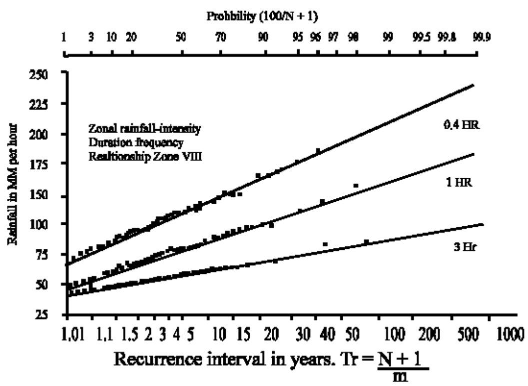

Fig. 10: Reginal intensity-duration-frequency relationships for zone VIII by the Maximum Entropy Method (M.E.M)

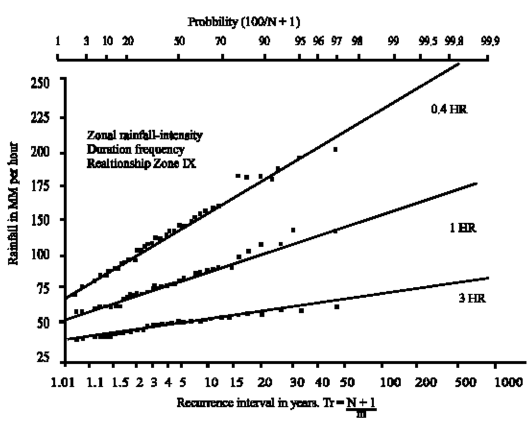

Fig. 11: Reginal intensity-duration-frequency relationships for zone IX by the Maximum Entropy Method (M.E.M)

Duration-frequency relationships with a return period of 50 or more years can be obtained for any part of Nigeria with some confidence.

CONCLUSION

This study, therefore, illustrates the emerging concept of Maximum Entropy as a powerful alternative technique for estimating the parameters of the Gumbel EVI distribution.

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