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An Application of Certain Third Order Difference Equation in Image Enhancement

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Abstract: Human life is bound to limits of time. Ageing is an inevitable process conditioned in linearity of time. Face is the mirror of this ageing process. The tools of image enhancement play a significant role through effective filtering techniques. So far, no researchers can find a method to develop image enhancement from difference equations. This research study illustrates a specific technique to remove speckle noise in any digital image using the solutions of difference equations. Here we propose a novel model to remove ageing marks from the face images. We bring out a series of mask from the oscillatory solution of a certain third order difference equation. This, for specific values, helps us to remove noise in a different way. Illustrations through common face outlook and ageing are considered.

Key words: Face enhancement, filters, speckle noise, mixedneutral difference equation, oscillation

INTRODUCTION

A common problem with image or video signals is the contamination with undesired noise which causes problems both for visual quality and automated analysis. Many automated analysis operations are highly sensitive to noise.There are different sources of noise in a digital image. The overall noise characteristics in an image depend on many factors, including sensor type, pixel dimensions, temperature, etc., Removing noise from image is often the first step in image analysis John (2011).

There are many filtering techniques which can remove noise from image but some of these techniques have limitations. The images taken by a camera can be modified to any extent in the digital world. Facial images are the most modified images because everybody wants their facial images to be good. Various image processing operations can be done on the facial images to achieve the modification to the required level.

Here in our research, we effectively reduce the ageing marks by reducing the difference equation to the new operator. The notion of nonlinear difference equations was discussed intensively by Agarwal (2000) and oscillatory properties were studied by Agarwal (2000).

Difference equations find a lot of applications in the natural sciences, technology and population dynamics (Kelley and Peterson, 2001). Recently there have been a lot of interests in the study of oscillatory and asymptotic behavior of solutions of difference equations with delay and neutral delay type (Thandapani and Selvaraj, 2004).We have utilized the basic results which was our earlier research contribution Selvaraj and Kaleeswari (2013a, b) in developing convergent, oscillatory and divergent sequences through difference sequences.

Wrinkles are the most prominent ageing marks which appear on the face. There are sophisticated algorithms available by Han *et al.* (2012) and Guo and Sim (2009) which ease the modification of ageing marks and blemishes in the facial images.The emergence of high speed hardware desires and various image processing algorithms have certainly enhanced the beautification process.We have proposed an algorithm which essentially detects the affected areas on the face especially around the eyes and forehead.

Our method works appropriately in real time and its result can be viewed quickly. This application is very helpful to those who are health conscious and those who want to see their faces without wrinkles and blemishes. It will also be useful to radiate both additive and speckle noises.

Noise types: The principal sources of noise in digital images arise during image acquisition and transmission. The performance of imaging sensors is affected by a

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variety of factors such as environmental conditions during image acquisition and by the quality of the sensing elements themselves. For instance, in acquiring images with a CCD camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image. Images are corrupted during transmission principally due to interference in the channel used for transmission. For example, an image transmitted using a wireless network might be corrupted as a result of lightning or other atmospheric disturbance.

There are parameters that define the spatial characteristics of noise and whether the noise is correlated with the image. Frequency properties refer to the frequency content of noise in the fourier sense. When the fourier spectrum of noise is constant the noise usually is called white noise. This terminology is a carryover from the physical properties of white light which contains nearly all frequencies in the visible spectrum in equal proportions.

A different type of noise that occurs in the coherent imaging of objects is called speckle noise. For low resolution objects it is multiplicative and occurs whenever the surface roughness of the object being imaged is of the order of the wavelength of the incident radiation. The noise magnitude depends in many cases on the signal magnitude itself. If the noise magnitude is much higher in comparison with the signal we can write $f = g(1+\gamma)\approx g\gamma$ where the noise $\gamma$and the input image g are independent variables. This model describes multiplicative noise. An example of multiplicative noise is the degradation of film material caused by the finite size of silver grains used in photosensitive emulsion.

Quantization noise occurs when insufficient quantization levels are used, for example, 50 levels for a monochromatic image. In this case false contours appear. Quantization noise can be eliminated simply. The term salt and pepper noise is used to describe saturated impulsive noise. An image corrupted with white and/or black pixels is an example. Salt and pepper noise can corrupt binary images.

MATERIALS AND METHODS

Basic models of removal for additive, multiplicative noises

Additive noise removal based on difference equation: The research in Kaleeswari *et al*. (2014) identifies the removal of additive noise in cameraman digital image using SK difference operator. In this filtering technique, the author has reduced the difference equation to the new operator. Then the coefficient of the consecutive values from the sequence could be taken as entries of a mask. The mask corresponding to n = 1 to the magic square of 4×4 gives an inference to the difference of the results obtained from the earlier mask. The sequence has solutions for difference equation which are bounded and normalized. This can be regarded as entries of magic square. Here, the distribution of pixel value during the three distinct stages of image production is described in the histogram. This enhances edges and brings some objects in the original by appealing to n digit coefficients.

Implement improved model to suit speckle nose removal: The study of neighbourhood processing attempts to specify the four varying stages as follows:

- It defines a centre point namely (x, y)
- Further, it performs an operation that involves only the pixels in a predefined neighbourhood about the specificcentre point
- In addition, it allows the result of that operation be the “response” of the process at that point
- It repeats the process for every point in the image

The process of moving the centre point creates new neighbourhoods, one for each pixel in the input image. The two principal terms namely neighbourhood processing and spatial filtering are used to identify this operation.

In this study, we structure the implement improved model to remove speckle noise from the images. The linear operations consist of multiplying each pixel in the neighbourhood by a corresponding coefficient and summing the results to obtain the response at each point (x, y). If the neighbourhood is of size m×n, mn coefficients are required. The coefficients are arranged as a matrix, called a mask. The process consists simply of moving the centre of the mask w from point to point in an image f. At each point (x, y),the response of the filter at that point is the sum of products of the filter coefficients and the corresponding neighbourhood pixels in the area.

Theoretical discussion: This study illustrates the main results which are used to create noise removal operator. We are concerned with the third order mixed type neutral difference Eq. 1 of the form:

$$\Delta\left(a_n\Delta\left(d_n\Delta\left(x_n+b_nx_{n-\tau_1}+c_nx_{n+\tau_2}\right)\right)\right) + q_nx^{\beta}_{n+1-\sigma_1}+p_nx^{\beta}_{n+1-\sigma_2}=0 \quad (1)$$

We assume the following conditions to hold:

- H_1: (a_n) is a positive non-decreasing sequence such that:

$$\sum_{n=n_0}^{\infty}\frac{1}{a_n}$$

- H_2: d_n is a positive non-decreasing sequence
- H_3: (p_n) and (q_n) are positive real sequences for $n\geq n_0$
- H_4: β is the ratio of odd positive integers, τ_1, τ_2, σ_1 and are non-negative integers
- H_5: (b_n), (c_n) are real sequences such that $0\leq b_n\leq b$ and $0\leq c_n\leq c$ $b+c<1$ with

Oscillatory solution: By a solution of Eq. 1 we mean a real sequence $\{x_n\}$ which is defined for all $n\geq n_0-\theta$ and satisfies Eq. 1 for all $n\in N$ where $\theta = \max\{\tau_1, \sigma_1\}$. A solution $\{x_n\}$ is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise it is called non-oscillatory. A difference equation is said to be oscillatory if all of its solutions are oscillatory. Otherwise it is non-oscillatory. Note: For simplicity, we use the following notations:

$$y_n = x_n + b_n x_{n-\tau_1} + c_n x_{n+\tau_2}$$

$$R_n = Q_n + P_n$$

$$Q_n = \min\left\{q_n, q_{n-\tau_1}, q_{n+\tau_2}\right\}$$

$$P_n = \min\left\{p_n, p_{n-\tau_1}, p_{n+\tau_2}\right\}$$

$$\eta_n = \left(\frac{d}{4}\right)^{\beta-1}\frac{k(n-\sigma_1)^{\beta}}{2^{\beta}}R_n \text{ for some } k\in(0,1) \text{ and } d>0$$

Preliminary lemmas: We need the following lemmas whose proofs can be found in Thandapani and Selvaraj, (2004) to prove the main results.

Lemma 1: Assume $A\geq 0, B\geq 0, \beta\geq 1$ and A, $B\in R$. Then $(A+B)\leq 2^{\beta-1}\left(A^{\beta}+B^{\beta}\right)$.

Lemma: 2 $\{x_n\}$Let be a positive solution of Eq. 1. Then there are only two cases for $n\geq n_1\in N$ sufficiently large:

$$y_n > 0, \Delta y_n > 0, \Delta(d_n \Delta y_n) > 0, \Delta\left(a_n\left(d_n \Delta y_n\right)\right) \leq 0$$

$$y_n > 0, \Delta y_n < 0, \Delta(d_n \Delta y_n) > 0, \Delta\left(a_n\left(d_n \Delta y_n\right)\right) \leq 0$$

Lemma 3: Let $y_n > 0, \Delta y_n > 0, \Delta^2 y_n > 0, \Delta^3 y_n \leq 0$ for all $n\geq N_1\in N$. Then for any $k\in(0, 1)$ and for some integer N_2, one has

$$\frac{y_{n+1}}{\Delta y_n} \geq \frac{(n-N)}{2} \geq \frac{kn}{2} \text{ for, } n\geq N_1 \geq N \tag{2}$$

Lemma 4: Let $\{x_n\}$be a positive solution of Eq. 1 and the corresponding y_n satisfies Lemma 2(ii). If

$$\sum_{n=n_0}^{\infty}\left(\frac{1}{d_n}\sum_{s=n}^{\infty}\left(\frac{1}{a_s}\sum_{t=s}^{\infty}\left(q_t+p_t\right)\right)\right)=\infty \tag{3}$$

holds, then$_{n\to\infty}$ $x_n = 0$

Important proposition:

Theorem 5: Assume that condition 3 holds $\sigma_1\geq\tau_1$ and $\beta\geq 1$ If there exists a positive real sequence $\{\rho_n\}$ and an integer $N_1\in N$ with:

$$\limsup_{n\to\infty}\sum_{s=N_1}^{n-1}\left[\rho_s\eta_s\frac{d_{s-\sigma_1}}{d_{s+1-\sigma_1}}-\frac{\left(1+b^{\beta}+\frac{c^{\beta}}{2^{\beta-1}}\right)a_{s-\sigma_1}\left(\Delta\rho_s\right)^2}{4\rho_s}\right]=\infty \tag{4}$$

holds, then every solution $\{x_n\}$of Eq. 3. oscillates or $\lim_{n\to\infty} x_n = 0$

Proof: Let $\{x_n\}$ be a non-oscillatory solution of Eq. 1. Without loss of generality, weassume that there exists an integer $N\geq n_0$ such that $x_n>0$, $x_{n-\sigma 1}>0$, $x_n+r_2\sigma_2>0$, $x_n\tau_2>0$, $x_n+\tau_2>0$ for all $n\geq N$. Then, $y_n>0$ and from equation, we have:

$$\Delta\left(a_n\Delta\left(d_n\Delta y_n\right)\right) = -q_n x^{\beta}_{n+1-\sigma_1} - p_n x^{\beta}_{n+1+\sigma_2} < 0 \tag{5}$$

for all $n\geq N$. Also from Eq. 4 for all, $n\geq N$we have:

$$\begin{aligned}
&\Delta\left(a_n\Delta\left(d_n\Delta y_n\right)\right)+q_n x^{\beta}_{n+1-\sigma_1}+p_n x^{\beta}_{n+1+\sigma_2}\\
&+b^{\beta}\Delta\left(a_{n-\tau_1}\Delta\left(d_{n-\tau_1}\Delta y_{n-\tau_1}\right)\right)\\
&+b^{\beta}q_{n-\tau_1}x^{\beta}_{n+1-\tau_1-\sigma_1}+b^{\beta}p_{n-\tau_1}x^{\beta}_{n+1-\tau_1+\sigma_2}\\
&+\frac{c^{\beta}}{2^{\beta-1}}\Delta\left(a_{n+\tau_2}\Delta\left(d_{n+\tau_2}\Delta y_{n+\tau_2}\right)\right)\\
&+\frac{c^{\beta}}{2^{\beta-1}}q_{n+\tau_2}x^{\beta}_{n+1+\tau_2-\sigma_1}+\frac{c^{\beta}}{2^{\beta-1}}p_{n+\tau_2}x^{\beta}_{n+1+\tau_2+\sigma_2}=0
\end{aligned} \tag{6}$$

Using Lemma 1 in Eq. 6, we have:

$$\Delta\left(a_n\Delta\left(d_n\Delta y_n\right)\right)+ b^\beta\Delta\left(a_{n-\tau_1}\Delta\left(d_{n-\tau_1}\Delta y_{n-\tau_1}\right)\right) +\frac{c^\beta}{2^{\beta-1}}\Delta\left(a_{n+\tau_2}\Delta\left(d_{n+\tau_2}\Delta y_{n+\tau_2}\right)\right) +\frac{Q_n}{4^{\beta-1}}y^\beta_{n+1-\sigma_1}+\frac{P_n}{4^{\beta-1}}y^\beta_{n+1+\sigma_2}\le 0 \quad (7)$$

By Lemma 2, there are two cases for (y_n). Assume case (i) holds for $(n\geq N_1\geq N)$. Since, $\Delta y_n>0$, we have $y_n+\sigma_2\geq y_{n-\sigma 1}$. Therefore, from Eq. 7, we have:

$$\Delta\left(a_n\Delta\left(d_n\Delta y_n\right)\right)+ b^\beta\Delta\left(a_{n-\tau_1}\Delta\left(d_{n-\tau_1}\Delta y_{n-\tau_1}\right)\right) +\frac{c^\beta}{2^{\beta-1}}\Delta\left(a_{n+\tau_2}\Delta\left(d_{n+\tau_2}\Delta y_{n+\tau_2}\right)\right) +\frac{R_n}{4^{\beta-1}}y^\beta_{n+1-\sigma_1}\le 0 \quad (8)$$

Define:

$$w_1(n)=\rho_n\frac{a_n\Delta\left(d_n\Delta y_n\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}}\text{for } n\geq N_1 \quad (9)$$

Then $w_1(n)>0$ for $n\geq N_1$. From Eq. 9, we can see that:

$$\Delta w_1(n)=\frac{\Delta\rho_n}{\rho_{n+1}}w_1(n+1)+\rho_n\frac{\Delta\left(a_n\Delta\left(d_n\Delta y_n\right)\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}} -w_1(n+1)\frac{\rho_n}{\rho_{n+1}}\frac{\Delta\left(d_{n-\sigma_1}\Delta y_{n-\sigma_1}\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}}$$

By Eq. 5, we have

$$a_{n-\sigma_1}\Delta\left(d_{n-\sigma_1}\Delta y_{n-\sigma_1}\right)\geq a_{n+1}\Delta\left(d_{n+1}\Delta y_{n+1}\right)$$

Therefore, from Eq. 9, we get:

$$\Delta w_1(n)\le\frac{\Delta\rho_n}{\rho_{n+1}}w_1(n+1)+\rho_n\frac{\Delta\left(a_n\Delta\left(d_n\Delta y_n\right)\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}} -\frac{\rho_n}{\rho^2_{n+1}}\frac{w_1^2(n+1)}{a_{n-\sigma_1}} \quad (10)$$

Next, we define:

$$w_2(n)=\rho_n\frac{a_{n-\tau_1}\Delta\left(d_{n-\tau_1}\Delta y_{n-\tau_1}\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}},\text{for } n\geq N_1 \quad (11)$$

Then $w_2(n)>0$ for $n\geq N_1$. Note that $\sigma 1\geq\tau_1$. Also from 5, we find that:

$$a_{n-\sigma_1}\Delta\left(d_{n-\sigma_1}\Delta y_{n-\sigma_1}\right)\geq a_{n+1-\tau_1}\Delta\left(d_{n+1-\tau_1}\Delta y_{n+1-\tau_1}\right)$$

Then from Eq. 11, we have:

$$\Delta w_2(n)\le\frac{\Delta\rho_n}{\rho_{n+1}}w_2(n+1)+\rho_n \frac{\Delta\left(a_{n-\tau_1}\Delta\left(d_{n-\tau_1}\Delta y_{n-\tau_1}\right)\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}} -\frac{\rho_n}{\rho^2_{n+1}}\frac{w_2^2(n+1)}{a_{n-\sigma_1}} \quad (12)$$

Also, we define:

$$w_3(n)=\rho_n\frac{a_{n+\tau_2}\Delta\left(d_{n+\tau_2}\Delta y_{n+\tau_2}\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}},\text{for } n\geq N_1 \quad (13)$$

Then, $w_3(n)>0$ for $n\geq N_1$ By Eq. 5, we get:

$$a_{n-\sigma_1}\Delta\left(d_{n-\sigma_1}\Delta y_{n-\sigma_1}\right)\geq a_{n+1+\tau_2}\Delta\left(d_{n+1+\tau_2}\Delta y_{n+1+\tau_2}\right)$$

From Eq. 13, we can find that:

$$\Delta w_3(n)\le\frac{\Delta\rho_n}{\rho_{n+1}}w_3(n+1)+\rho_n \frac{\Delta\left(a_{n+1+\tau_2}\Delta\left(d_{n+1+\tau_2}\Delta y_{n+1+\tau_2}\right)\right)}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}} -\frac{\rho_n}{\rho^2_{n+1}}\frac{w_3^2(n+1)}{a_{n-\sigma_1}} \quad (14)$$

Therefore, Eq. 10, 12 and 14 imply that

$$\Delta w_1(n)+b^\beta\Delta w_2(n)+\frac{c^\beta}{2^{\beta-1}}\Delta w_3(n)\le -\rho_n\frac{R_n}{4^{\beta-1}}\frac{y^\beta_{n+1-\sigma_1}}{d_{n-\sigma_1}\Delta y_{n-\sigma_1}} +\left(\frac{\Delta\rho_n}{\rho_{n+1}}w_1(n+1)-\frac{\rho_n}{\rho^2_{n+1}}\frac{w_1^2(n+1)}{a_{n-\sigma_1}}\right) +b^\beta\left(\frac{\Delta\rho_n}{\rho_{n+1}}w_2(n+1)-\frac{\rho_n}{\rho^2_{n+1}}\frac{w_2^2(n+1)}{a_{n-\sigma_1}}\right) +\frac{c^\beta}{2^{\beta-1}}\left(\frac{\Delta\rho_n}{\rho_{n+1}}w_3(n+1)-\frac{\rho_n}{\rho^2_{n+1}}\frac{w_3^2(n+1)}{a_{n-\sigma_1}}\right) \quad (15)$$

Since, $\{a_n\}$is non-decreasing and $\Delta^2 y_n > 0$ for $n \geq N_1$, we have $\Delta^3 yn \leq 0$ for $n \geq N_1$. Then by Lemma 3, we find for any $k \in (0, 1)$ and n for sufficiently large:

$$\frac{y_{n+1-\sigma_1}}{\Delta y_{n-\sigma_1}} \geq \frac{k(n-\sigma_1)}{2} \frac{d_{n-\sigma_1}}{d_{n+1-\sigma_1}} \tag{16}$$

Since:

$$y_n > 0, \Delta y_n < 0, \Delta(d_n \Delta y_n) > 0$$

for $n \geq N_1$ we have:

$$y_n = y_{N_1} + \sum_{s=N_1}^{n-1} \Delta y_s \geq (n - N_1)\Delta y_{N_1} \geq \frac{1n}{2} \tag{17}$$

for some i>0 and n for sufficiently large. From Eq. 16, 17 and $\beta \geq 1$,we have:

$$\frac{y^{\beta}_{n+1-\sigma_1}}{\Delta y_{n-\sigma_1}} \geq \frac{1^{\beta-1}(n-\sigma_1)}{2^{\beta}} \frac{d_{n-\sigma_1}}{d_{n+1-\sigma_1}} \tag{18}$$

Substitutingthe inequality 18 inthe inequality 15 and summing the resulting inequality from $N_2 \geq N_1$to n-1, we obtain:

$$\sum_{s=N_1}^{n-1} \left[\rho_s \eta_s \frac{d_{s-\sigma_1}}{d_{s+1-\sigma_1}} - \frac{\left(1 + b^{\beta} + \frac{c^{\beta}}{2^{\beta-1}}\right) a_{s-\sigma_1} (\Delta \rho_s)^2}{4\rho_s} \right] \tag{19}$$

$$\leq w_1(N_2) + b^{\beta} w_2(N_2) + \frac{c^{\beta}}{2^{\beta-1}} w_3(N_2)$$

Taking lim sup for the above inequality, we get a contradiction to 4. Assume that Lemma 2(ii) holds. Then by Lemma 4, we can obtain. $\lim_{n\to\infty} x_n = 0$ Hence, the proof is complete. Letρ_n = n and β = 1 Then from Theorem 5, we obtain the following corollary.

Corollary: 6 Assume that condition 3 holds and $\sigma_1 \geq \tau_1$. If there is an integer $N_1 \in N$ with:

$$\lim_{n\to\infty} \sup \sum_{s=N_1}^{n-1} \left[s\eta_s \frac{d_{s-\sigma_1}}{d_{s+1-\sigma_1}} - \frac{(1+b+c)}{4s} a_{s-\sigma_1} \right] = \infty \tag{20}$$

holds, then every solution $\{x_n\}$of Eq. 1 oscillates or $\lim_{n\to\infty} x_n = 0$.

MATERIALS AND METHODS

Here we have illustratedthe creation of noise removal operator. Let us consider the third order difference Eq. 21:

$$\Delta^3\left(x_n + \frac{1}{4}x_n + \frac{1}{4}x_{n+1}\right) + \left(\frac{16}{3}\right)9^n x^3_{n+1} + (144)9^n x^3_{n+2} = 0 \tag{21}$$

Let:

$$a_n = d_n = 1, b_n = c_n = \frac{1}{4}, q_n = \left(\frac{16}{3}\right)9^n, p_n = (144)9^n$$

and:

$$\tau_1 = 0, \tau_2 = 1, \sigma_1 = 0, \sigma_2 = 1$$

Then conditions 3 and 4 hold. Therefore all the conditions of theorem 5 hold and hence every solution of equation (E1) is oscillatory or tends to zero as $n \to \infty$. One such solution is $x_n = 1/3^n$. Now Eq (E1) can be written as

$$\frac{1}{4}x_{n+4} - \frac{1}{4}x_{n+3} - \frac{6}{4}x_{n+2} + \frac{11}{4}x_{n+1} - \frac{5}{4}x_n + \left(\frac{16}{3}\right)9^n x^3_{n+1} + (144)9^n x^3_{n+2} = 0 \tag{22}$$

which is equivalent to the following partial differential equation:

$$\frac{1}{4}\left[\frac{\partial^4 z}{\partial x_n^4} - \frac{\partial^3 z}{\partial x_n^3} - 6\frac{\partial^2 z}{\partial x_n^2} + 11\frac{\partial z}{\partial x_n} - 5x_n\right] + \left(\frac{16}{3}\right)9^n \left(\frac{\partial z}{\partial x_n}\right)^3 + (144)9^n \left(\frac{\partial^2 z}{\partial x_n^2}\right)^3 = 0$$

From our earlier researches, for n = 1, we have obtained a mask:

$$A = \begin{pmatrix} 0 & -26 & 0 \\ -10 & 1 & 26 \\ 0 & 9 & 0 \end{pmatrix}$$

which is used as an operator in the removal of noise. While combining the entries of the above mask with the corresponding coefficients of Equation (E2), we get a new noise removal operator:

$$B = \begin{pmatrix} 1 & 26 & 0 \\ 10 & -59 & -26 \\ 0 & -9 & 1 \end{pmatrix}$$

Novel speckle noise removal model: In this study, we have implemented noise removal operator to remove speckle noises for the new model. Normally wrinkles appear to people who get older but there are some cases where some part of the skin gets wrinkles even before ageing. People with these wrinkles will hesitate to put their images in its original form. It would be a boon for them if their wrinkles get removed from their images and they look younger. Much research has been done for a long period of time in this area of beautification of images (Lee *et al*., 2009). In our research, we have partitioned the image into segments by focusing on the areas around the eyes, forehead and the cheek. Further we have applied our operator on the segments.

Model for removal of additive noise and multiplicative noise by successive application of specialized sequence filters is illustrated:

- Step 1: Divide the image into 256×256 cells
- Step 2: Get the pixel values
- Step 3: Apply the windowing technique to the operator matrix A in the eight neighbourhood pixels
- Step 4: Record the resulted pixels in a new frame
- Step 5: Now apply eight neighbourhood noise removal to remove multiplicative noise by the operatorB
- Step 6: The image is screened at the threshold value to get the optimum clarity in the image with brightness and colour
- Step 7: Noise removal test is performed at the steps 3 and 5. The result is compared with the original image (in particular ageing symptoms)

Note that the image is split into pieces to execute the algorithm and merge together. The algorithm for one segment of the image used in the above model is as follows:

Algorithm

```
A=imread(original image); k=3;
fori=1;
 k=k*3;  % because solution of (E1) is i/3^n
end
px=[1 26 0;10 -59 -26;0 -9 1];
icAx=filter2(px,A);
py=px';
icAy=filter2(py,A);
RA=sqrt(icAx.^2+icAy.^2);
L1=RA/k;
```

Fig. 1: Orignal image

Fig. 2: Result image

For implementing above model, let us consider (Fig. 1). After applying our model, we obtain the resultant image (Fig. 2). Intensity transformation functions based on information extracted from image intensity histograms play a basic role in image processing in areas such as enhancement, compression, segmentation and

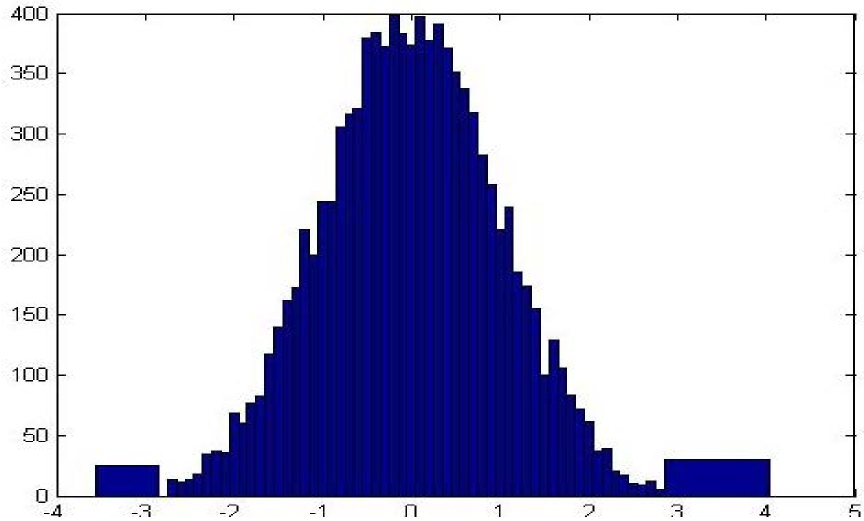

Fig. 3: Orignal image histogram

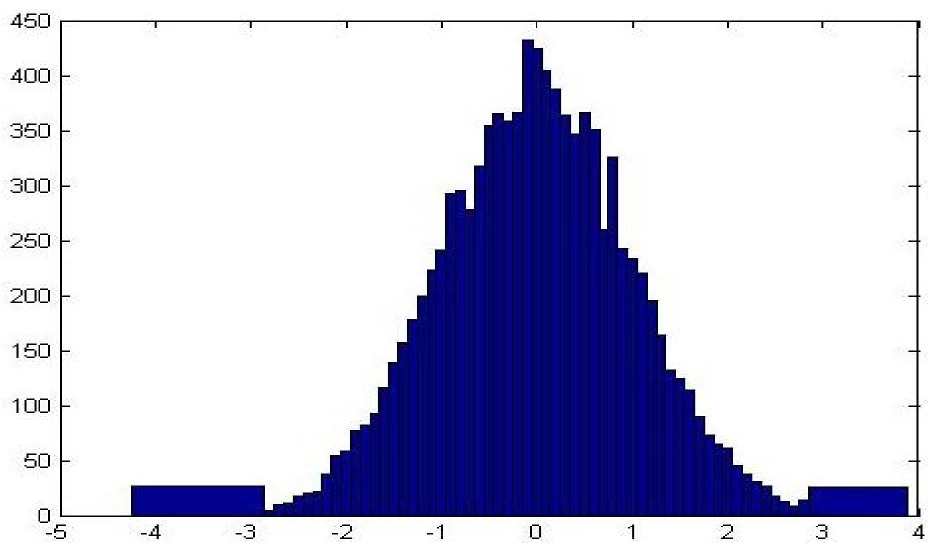

Fig. 4: Result image histogram

description. Histo grams are simple to calculate in software and also lend themselves to economic hardware implementations, thus making them a popular tool for real-time image processing.

The histogram of a digital image with intensity levels in the range [0, L-1] is a discrete function $h(r_k) = nk$ where r_k is the kth intensity value and is the number of pixels in the image with intensity r_k. It is common practice to normalize a histogram by dividing each of its components by the total number of pixels in the image, denoted by the product MN, where as usual, M and N are the row and column dimensions of the image. Thus, a normalized histogram is given by $p(r_k) = n_k/MN$, k = 0, 1, 2,....,L-1. $P(r_k)$ is an estimate of the probability of occurrence of intensity level r_k in an image. The sum of all components of a normalized histogram is = 1. Figure 3 and 4 are the histograms of original image (Fig. 1) and result image (Fig .2), respectively. We have considered the colour image (Fig. 5) for further analysis and applied our model and got (Fig. 6) as result.

Fig. 5: Orignal image

Fig. 6: Result image

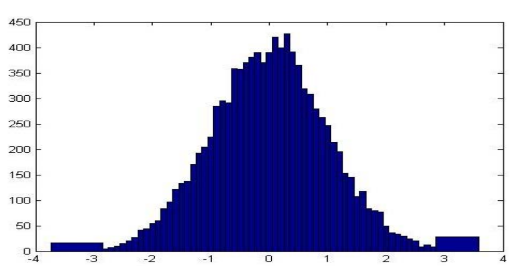

Fig. 7: Orignal image histogram

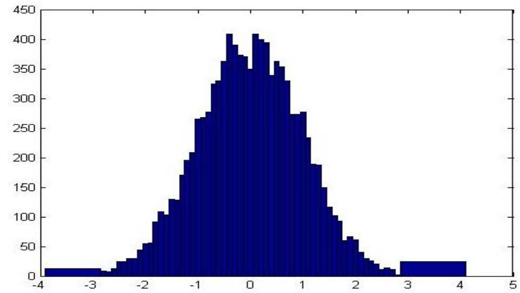

Fig. 8: Result image histogram

Figure 7 and 8 show the histograms of original image (Fig. 5) and result image (Fig. 6), respectively.

RESULTS AND DISCUSSION

PSNR or peak-to-noise ratio is used to evaluate the quality of the watermarked image after embedding the secret message in the image. Table 1 describes the PSNR value to test the image quality: As the PSNR ratio of our operator is greater, we conclude that our operator is capable of removing noise in this particular image and we show in general context of blunt images. This we feel, may help diagnostic techniques in medical imaging and other video imagery. Though the age marks are removed, the brightness of result image differs significantly from that of the original image. By improving the statistical parameters, we wish to get better results.

Comparison with other models

Active shape models: Active Shape Models (ASM) are statistical models of the shape of objects which iteratively deform to fit to an example of the object in a new image, developed by Cootes *et al*. (1995) and modified later to overcome many of its limitations such as computational complexity (Celiktutar *et al*., 2013) and convergence issues. Shapes which are labeled with points that are given to training using the point distribution model and are controlled by shape model. ASM algorithm aims to match the model to a given new image. In short ASM represents a parametric deformable model where a statistical model of the global shape from the training set is to be built.

The aim of active shape model is to locate automatically the landmark points.When considering the face models, the landmark points consists of the points which lie on the shape boundaries of facial features such as eyes, lips, nose, mouth and eyebrows. The ASM works by alternating the following steps:

Generate a suggested shape by looking in the image around each point for a better position for the point. This

Table 1: Image quality tested (PSNR)

Comparision images	Figure No.	Row	Column	PSNR value
Original image and result image	Fig.1 and 2	262	588	10.5987
Original image and result image	Fig.5 and 6	288	612	10.7921

is commonly done using what is called a "profile model", which looks for strong edges to match a model template for the point. Conform the suggested shape to the point distribution model, commonly called a "shape model" in this context. The technique has been widely used to analyse images of faces, mechanical assemblies and medical images (2D and 3D). It is closely related to the active appearance model. It is also known as a "Smart Snakes" method, since it is an analogue to an active contour model which would respect explicit shape constraints.

Edge preserved smoothing using weighted least square method: Removing age marks such as wrinkles in real time is achieved by using advanced form of edge preserved smoothing filter. This filter uses weighted least square method for smoothing. Multiscale is always better to work on details at different scales rather than working on single scale. Multiscale image processing is achieved by using any of the multiscale decompositions such as Laplacian pyramid Burt and Adelson (1983). These pyramids are created by linear filters which produces halo artifacts near edges. Edge preserved smoothing filters are used to reduce these kinds of artifacts.

Advanced versions of traditional edge preserved smoothing filters can reduce the artifacts produced near the edges. The methods namely robust smoothing Black *et al.* (1998), anisotropic diffusion and bilateral filter are also used to reduce aircrafts. The method that is based on the weighted least squared framework was used originally in the denoising stage of images to reduce the ringing effect. We found that this method is very much suitable to reduce the ageing marks very effectively without affecting the face image.

Edge preserved smoothing should smooth as much as possible but at the same time as same as the original image. If the original image is given as w and the smoothed image is given as v then the typical edge preserved smoothing filter is obtained by minimizing the function:

$$\sum_{l}\left(w_l - v_l\right)^2 + \gamma\left(b_{x,l}(v)\left(\frac{\partial w}{\partial x}\right)_l^2 + b_{y,l}(v)\left(\frac{\partial w}{\partial y}\right)_l^2\right) \quad (23)$$

Minimizing the first term gives the minimum distance between the original image and smoothed image. The second term achieves the smoothness by minimizing the partial derivatives of the original image w. Smoothness is controlled by the smoothness weights b_x and b_y. The γ is the term which is playing in between the difference of images and smoothness value. If $\gamma$is increased, we will get a smoother image. In comparison to our model, the above models did not use pixel values and neighbourhood impact on any central cell, but we have used the windowing technique.The proportionate contribution to any particular cell from neighbourhood removes both additive and multiplicative noises. Appropriate threshold value is guaranteed by the PSNR comparison to the original and the image after the application operator.

Meritsof our model: Our model was successful in identifying the marks and effectively reduced the intensity of those marks. Easy and fast applications of difference operators enable the removal of speckle noise in the image.

CONCLUSION

While conducting our research work on different types of solutions of difference equations of second and higher order, we met with varied specialized sequences. These sequences have been viewed to create mask while operating on images with noises in the removal process. As such this is an improvement of earlier operators such as sobel and others in removing additive noises in coloured and grayscale images. Our discussion on the removal of speckle noise is designed as a successive operator on the above images. These two successive specialized sequence solutionsof difference equations help us in the future extraction in the domain of images with age marks and improves the mutual vision in cultural sculpture and other artistic articles exhibiting arts and culture. Experiments have been conducted in face analysis in two different samples for brightness aspect. Inferences on statistical parameters have been tabulated to support our claim.

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