

DT-CWT and IFS Based Fractal Image Compression

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Abstract: The significance of images has increased drastically in all the technological fields. Though the rapid decoding algorithms exist, the encoding process is extremely time consuming. But, owing to large encoding time of Fractals image coder, its application is restricted. Hence, a fast encoder is needed to extend the application of Fractals technology, to fast communication such as World Wide Web. Here, a fast image coder is proposed which reduces the coding considerably time of image. In this study a fast fractal encoding system is proposed using Dual Tree Complex Wavelet Transform (DTCWT) and reduce the encoding time with increased quality of image can be reproduced. The proposed simple SPIHT (Set partitioning in hierarchical Trees) algorithms used and which is based on not only the relationship between the bit-planes and the target bit-rate. It gives the relationship between the initial threshold and the target bit-rate, can effectively reduce the computation time and required memory space also. The simulation results using MATLAB is compared with pure SPIHT based Fractals Image coder using DWT.

Key words: Dual Tree Complex Wavelet Transform (DTCWT), Set Partitioning in Hierarchical Trees (SPIHT), Peak Signal to Noise Ratio (PSNR), Mean square Error (MSE), owing

INTRODUCTION

In multimedia, the key technologies are Digital image compression. Uncompressed multi-media (graphics, audio and video) data requires considerable storage capacity and transmission band width. Internet teleconferencing, Processing of Medical images including compression, should not interfere the information carried by the images without delay, because those medical images require special treatment for fast and correct diagnosis, despite rapid progress in mass-storage density. Though a variety of powerful and sophisticated wavelet-based schemes for image compression, the need for more efficient ways to encode signals and images. For efficient encoding can get it from EZW algorithm, SPIHT algorithm, WDR algorithm (Said and Pearlman, 1996; Brahimi *et al.*, 2009; Jacquin, 1992; Selesnick *et al.*, 2005) have been developed and implemented. In the field of image compression has many advantages with the wavelet transform has emerged as a advance technology. Wavelet-based coding provides substantial improvements in the PSNR of picture quality with higher compression ratios. Although, DWT provide high coding efficiency for natural (smooth) images in Image compression algorithms, it has three major disadvantages that weaken its application. These disadvantages are described.

Low of shift sensitivity with different scales the distribution of energy between DWT coefficients are

unpredictable, it means that small shifts in the input signal can cause an variance. It is serious disadvantaged by the shift sensitivity that arises from down samplers in the DWT implementation. Low directional selectivity: only a few feature orientations in the spatial domain, when the m-Dimensional transform ($m > 1$) coefficients reveals that orientation consider the transform as poor directional selectivity.

In many signal processing applications like an image compression and power measurement are required the phase information and it is valuable e.g.. for a complex valued signal or vector, this phase value evaluate by its real and imaginary projections. Processing the image with 2-D DWT increases phase size and adds phase distortion in Digital image as a data matrix with a finite support in 2-D. So, it is a well-known way of providing shift invariance is to use the Dual Tree Complex Wavelet Transform (DT-CWT) (Abdullah, 2008; Selesnick *et al.*, 2005).

One way of decreasing the encoding time is by using Genetic Algorithm (GA) with stochastic optimization methods which recent topics of GA-based methods are proposed to improve the efficiency (Barnsley, 1993; Jacquin, 1992; Gonzalez, *et al.*, 2004). The idea of special correlation of an image is used in these methods while the chromosomes in GA consist of all range blocks which leads to high encoding speed for the fractal image compression. Other researchers focused on tree structure

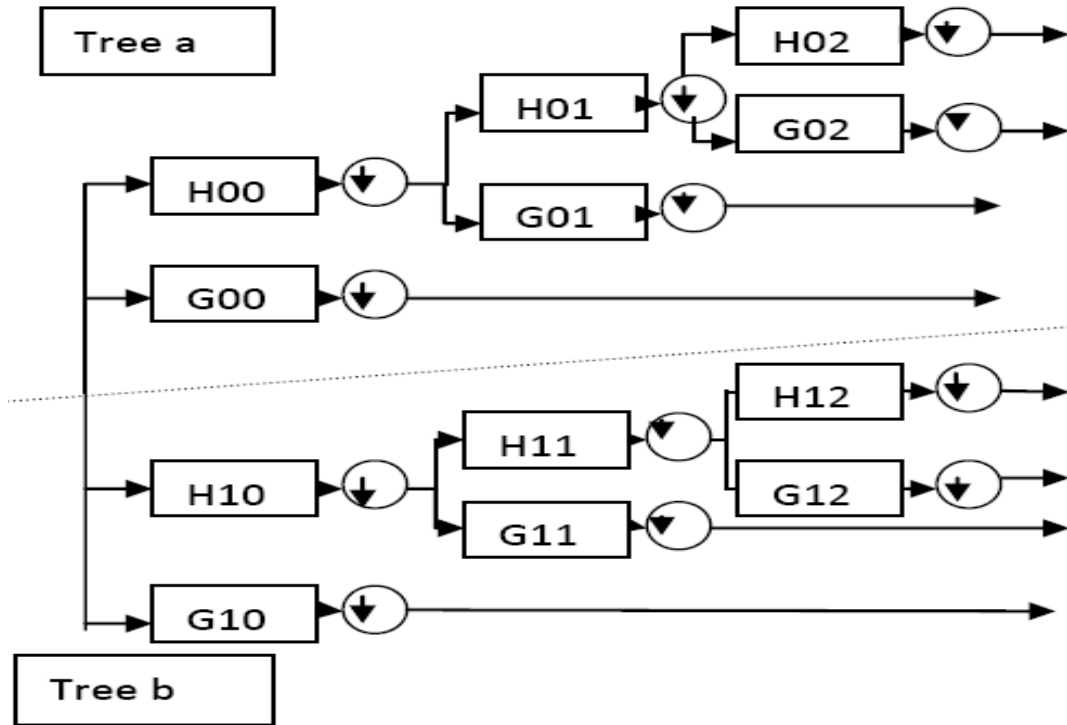


Fig. 1: the dual-tree complex wavelet transformation , comparison two trees oof real filters a and b

search methods (Sendur and Selesnick, 2002; Hurtgen Stiller, 1993) of the search process give the improvements and parallel search methods (Hufnagl and Uhl, 2000; Vidya *et al.*, 2000) or quad tree partitioning of range blocks (Fisher, 1994) to make it faster.

Dual-tree complex wavelet trans-form theories: The DT-CWT is a 2 parallel wavelet filter bank combination as tree b and tree a. It is minimizes the aliasing effects by down sampling while designing the filters with different delays. The filters are designed in a specific way of tree a can be interpreted as the real part of a complex wavelet transform in the sub-band images and the tree b can be interpreted as the imaginary part as in Fig. 1 where downward arrow shows down sampling by 2 operation. The DT-CWT is nearly shift- invariant, when compare to the classic DWT.

In this filter can achieve correct relative signal delay by the total delay difference for a given level and all previous levels must sum to one sample period at the input sample rate of the given level. So that, the filters are below level 2 in one tree must provide delays that are half a sample different (at each filter's input rate) from those in the opposite tree (Fisher, 1994). This requires odd-length filters in one tree and even-length filters in the other tree

for linear phase filters. Note that the filters in the first stage of each tree are different from the filters in all the later stages. Further there is no complex arithmetic involved in any of the trees. The complex coefficients are simply obtained from the Eq. 1:

$$x_J^C = x_{\frac{x_{0...01a}}{j}}(k) + ix_{\frac{x_{0...01a}}{j-1}}(k) \quad (1)$$

where, $i = \sqrt{-1}$ and complex wavelet basis function are given by:

$$\Psi_{jk}^C(n) = \Psi_{jka}(n) + j\Psi_{jkb}(n) \quad (2)$$

The inverse DTCWT is calculated as Eq. 2 normal inverse wavelet transforms, one corresponding to each tree and the results of each of the Eq. 2 inverse transforms are then averaged to give the reconstructed signal. Again, there is no complex arithmetic needed, since, $x_J^C(k)$ the coefficients are split up into $X_{x_{0...01a}}(k)/J$ and $X_{x_{0...01b}}(k)/J$ before they are used in the corresponding inverse transforms.

MATERIALS AND METHODS

Principle of fractal coding: In the encoding phase of fractal image compression, the image of size $N \times N$ is first

partitioned into non overlapping range blocks $\{R, R, R, \dots\}$ of a predefined size $B \times B$. Then, a search codebook is created from the image taking all the square blocks (domain blocks) $\{D, D, D, \dots\}$ of size $2B \times 2B$ with integer step L in horizontal or vertical directions. The range-domain matching process initially consists of a shrinking operation in each domain block that averages its pixel intensities forming a block of size $B \times B$. For a given range R , the encoder must search the domain pool for best affine transformation, which minimizes the distance between the image (R) and the image (D), (i.e. $(D) = (R)$). The distance is taken in the luminance dimension not the spatial dimensions. Such a distance can be defined in various ways, but to simplify the computations it is convenient to use the Root Mean Square (RMS) metric. For a range block with n pixels, each with intensity r and a decimated domain block with n pixels, each with intensity d the objective is to minimize the quality in (A) as follows:

$$E(R_i, D_i) = \sum_{i=1}^n (sd_i - o - r_i)^2 \quad (3)$$

This occurs when the partial derivatives with respect to s and o are zero. Solving the resulting equations will give the best coefficients s and o .

$$s = \frac{\sum_{i=1}^n d_i r_i - \sum_{i=1}^n d_i \sum_{i=1}^n r_i}{n \sum_{i=1}^n d_i^2 - (\sum_{i=1}^n d_i r_i)^2} \quad (4)$$

$$o = \frac{1}{n} \left(\sum_{i=1}^n n - s \sum_{i=1}^n d_i \right) \quad (5)$$

The parameters that need to be placed in the encoded bit streams are s , o , index of the best matching domain and rotation index. The range index can be predicted from the decoder if the range blocks are coded sequentially. The coefficient s represents a contrast factor, while the coefficient o represents brightness offset. At decoding phase, it is found that if the transforms are performed iteratively, beginning from an arbitrary image of equal size, the result will be an attractor resembling the original image at the chosen resolution.

Fractal image compression algorithm: The fundamental idea of fractal image compression is based on an Iteration Function System (IFS) in which the governing theorems are the Collage Theorem and the Contractive Mapping

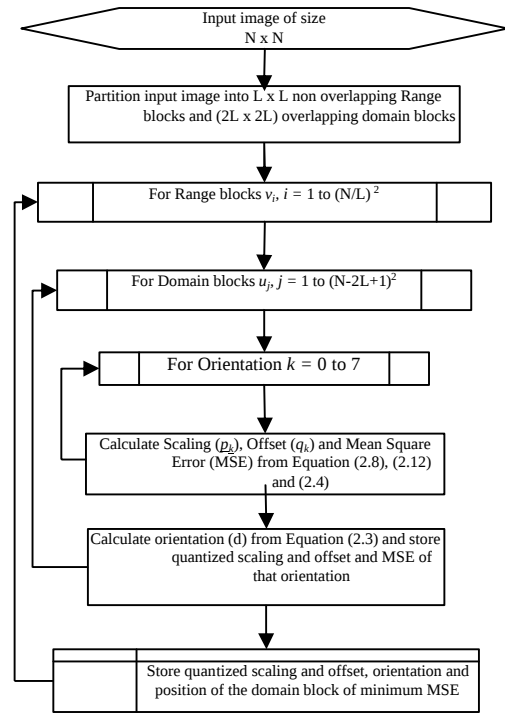


Fig. 2: Flow chart for FIC encoder

Fixed-Point Theorem. The encoding unit of FIC for given gray level image of size $N \times N$ is $(N/L)^2$ of non-overlapping range blocks of size $L \times L$ which forms the range pool R . For each range block v in R , one search in the $(N - 2L + 1)^2$ overlapping domain blocks of size $2L \times 2L$ which forms the domain pool D to find the best match. The parameters describing this fractal affine transformation of domain block into range block from the fractal compression code of v . The parameters of fractal affine transformation is Φ of domain block into range block having domain block coordinates (t_x, t_y) , Dihedral transformation- d , contrast scaling- p , brightness offset- q . The affine transformation is illustrated as flowchart in Fig. 2 for this fractal:

$$\Phi \begin{bmatrix} x \\ y \\ u(x,y) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & p \end{bmatrix} \begin{bmatrix} x \\ y \\ u(x,y) \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ q \end{bmatrix}$$

where the 2×2 sub-matrix:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is one of the dihedral transformation:

$$\begin{aligned}
T_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \\
T_2 &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, T_3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \\
T_4 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, T_5 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \\
T_6 &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, T_7 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}
\end{aligned}$$

$$s = \frac{\sum_{i=1}^n d_i r_i - \sum_{i=1}^n d_i \sum_{i=1}^n n}{n \sum_{i=1}^n d_i^2 - \left(\sum_{i=1}^n d_i r_i \right)^2}$$

$$o = \frac{1}{n} \left(\sum_{i=1}^n n - s \sum_{i=1}^n d_i \right)$$

The above parameters are found using the following procedure. The domain block is first down-sampled to $L \times L$ and denoted by u . The down-sampled block is transformed subject to the eight transformations T_k : $k = 0, \dots, 7$ in the Dihedral on the pixel positions and are denoted by u_k , $k = 0, 1, \dots, 7$, where $u_0 = u$. The transformations T_1 and T_2 correspond to the flips of u along the horizontal and vertical lines, respectively. T_3 is the flip along both the horizontal and vertical lines. T_4 - T_7 are the transformations of T_0 - T_3 performed by an additional flip along the main diagonal line, respectively. For each domain block, there are eight separate MSE computations required to find the index d such that:

$$d = \arg \min \{ \text{MSE}((p_k u_k + q_k), v) : k = 0, 1, \dots, 7 \}$$

Where:

$$\text{MSE}(u, v) = \frac{1}{L^2} \sum_{i,j=0}^{L-1} (u(i, j) - v(i, j))^2$$

Here, p_k and q_k can be computed directly as:

$$\begin{aligned}
p_k &= \frac{\left[L^2 \langle u_k, v \rangle - \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j) \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} v(i, j) \right]}{\left[L^2 \langle u_k, u_k \rangle - \left(\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j) \right)^2 \right]} \\
q_k &= \frac{1}{L^2} \left[\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} v(i, j) - p_k \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} u_k(i, j) \right] \quad (6)
\end{aligned}$$

As u runs over all of the domain blocks in D to find the best match, the terms t_x and t_y can be obtained together with d and the specific p and q corresponding this d , the affine transformation Eq. 1 is found for the given range block v . In practice, t_x , t_y , d , p and q can be encoded using $\log_2(N)$, $\log_2(N)$, 3, 5 and 7 bits, respectively which are regarded as the compression code of v . Finally, the encoding process is completed as v runs over all of the $(N/L)^2$ range blocks in R .

From Fig. 3-5 find the MSE vs. quantization parameter for randomly selected range block of size 8×8 from 256×256 Lena image (Boat 512×512 and Barbara 512×512). From Fig. 3, choosing 5 bits and 7 bits as quantization parameter for scale and offset value are justified respectively. To decode, the compression codes to obtain a new image and proceeds recursively by chooses any image as the initial one and makes up the $(N/L)^2$ affine transformations. According to Partitioned Iteration Function Theorem (PIFS) (Chang and Girod, 2007), the sequence of images will converge. According to user's application the stopping criterion of the recursion is designed based on the optimization technique like Genetic Algorithm, PSO and etc. The final image is the retrieved image of fractal coding.

Simple SPIHT algorithm: Ac Set Partitioning in Hierarchical Trees (SPIHT) algorithm, introduced by Fang *et al.* (2000), Chuo-Ling by Said and Pearlman is a highly renewed version of the EZW algorithm. Some of the best results that obtain highest PSNR values for given compression ratios for a wide variety of images have been obtained with SPIHT. The SPIHT multistage encoding process employs three lists and sets:

- The List of Insignificant Pixels (LIP) contains individual coefficients that have magnitudes smaller than the threshold.
- The List of Insignificant Sets (LIS) contains sets of wavelet coefficients that are defined by tree structures and are found to have magnitudes smaller than the threshold (insignificant). The sets exclude the coefficients corresponding to the tree and all subtree roots and they have at least four elements
- The List of Significant Pixels (LSP) is a list of pixels found to have magnitudes larger than the threshold (significant)

For a given entry (i, j) , define the set $D(i, j)$ as follows. If m is either at the 1st level or at the all low-pass level, then $D(i, j)$ is the empty set; otherwise, if m is at the j th level for $j > 1$, then $D(i, j) = \{\text{Descendents of entry}(i, j-P)\}$. The significance function S is defined by:

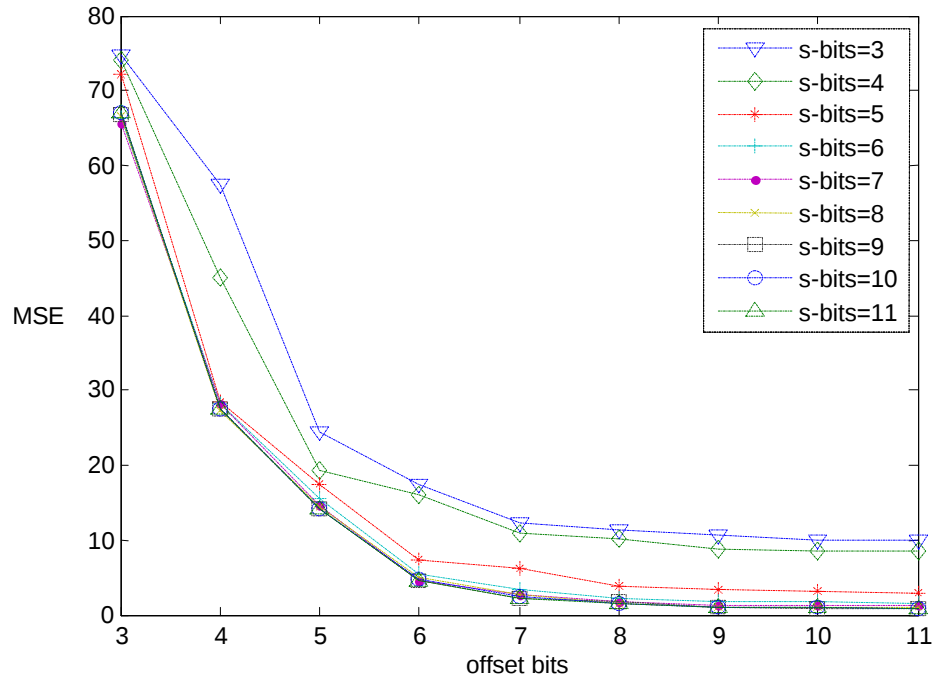


Fig. 3: MSE vs quantization parameters for lena 256×256

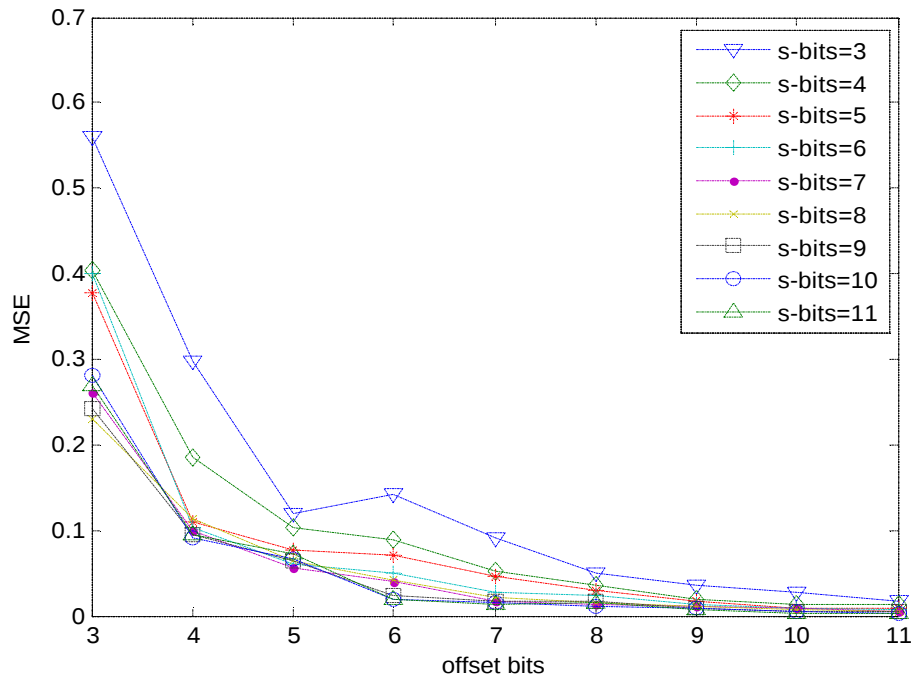


Fig. 4: MSE vs quantization parameters for Babraa 512×512

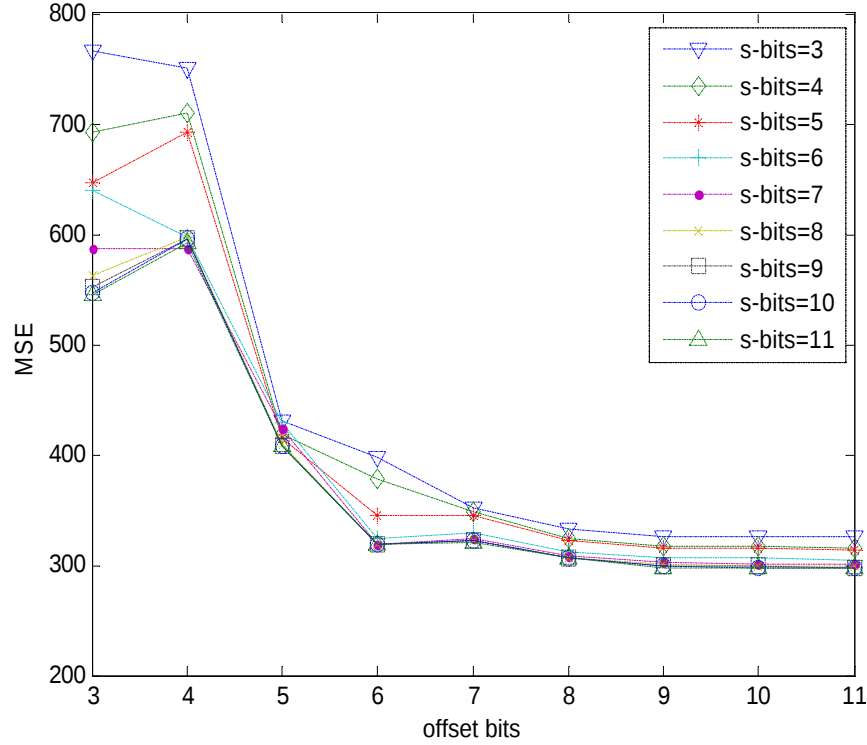


Fig. 5: MSE vs. quantization parameters for Boat 512×512

$$S(i, j) = \begin{cases} 1 & \max_{n(D(i, j))} |w(i, j)| \geq T \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where, T denotes the threshold for different target bit rates. Since, the SPIHT algorithm relies on Spatial Orientation Trees (SOT) defined on dyadic subband structure, there are a few problems that arise.

First, in the original SPIHT coding process, a lesser number of bit planes are discarded for higher target bit-rates. Inversely, more bit planes are discarded for lower target bit-rates. If it can determine the relationship between the bit-planes and target bit-rates, this work can immediately discard the appropriate num of bit-planes to achieve different target bit rates.

Second, at low bit-rates (implying that some bit planes are to be discarded), if a subband coefficient is slightly lower than $2n$ and considerably $>2n-1$ this work should then regard the coefficient as being “significant,” and one bit is used to describe its significance (Ding and Yang, 2008). With the above two problems, based on these ideas discussed in (Brahimi *et al.*, 2009; Jacquin, 1993, 1992). This study presents the simplified SPIHT encoding procedure listed as follows:

1) Initialization:

2) Sorting Pass:

2.1) for each entry (i, j) in the LIP: output $S(i, j)$; if $S(i, j) = 1$, then output the sign and delete (i, j) node.

2.2) for each entry (i, j) in the LIS:

2.2.1) if the entry is of type A. Then output $S(D(i, j))$; if $S(D(i, j)) = 1$, then

(a) for each $(k, l) \in O(i, j)$, compute $S(k, l)$; if $S(k, l) = 1$, then output the sign and delete (i, j) node;

$S(k, l) = 0$, then add (k, l) to the end of the LIP; if $L(k, l) \neq \emptyset$, then move (i, j) to the end of the LIS as an entry of type B; go to Step 2.2.2;

otherwise, remove entry (i, j) from the LIS.

2.2.2) if the entry is of type B, then output $S(L(i, j))$; if $S(L(i, j)) = 1$ then

(a) add each $(k, l) \in O(i, j)$ to the end of the LIS as an entry of type A;

remove (i, j) from the LIS.

2.3) Threshold Update: decrease n by 1;

2.4) go to step 2

Based on this algorithm, this study can obtain a new image compression process. First, decompose the input image using dual-tree complex wavelet; second, use the above algorithm to encode and quantize the wavelet coefficients and then obtain the compressed images; the last, at the receiving end This study can obtain the original image from inverting quantize and reconstructing the image. General flowchart is as shown in Fig. 6 and 7.

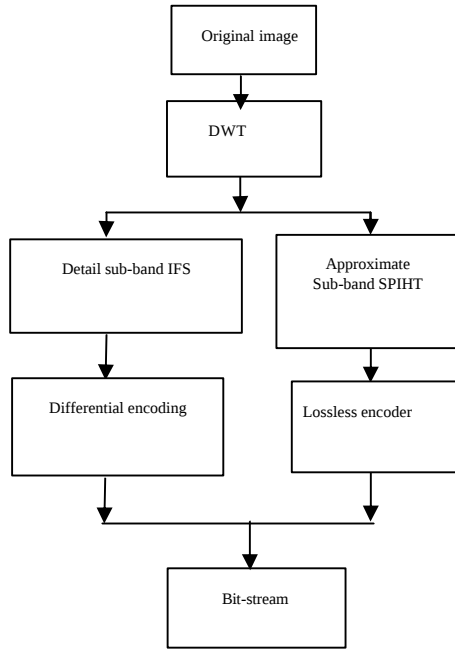


Fig. 6: Block diagram of proposed Encoder

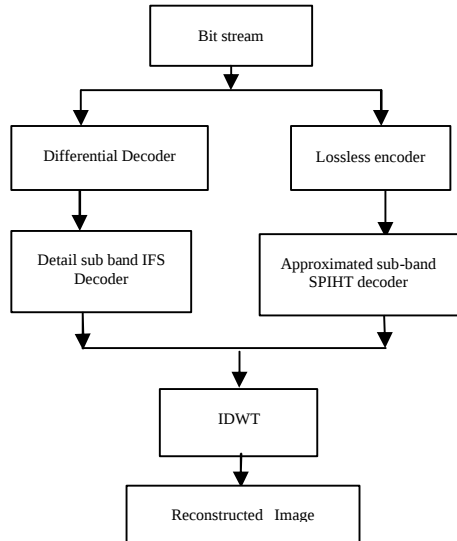


Fig. 7: Block diagram for proposed decoder

RESULTS AND DISCUSSION

In this study made a simulation test to the image and compared with DWT method, achieved a relatively good results in concern with the effectiveness of the method. The comparison is mainly containing two aspects: one is the reconstruction of input image and the other one is the Peak Signal to Noise Ratio (PSNR). Experiment Environment: Computer frequency 2.50 GHZ, internal



Fig. 8: Reconstruction of Barbara, Lena and Boat images using DT-CWT and DWT

storage 2.0GB, Software Environment MATLAB 200(a). In the experiment, this study compared the quality of reconstruction image. Simulation results are shown below: To examine the quality of the re-constructed image, in which the constraint condition affects on this Image. In this study the Peak Signal to Noise Ratio (PSNR) of these three reconstructed images is calculated, it can be expressed using the following equation:

$$\sigma^2 = \frac{1}{MN} \sum_{i=0}^{I=M-1} \sum_{j=0}^{J=N-1} (X_{i,j} - \widehat{X}_{i,j})^2 \quad (8)$$

$$\text{PSNR} = 20 \log_{10} \frac{255}{\sigma^2} \quad (9)$$

M and N are rows and columns of the image, $x_{i,j}$ is pixel value of the reconstruction image, $x_{i,j}$ is pixel value of the original image.

A new image coder is proposed in this study. Here, speed of wavelet is more when compare with fractal Image compression with the advantages quality of an image. First of all Discrete wavelet transform is applied on the full images as given in Fig. 8. By doing this here study get four Quadrant frequency matrix named as approximate sub-band and detail sub-band. The iterated Function system algorithm is applied on the detail sub-band coefficients. Here affine transform is used to get IFS. Finally, 8X8 images and set of IFS Parameters is achieved, which is encoded using differential encoding and transmitted to the decoder section. At the same time DT-CWT algorithm is applied on approximate sub-band and it is further encoded using SPIHT coding and transmitted to the decoder section. At the decoder section the reverse process is carried out and image is reconstructed as shown in Fig. 8. So with acceptable speed and high PSNR of the image is reproduced here.

Table 1: Comparison for DT-CWT and DWT

Images	Transform methods	PSNR
LENA	DT-CWT	29.22
	DWT	28.31
BOAT	DT-CWT	27.87
	DWT	27.01
BARBARA	DT-CWT	28.32
	DWT	27.83

Table 2: Comparison for image compressions of proposed against existing technique

Image	Parameters for image	DT-CWT		DWT	
		Proposed new fractal image coder	Tested image form the references	Tested Fractal image form paper	
Lena	Compression ratio	14.76:1	10:1	10:1	
	Coding time in sec	305	280	283	
	PSNR in db	29.2243	23.9112	28.3132	
Boat	Compression ratio	14.76:1	10:1	10:1	
	Coding time in sec	315	288	285	
	PSNR in db	29.4143	27.8743	33.9123	
Barbara	Compression ratio	14.76:1	10:1	10:1	
	Coding time in sec	311	286	281	
	PSNR in db	29.4143	28.3254	27.8365	

The Proposed Image coder has been simulated using MATLAB and encoding coding time compared to Pure Fractal image coder is achieved at the same time it retains all the visual qualities of the pure Fractal image coder with acceptable speed with DWT when using of DT-CWT. Simulated and comparison result is given in Fig. 8. The compression ratio values are tabulated in Table 1 and 2.

CONCLUSION

The proposed algorithm could be used for fast encoding and decoding while reconstruction of a quality image. Further the above proposed FIC can be speed up by any algorithm. It is also can be implemented, using code composer studio with 6711 processor for hardware related work.

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REFERENCES

- Abdullah, H.N., 2008. SAR image denoising based on dual-tree complex wavelet transform. *J. Eng. Applied Sciences*, 3: 587-590.
- Barnsley, M., 1993. *Fractals Everywhere*. 2nd Edn., Academic Press, San Diego, CA., USA., ISBN-10: 0120790610, pp: 550.
- Brahimi, T., A. Melit and F. Khelifi, 2009. An improved SPIHT algorithm for lossless image coding. *Digital Signal Process.*, 19: 220-228.
- Chang, C.L. and B. Girod, 2007. Direction-adaptive discrete wavelet transform for image compression. *IEEE. Trans. Image Proc.*, 16: 1289-1302.
- Ding, J.R. and J.F. Yang, 2008. A simplified SPIHT algorithm. *J. Chinese Inst. Eng.*, 31: 715-719.
- Fang, L.H., M.G. Feng and X.H. Jie, 2010. Images compression using dual tree complex wavelet transform. *Proceedings of the 2010 International Conference on Information Science and Management Engineering*, August 7-8, 2010, IEEE, Xi'an, China, ISBN:978-1-4244-7670-1, pp: 559-562.
- Fisher, Y., 1994. *Fractals Image Compression-Theory and Application*. Springer, New York, USA.,.
- Gonzalez, R.C., R.E. Woods and S.L. Eddins, 2004. *Digital Image Processing Using MATLAB*. Pearson Prentice Hall, New Jersey, USA., ISBN-13: 978-0130085191, Pages: 609.
- Hufnagel, C. and A. Uhl, 2000. Algorithms for fractal image compression on massively parallel simd arrays. *Real-Time Imag. J.*, 6: 267-281.
- Hurtgen, B. and C. Stiller, 1993. Fast hierarchical codebook search for fractal coding of still images. *Proceedings of the Conference on Video Communications and PACS for Medical Applications Berlin-DL Tentative*, October 29, 1993, International Society for Optics and Photonics, Berlin, Germany, pp: 397-408.
- Jacquin, A.E., 1992. Image coding based on a fractal theory of iterated contractive image transformations. *IEEE Trans. Image Process.*, 1: 18-30.
- Jacquin, A.E., 1993. Fractal image coding: A review. *Proc. IEEE*, 81: 1451-1465.
- Said, A. and W.A. Pearlman, 1996. A new, fast and efficient image codec based on set partitioning in hierarchical trees. *IEEE Trans. Circuits Syst. Video Technol.*, 6: 243-250.
- Selesnick, I.W., R.G. Baraniuk and N.C. Kingsbury, 2005. The dual-tree complex wavelet transform. *IEEE Signal Process. Mag.*, 22: 123-151.

- Sendur, L. and I.W. Selesnick, 2002. Subband adaptive image denoising via bivariate shrinkage. Proceedings of the International Conference on Image Processing, September 22-25, 2002, IEEE, New York, USA., ISBN:0-7803-7622-6, pp: 577-580.
- Vidya, D., R. Parthasarathy, T.C. Bina and N.G. Swaroopa, 2000. Architecture for fractal image compression. J. Syst. Archit., 46: 1275-1291.