

Isomorphism and Anti-Isomorphism in Interval Valued of Q-Fuzzy Subhemirings of a Hemiring

¹T. Gandhimathi ²N. Anitha and ³M. Latha

¹Department of Mathematics, P.A. College of Engineering and Technology, 642002 Pollachi, India

²Department of Mathematics, Periyar University PG Extension Centre, 636705 Dharmapuri

³Department of Mathematics, Karpagam University India, 642021 Coimbatore, India

Abstract: In this study, we study some of the properties of interval valued Q-fuzzy subhemiring of a hemiring under isomorphism and anti-isomorphism. Also prove that the homomorphic image and anti-homomorphic image of an interval valued Q-fuzzy normal subhemiring is an interval valued Q-fuzzy subhemiring.

Key words: Interval valued fuzzy subset, interval valued Q-fuzzy subhemiring, interval valued Q-fuzzy normal subhemiring, homomorphic

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring $(R, +, \cdot)$. Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras $(R, +, \cdot)$ share the same properties as a ring except that $(R, +)$ is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra $(R, +, \cdot)$ is said to be a semi ring $(R, +)$ and $(R, \cdot)$ are semi groups satisfying $a(b+c)=a.b+a.c$ and $(b+c).a = b.a+c.a$ for all a, b and c in R . A semi ring R is said to be additively commutative if $a+b = b+a$ for all a, b and c in R . A semi ring R may have an identity 1, defined by $1.a = a = a.1$ and a zero 0, defined by $0+a = a = a+0$ and $a.0 = 0 = 0.a$ for all a in R . A semi ring R is said to be a hemi ring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh (1975), Guinness (1976), Jahn, in the seventies, in the same year, an Interval Valued Fuzzy set (IVF) is defined by an interval-valued membership function. Jun and Kim (2002). defined an interval valued fuzzy R-subgroups of nearrings. Akram and Dar (2005) have introduced the notions of fuzzy algebras and d-ideals in d-algebras. The concept of anti fuzzy sub groups of groups, lower level subgroups was introduced by Sebastian (1994). Solairaju and Nagarajan (2009a, b). defined the characterization of interval valued Anti fuzzy Left h-ideals over hemirings. The properties of interval valued fuzzy Rw open maps, closed maps and Rw homeomorphism in an interval

valued topological spaces were discussed in Indira *et al.* (2013) and Palaniappan (2007). Rosenfeld (1971) defined fuzzy groups. Kazanci *et al.* (2007) has introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju and Nagarajan, have given a new structure in the construction of Q-fuzzy groups and subgroups (Solairaju and Nagarajan, 2009a, b). In this study, we introduced the concept of interval valued Q-fuzzy subhemiring of a hemiring under isomorphism and antiisomorphism and established some results.

Preliminaries

Definition: Let X be any nonempty set. A mapping is called an interval valued fuzzy subset (briefly, IVFS) of X , where $D[0,1]$ denoted the family of all closed subintervals of $[0,1]$ and $[M](x) = [M^-(x), M^+(x)]$ for all x in X , where M^- and M^+ are fuzzy subsets of X such that, $M^- \leq M^+$ for all x in X . Thus $[M](x)$ is an interval (a closed subset of $[0,1]$ and not number from the interval $[0,1]$ as in the case of fuzzy subset. Note that $[0] = [0,0]$ and $[1] = [1,1]$.

Remark: Let D^X be the set of all interval valued fuzzy subsets of X , where D means $D[0,1]$.

Definition: Let the $[M] = \{[x, [M^-(x), M^+(x)]] / x \in X\}$, $[N] = \{[x, [N^-(x), N^+(x)]] / x \in X\}$ be any two interval valued fuzzy subsets of x . We define the following relations and operations: $[M] \subseteq [N]$ if and only if $M^-(x) \leq N^-(x)$ and $M^+(x) \leq N^+(x)$, for all x in X . $[M] = [N]$ if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$ for all x in X .

$$[M] = \cap [N] = \{ \langle x, [\min \{M^-(x), N^-(x)\}, \min \{M^+(x), N^+(x)\}] \rangle / x \in X \}.$$

$$[M] \cup [N] = \{ \langle x, [\max \{M^-(x), N^-(x)\}, \max \{M^+(x), N^+(x)\}] \rangle / x \in X \}.$$

$$[M]^c = [1] - [M] = \{ \langle x, [1 - M^+(x), 1 - M^-(x)] \rangle / x \in X \}$$

Definition: Let X be a non-empty set and Q be a non-empty set. A Q -fuzzy subset A of X is function

Definition: Let $(R, +, \cdot)$ be a hemiring. A interval valued Q -fuzzy subset $[M]$ of R is said to be an interval valued Q -fuzzy subhemiring (IVFSHR) of R if the following conditions are satisfied:

- $[M](x+y, q) \geq \min \{ [M](x, q), [M](y, q) \}$
- $[M](xy, q) \geq \min \{ [M](x, q), [M](y, q) \}$, for all x and y in R and q in Q

Definition: Let $(R, +, \cdot)$ be a hemiring. A interval valued Q -fuzzy subhemiring $[A]$ of R is said to be an interval valued Q -fuzzy normal subhemiring (IVFNSHR) of R if $[A](xy, q) = [A](yx, q)$ for all x and y in R and q in Q

Definition: Let X and X' be any two sets. Let $f: X \rightarrow X'$ be any function and $[A]$ be an interval valued Q -fuzzy subset in X , $[V]$ be an interval valued Q -fuzzy subset in, defined by for all x in X and y in X' and q in Q . Then $[A]$ is called a pre-image of $[V]$ under and is denoted by $f^{-1}([V])$

Definition: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Then the function $f: R \rightarrow R'$ is called a hemiring homomorphism if it satisfies the following axioms:

- $f(x+y) = f(x)+f(y)$
- $f(xy) = f(x)f(y)$ for all x and y in R

Definition: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Then the function f is called a hemiring anti-homomorphism if it satisfies the following axioms:

- $f(x+y) = f(y)+f(x)$
- $f(xy) = f(x)f(y)$ for all x and y in R

Definition: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Then the function $f: R \rightarrow R'$ be a hemiring homomorphism. If f is one-to-one and onto, then f is called a hemiring isomorphism.

Definition: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Then the function $f: R \rightarrow R'$ be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a hemiring anti-isomorphism.

PROPERTIES OF INTERVAL VALUED Q-FUZZY SUBHEMIRINGS

Theorem: Let $[A]$ be an interval valued Q -fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q -fuzzy subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q -fuzzy subhemiring of a hemiring H . Then we have, $([A] \circ f)(x+y, q) = [A](f(x+y), q) = [A](f(x), q) + (f(y), q) \geq \min \{ ([A](f(x), q), [A](f(y), q)) \} = \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$. which implies that $([A] \circ f)(x+y, q) \geq \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$ and $([A] \circ f)(xy, q) = [A](f(xy), q) = [A](f(x), q)(f(y), q) \geq \min \{ ([A](f(x), q), [A](f(y), q)) \} = \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$. which implies that $([A] \circ f)(xy, q) \geq \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$. Therefore $([A] \circ f)$ is an interval valued Q -fuzzy subhemiring of R .

Theorem: Let $[A]$ be an interval valued Q -fuzzy subhemiring of hemiring H and f is an anti-isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q -fuzzy subhemiring of R .

Proof: Let x and y in R . Then we have, $([A] \circ f)(x+y, q) = [A](f(x+y), q) = [A](f(y), q) + (f(x), q) \geq \min \{ ([A](f(x), q), [A](f(y), q)) \} = \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$. which implies that $([A] \circ f)(x+y, q) \geq \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$. And, $([A] \circ f)(xy, q) = [A](f(yx), q) = [A](f(y), q)f(x, q) \geq \min \{ ([A](f(x), q), [A](f(y), q)) \} = \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$. which implies that $([A] \circ f)(xy, q) \geq \min \{ ([A] \circ f)(x, q), ([A] \circ f)(y, q) \}$. Therefore $([A] \circ f)$ is an interval valued Q -fuzzy subhemiring of the hemiring R .

Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic image of an interval valued Q -fuzzy normal subhemiring of R is an interval valued Q -fuzzy subhemiring of R'

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Let $[A]$ is an interval valued Q -fuzzy normal subhemiring of R . We have to prove that $[V]$ is an interval valued Q -fuzzy normal subhemiring of $(R) = R'$

Now, for $f(x), f(y)$ in R and q in Q . Clearly, $[V]$ is an interval valued Q fuzzy subhemiring of $f(R) = R'$. Since $[A]$ is an interval valued Q -fuzzy subhemiring of R . Now, $[V](f(x), q)(f(y), q) = [V](f(xy), q) \geq [A](xy, q) = [A](yx, q) \leq [V](f(yx), q) = [V](f(y), q)(f(x), q)$. Hence $[V]$ is an interval valued Q -fuzzy normal subhemiring of the hemiring.

Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The homomorphic preimage of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy normal subhemiring of R .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a homomorphism. Let $[V]$ is an interval valued Q-fuzzy normal subhemiring of $f(R)$. We have to prove that $[A]$ is an interval valued Q-fuzzy normal subhemiring of R . Let x and y in R and q in Q . Then clearly, $[A]$ is an interval valued Q-fuzzy subhemiring of the hemiring R . Now, $[A](xy, q) = [V](f(xy), q) = [V]\{(f(x), q)(f(y), q)\} = [V](f(y), q)(f(x), q) = [V](f(yx), q) = [A](yx, q)$ which implies that $[A](xy, q) = [A](yx, q)$, for all x and y in R and q in Q . Hence $[A]$ is an interval valued Q-fuzzy normal subhemiring of the hemiring R .

Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The anti-homomorphic image of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy subhemiring of

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be a anti homomorphism. Let $[A]$ is an interval valued Q-fuzzy normal subhemiring of R . We have to prove that $[V]$ is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$. Now, for $f(x), f(y)$ in R' and q in Q . Clearly, $[V]$ is an interval valued Q-fuzzy subhemiring of R' . Since $[A]$ is an interval valued Q-fuzzy subhemiring of R . Now, $[V](f(x), q)(f(y), q) = [V](f(yx), q) \geq [A](yx, q) = [A](xy, q) \leq [V]\{(f(xy), q)\} = [V](f(y), q)(f(x), q)$ which implies that $[V]((f(x), q)(f(y), q)) = [V]((f(y), q)(f(x), q))$. Hence $[V]$ is an interval valued Q-fuzzy normal subhemiring of the hemiring R' .

Theorem: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. The anti homomorphic preimage of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy normal subhemiring of R' .

Proof: Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two hemirings. Let $f: R \rightarrow R'$ be anti homomorphism. Let $[V]$ is an interval valued Q-fuzzy normal subhemiring of $f(R) = R'$. We have to prove that $[A]$ is an interval valued Q-fuzzy normal subhemiring of R . Let x and y in R and q in Q . Then clearly

$[A]$ is an interval valued Q-fuzzy subhemiring of the hemiring R . since $[V]$ is an interval valued Q-fuzzy normal subhemiring of the hemiring R' . Now, $[A](xy, q) = [V]\{(f(xy), q)(f(x), q)\} = [V]((f(x), q)(f(y), q)) = [V](f(yx), q) = [A](yx, q)$ which implies that $[A](xy, q) = [A](yx, q)$, for all x and y in R and q in Q . Hence $[A]$ is an interval valued Q-fuzzy normal subhemiring of the hemiring R .

Theorem: Let $[A]$ be an interval valued Q-fuzzy normal subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q-fuzzy normal subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q-fuzzy normal subhemiring of a hemiring H . Then clearly, $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of the hemiring R . Then we have $([A] \circ f)(xy, q) = [A](f(xy), q) = [A]((f(x), q)(f(y), q)) = [A]((f(y), q)(f(x), q)) = [A](f(yx), q) = ([A] \circ f)(yx, q)$ which implies that $([A] \circ f)(xy, q) = ([A] \circ f)(yx, q)$ therefor $([A] \circ f)(yx, q)$. Therefore $([A] \circ f)$ is an interval valued Q-fuzzy normal subhemiring of R .

Theorem: Let $[A]$ be an interval valued Q-fuzzy normal subhemiring of hemiring H and f is an anti-isomorphism from a hemiring R onto H . Then $[A] \circ f$ is an interval valued Q-fuzzy normal subhemiring of R .

Proof: Let x and y in R and $[A]$ be an interval valued Q-fuzzy normal subhemiring of a hemiring H . Then clearly $([A] \circ f)$ is an interval valued Q-fuzzy subhemiring of the hemiring R . $([A] \circ f)((f(x), q)(f(y), q)) = [A](f(xy), q) = [A](f(y), q)(f(x), q) = [A]((f(x), q)(f(y), q)) = [A](f(yx), q) = ([A] \circ f)(yx, q)$ which implies that $([A] \circ f)(xy, q) = ([A] \circ f)(yx, q)$, for all x and y in R and q in Q . Hence $([A] \circ f)$ is an interval valued Q-fuzzy normal subhemiring of hemiring R .

CONCLUSION

In this study, some properties of interval valued Q-fuzzy subhemiring of a hemiring under isomorphism and anti-isomorphism were discussed. Some results of Interval Valued Q-Fuzzy Subhemirings of a hemirings were proved.

REFERENCES

- Akram, M. and K.H. Dar, 2005. On fuzzy d-algebras. Punjab Univ. J. Math., 37: 61-76.
- Guinness, I.G., 1976. Fuzzy membership mapped onto intervals and many-valued quantities. Math. Logic Q., 22: 149-160.
- Indira, R., K. Arjunan and N. Palaniappan, 2013. Notes on IV-fuzzy rw-Closed IV-fuzzy rw-Open sets in IV-fuzzy topological space. Int. J. Fuzzy Math. Syst., 3: 23-38.
- Jun, Y.B. and K.H. Kim, 2002. Interval-valued fuzzy R-subgroups of near-rings. Indian J. Pure Appl. Math., 33: 71-80.
- Kazanci, O., S. Yamak and S. Yılmaz, 2007. On Intuitionistic Q-fuzzy R-subgroups of near-Rings. Int. Math. Forum, 2: 2899-2910.

- Palaniappan, N.K., 2007. Arjunan operation on fuzzy and anti fuzzy ideals. *Antartical J. Math.*, 4: 59-64.
- Rosenfeld, A., 1971. Fuzzy groups. *J. Math. Anal. Appl.*, 35: 512-517.
- Sebastian, S., 1994. Erratum: Fuzzy subgroups and anti fuzzy subgroups: *Fuzzy Sets and Systems*. *Fuzzy Sets Syst.*, 64: 127-128.
- Solairaju, A. and R. Nagarajan, 2009a. A new structure and construction of Q-fuzzy groups. *Adv. Fuzzy Math.*, 4: 23-29.
- Solairaju, A. and R. Nagarajan, 2009b. Characterization of interval valued anti fuzzy left h-ideals over Hemi Rings. *Adv. Fuzzy Math.*, 4: 129-136.
- Zadeh, L.A., 1975. The concept of a linguistic variable and its application to approximation Reasoning-1. *Inf. Sci.*, 8: 199-249.